

*AAPPS-DPP2020 as e-conference*  
4th Asia Pacific Conference on Plasma Physics  
26-31, October, 2020  
Division of Plasma Physics, Association of Asia-Pacific Physical Societies



# Nonlinear radial envelope evolution equations and energetic particle transport in tokamak plasmas\*

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This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

AAPPS-DPP2020 as e-conference - October 30<sup>th</sup> 2020

# Abstract\*



Work submitted to J. Phys: Conf. Ser. [F. Zonca et al. P-15 Varenna 2020]

The role of energetic particles (EPs) in fusion plasmas is unique as they could act as mediators of cross-scale couplings [1,2]. Energetic particle driven shear Alfvén waves (SAWs), on one hand, could provide a nonlinear feedback onto the macro-scale system via the interplay of plasma equilibrium and fusion reactivity profiles. Meanwhile, EP-driven instabilities could also excite singular radial mode structures at SAW continuum resonances, which, by mode conversion, yield microscopic fluctuations that may propagate and be absorbed elsewhere, inducing nonlocal behaviors that require a global analysis.

Energetic particle transport must be described in phase space because of the underlying kinetic nature of wave-particle interactions and fluctuation excitations. The proper structures to describe such transport processes are phase space zonal structures (PSZS) [3]. Energetic particles, furthermore, may linearly and nonlinearly excite zonal field structures (ZFS), acting, thereby, as generators of nonlinear equilibria, or zonal states (ZS) that generally evolve on the same time scale of the underlying fluctuations. These issues are presented within a general theoretical framework. In particular, we present the nonlinear envelope equations that are needed to solve for the self-consistent evolution of the SAW fluctuation spectrum driven by EPs; and the PSZS transport equations, which determine the renormalized response of EPs including fluctuation induced transport [4].

## References:

- [1] L. Chen and F. Zonca, Rev. Mod. Phys. **88**, 015008 (2016).
- [2] F. Zonca et al., Plasma Phys. Control. Fusion **57**, 014024 (2015)
- [3] F. Zonca et al., New J. Phys. **17**, 013052 (2015)
- [4] M. V. Falessi and F. Zonca, Phys. Plasmas **26**, 022305 (2019).

\*Work supported by the EUROfusion Consortium under grant agreement number 633053 and by US DoE and NSFC grants -- EUROfusion Project ENR-MFE19.ENEA-05 (MET). <https://www.afs.enea.it/zonca/METproject/>  
Principal Investigator: Fulvio Zonca. Available online at <https://www.afs.enea.it/zonca/CNPS/Publications/>

# Summary

- This work provides a general description of the self-consistent energetic particle phase space transport in burning plasmas, based on nonlinear gyrokinetic theory [cf. F. Zonca P-15; M. V. Falessi I-14 Varenna20]
- The self consistency is ensured by
  - the evolution equations of the Alfvénic fluctuations are given by nonlinear radial envelope evolution equations  $\Rightarrow$  NLSE-like!
  - the energetic particle fluxes in the phase space are explicitly constructed from long-lived phase space zonal structures, which are undamped by collisionless processes
- As a result, this work provides a viable route to computing fluctuation induced energetic particle transport on long time scales in realistic tokamak plasmas.

$\Rightarrow$  Derived for the first time!

# Motivation and Background

- High temperature fusion plasmas are weakly collisional:  
this naturally introduces
  - ⇒ importance of phase space structures for transport processes
  - ⇒ deviation of the system from local thermodynamic equilibrium
  - ⊙ these issues are addressed here
  
- This work:
  - ⊙ Develops a first principle based reduced model for fluctuation induced transport in fusion plasmas
    - ⇒ Describes evolution equation for spectral density for low-frequency fluctuations (NL Schrödinger-like equation)
    - ⇒ Closes the system with evolution (transport) equations for phase space zonal structures (renormalization of particle response)
    - ⇒ Reduced Dyson Schrödinger transport Model (DSM)

# Evolution of the fluctuation spectrum

- Perpendicular pressure balance allows to explicitly solve for  $\delta B_{\parallel}$  [C&Z RMP16]  $\Rightarrow$  standard NLGK ordering

$$\nabla_{\perp} (B_0 \delta B_{\parallel} + 4\pi \delta P_{\perp}) \simeq 0 ,$$

- Introducing PSZS,  $\bar{F}_0$  (nonlinear equilibrium) [C&Z RMP16], and nonadiabatic particle response,  $\delta g$ , (to be discussed later), other two closing field equations are:

$\Rightarrow$  quasineutrality condition

$$\sum \left\langle \frac{e^2}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right\rangle_v \delta \phi + \nabla \cdot \sum \left\langle \frac{e^2}{m} \frac{2\mu}{\Omega^2} \frac{\partial \bar{F}_0}{\partial \mu} \left( \frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \nabla_{\perp} \delta \phi + \sum \langle e J_0(\lambda) \delta g \rangle_v = 0 .$$

⇒ gyrokinetic vorticity equation [C&Z RMP16]

$$\begin{aligned}
& B_0 \left( \nabla_{\parallel} + \frac{\delta \mathbf{B}_{\perp} \cdot \nabla}{B_0} \right) \left( \frac{\delta J_{\parallel}}{B_0} \right) - \nabla \cdot \sum \left\langle \frac{e^2 2\mu}{m \Omega^2} \left( B_0 \frac{\partial \bar{F}_0}{\partial \mathcal{E}} + \frac{\partial \bar{F}_0}{\partial \mu} \right) \left( \frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \nabla_{\perp} \frac{\partial}{\partial t} \delta \phi \\
& + \sum e c \mathbf{b}_0 \times \nabla \left\langle \frac{2\mu}{\Omega^2} \bar{F}_0 \left( \frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \cdot \nabla \nabla_{\perp}^2 \delta \phi + \frac{c}{B_0} \mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \sum \langle m (\mu B_0 + v_{\parallel}^2) J_0 \delta g \rangle_v \\
& + \delta \mathbf{B}_{\perp} \cdot \nabla \left( \frac{J_{\parallel 0}}{B_0} \right) + \sum e \left\langle J_0 \left[ \frac{c}{B_0} \mathbf{b}_0 \times \nabla (J_0 \delta \phi) \cdot \nabla \delta g \right] - \frac{c}{B_0} \mathbf{b}_0 \times \nabla \delta \phi \cdot \nabla (J_0 \delta g) \right\rangle_v \\
& + \frac{c}{B_0} \mathbf{b}_0 \times \nabla \delta \phi \cdot \nabla \left[ \nabla \cdot \sum \left\langle \frac{e^2 2\mu}{m \Omega^2} \frac{\partial \bar{F}_0}{\partial \mu} \left( \frac{1 - J_0^2}{\lambda^2} \right) \right\rangle_v \nabla_{\perp} \delta \phi \right] = 0 .
\end{aligned}$$



- Consistent with the ordering  $|\gamma_{Ln}| \sim \tau_{NLn}^{-1} \ll |\omega_n|$  [C&Z RMP16], we can average quasineutrality and nonlinear vorticity equations over linear parallel mode structures, yielding for  $\mathbf{A}_n(r, t) \equiv \hat{\mathbf{e}}_n A_n(r, t)$

$$\hat{\mathbf{e}}_n^+ \cdot \mathbf{D}(r, t, k_{nr}, \omega_n) \cdot \mathbf{A}_n(r, t) e^{iS_n(r, t)} = \hat{\mathbf{e}}_n^+ \cdot \mathbf{F}(r, t),$$

linear response nonlinear response

- Introduce  $\delta\hat{\phi}_{\parallel n} \equiv \delta\hat{\phi}_n - \delta\hat{\psi}_n$ , radial envelope,  $A_n$  and polarization vector  $\hat{\mathbf{e}}_n$

$$\begin{pmatrix} e\delta\hat{\psi}_n(r, \vartheta; t)/T_{0i} \\ e\delta\hat{\phi}_{\parallel n}(r, \vartheta; t)/T_{0i} \end{pmatrix} \equiv A_n(r, t) e^{iS_n(r, t)} \begin{pmatrix} e_1(r, t) y_1(r, \vartheta) \\ e_2(r, t) y_2(r, \vartheta) \end{pmatrix}.$$

with eikonal representation and normalizations

$$\int_{-\infty}^{\infty} |y_{1,2}(r, \vartheta)|^2 d\vartheta = 1. \quad \hat{\mathbf{e}}_n^+ \cdot \hat{\mathbf{e}}_n = 1.$$

- The **nonlinear term** (including ext. forcing) can be **formally written** as

$$e^{-iS_n} \hat{\mathbf{e}}_n^+ \cdot (\mathbf{F} - \mathbf{F}_{\text{ext}}) = (C_{n,0} + C_{0,n}) \circ A_n(r, t) A_z(r, t) + \sum_{\substack{n', n'' \neq n \\ n' + n'' = n}} C_{n', n''} \circ A_{n'}(r, t) A_{n''}(r, t)$$

where  $C_{n', n''}$  imply nonlocal interactions in the  $n$  toroidal mode number space and whose composition with  $A_n(r, t)$  and/or  $A_z(r, t)$  is denoted by  $\circ$ .

- **Consistent with the GFLDR theory** [C&Z RMP16],

$$e^{-iS_n} \hat{\mathbf{e}}_n^+ \cdot \mathbf{F} / A_n = i\Lambda^{NL} - \delta\bar{W}_f^{NL} - \delta\bar{W}_k^{NL} + e^{-iS_n} \hat{\mathbf{e}}_n^+ \cdot \mathbf{F}_{\text{ext}} / A_n .$$

$\Lambda$  describes the generalized inertia due to the e.m. field behaviors on short length scales, while  $\delta\bar{W}_f$  and  $\delta\bar{W}_k$  account for “fluid” and “kinetic” potential energy fluctuations due to meso- and macro-scale responses.

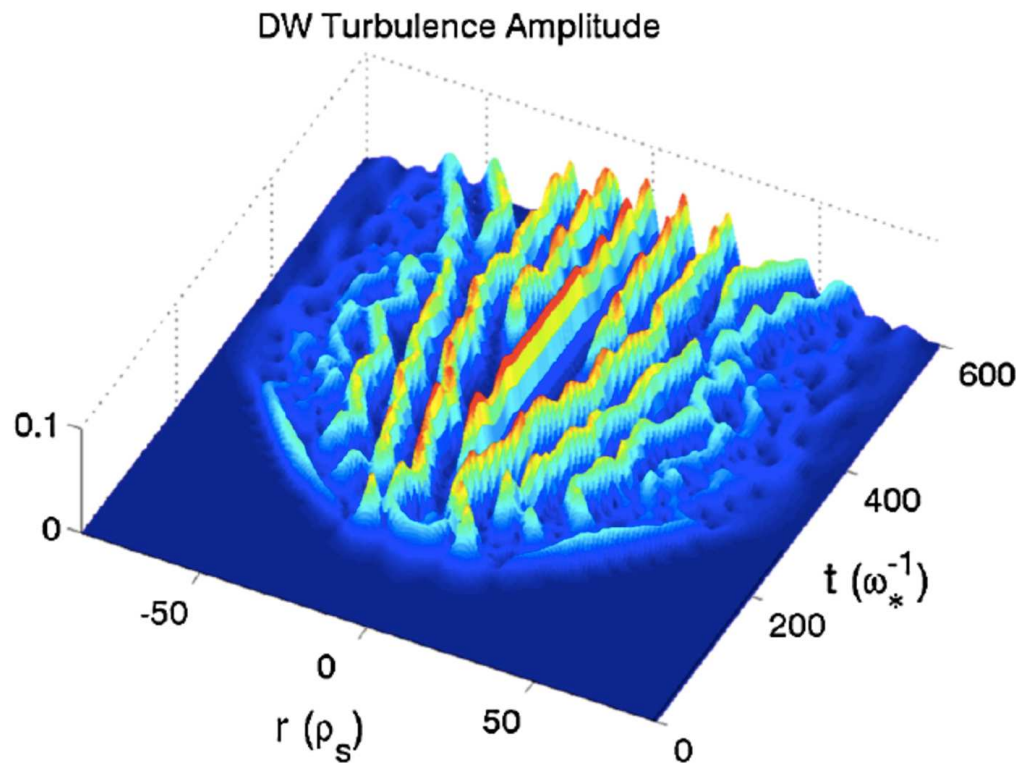


- Nonlinear envelope equation can be written in NLSE-like form

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left( \frac{\partial D_{Rn}^0}{\partial \omega_n} A_n^2 \right) - \frac{\partial}{\partial r} \left( \frac{\partial D_{Rn}^0}{\partial k_{nr}} A_n^2 \right) + 2D_{An}^1 A_n^2 - 2iD_{Rn}^1 A_n^2 \\
 & + iA_n \left( \frac{\partial^2 D_{Rn}^0}{\partial k_{nr}^2} + 2 \frac{\partial \hat{e}_n^+}{\partial k_{nr}} \cdot \mathbf{D}_{Rn}^0 \cdot \frac{\partial \hat{e}_n}{\partial k_{nr}} \right) \frac{\partial^2 A_n}{\partial r^2} = -2ie^{-iS_n} A_n \hat{e}_n^+ \cdot \mathbf{F} \\
 & - \left( \hat{e}_n^+ \cdot \frac{d}{dt} \hat{e}_n - \frac{d}{dt} \hat{e}_n^+ \cdot \hat{e}_n \right) \frac{\partial D_{Rn}^0}{\partial \omega_n} A_n^2 + \left( \frac{\partial \hat{e}_n^+}{\partial \omega_n} \cdot \mathbf{D}_{Rn}^0 \cdot \frac{\partial \hat{e}_n}{\partial t} - \frac{\partial \hat{e}_n^+}{\partial t} \cdot \mathbf{D}_{Rn}^0 \cdot \frac{\partial \hat{e}_n}{\partial \omega_n} \right) A_n^2 \\
 & - \left( \frac{\partial \hat{e}_n^+}{\partial k_{nr}} \cdot \mathbf{D}_{Rn}^0 \cdot \frac{\partial \hat{e}_n}{\partial r} - \frac{\partial \hat{e}_n^+}{\partial r} \cdot \mathbf{D}_{Rn}^0 \cdot \frac{\partial \hat{e}_n}{\partial k_{nr}} \right) A_n^2 .
 \end{aligned}$$

- The NLSE-like structure is of crucial importance for proper analysis of structure formation in strongly magnetized toroidal plasmas, where wave packets can be focused/defocused and back scattered by both nonlinearities as well as by radial nonuniformities [C&Z RMP16].

- This goes beyond the standard wave kinetic equation that is typically adopted in literature, is of fundamental importance not only in the description of EP induced avalanches, such as in the case of energetic particle modes (EPM) [Zonca et al 05] but also for the interaction of zonal fields and drift wave turbulence [Guo et al 09].

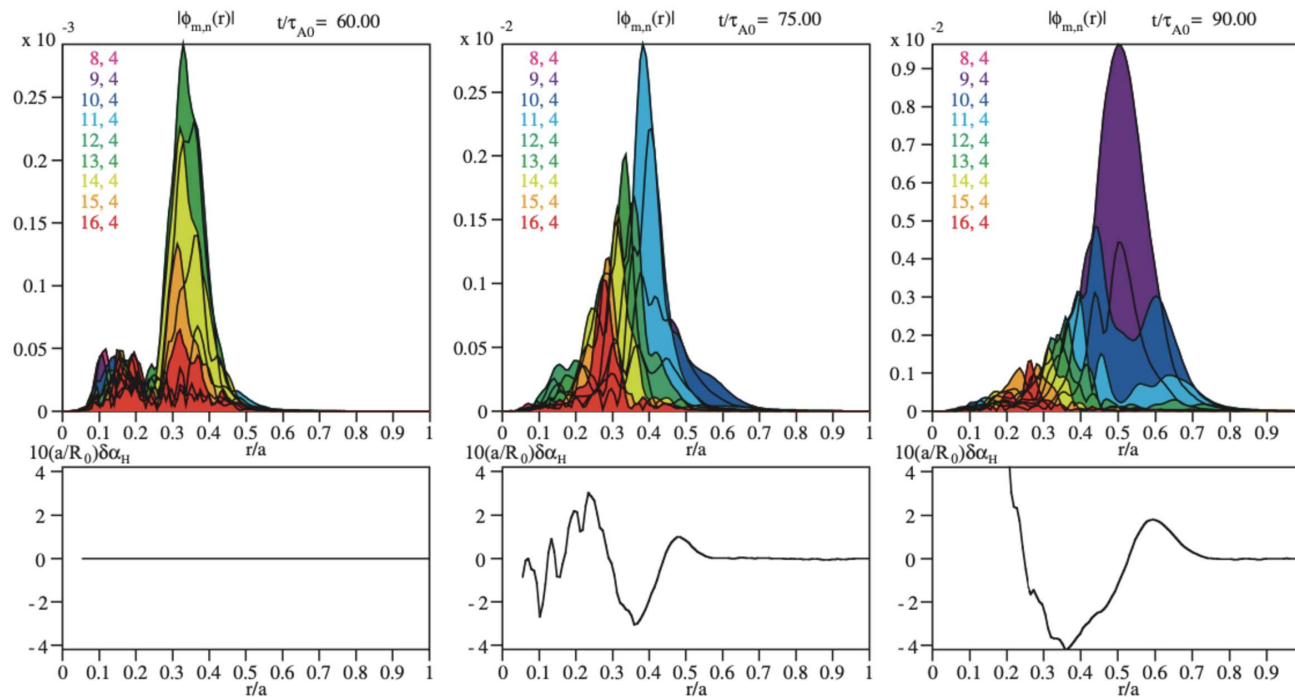


**DW turbulence spreading** in the presence of growth or damping, dissipation and finite system size effects [courtesy of Z. Guo PRL 103, 055002 (2009)]

- The NLSE-like equation for symmetry breaking fluctuations is closed by the nonlinear evolution equation for  $\delta\phi_z$  (quasineutrality; cf. above) and  $\delta A_{\parallel z}$  [C&Z RMP16] [M.V. Falessi et al. I-14 Varenna 2020]

$$\frac{\partial}{\partial t} \delta A_{\parallel z} = \left( \frac{c}{B_0} \mathbf{b}_0 \times \nabla \delta A_{\parallel} \cdot \nabla \delta \psi \right)_z .$$

Evidence of EP avalanche [Z. et al Nucl. Fusion 45 (2005) 477 – 484]



# Phase space zonal structures and transport

- ⊙ There is **more than radial corrugations** of equilibrium profiles
  - ⇒ Fluctuations force the system away from **reference state**
  - ⇒ Collisions tend to restore **local thermodynamic equilibrium**
  - ⇒ Need to **consider these processes in phase space** on the same footing
- ⊙ We can describe **reference state evolution** by means of **phase space zonal structures (PSZS)**, defined as those unaffected by fast collisionless damping
  - ⇒ **important on transport time scale** [M.V. Falessi I-14 Varenna 2020].
- ⊙ Since **PSZS are undamped** by (fast) collisionless dissipation mechanisms, they are naturally expressed as **functions of invariants of motion** (nearly integrable Hamiltonian system). Separating fast ( $[\dots]_F$ ) from slow variations,

$$F_z \equiv \bar{F}_0 + e^{-iQ_z} \left( \overline{e^{iQ_z} \delta F_z} \Big|_F + \delta \tilde{F}_{Bz} \right),$$

macro- ⊕ meso-scale (CGL)      micro-scale      collisionless damped

where  $\overline{[\dots]} = \oint dl/v_{\parallel} [\dots] / \oint dl/v_{\parallel}$ ,  $\overline{[\dots]} = 0$ ,  $F_z$  is the  $n = 0$  gyrocenter particle distribution function.  $e^{-iQ_z} \Rightarrow$  **nonlocal** (integral) **particle response**

- Evolution equation for the PSZS (CGL nonlinear equilibrium) is given by

$$\begin{aligned}
 \overline{\partial_t e^{iQ_z} \bar{F}_0} &= - \overline{e^{iQ_z} \frac{F(\psi)}{B_0} \partial_t \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} \bar{F}_0} \Big|_S - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\psi}_z \delta F_z} \right]_S \\
 &\quad - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\mathcal{E}}_z \delta F_z} \right]_S - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\psi} \delta F} \right]_{zS} - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\mathcal{E}} \delta F} \right]_{zS} \\
 &\quad + \overline{e^{iQ_z} [C_g + \mathcal{S}]} \Big|_{zS} .
 \end{aligned}$$

[M.V. Falessi I-14 Varenna 2020]

- Introducing

$$\begin{aligned}
 &\overline{\delta g_{Bz}} \Big|_S + \overline{\delta g_{Bz}} \Big|_F \\
 &\equiv \overline{e^{iQ_z} \delta F_z} \Big|_F - \frac{e}{m} \overline{e^{iQ_z} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} \Big|_{\bar{\psi}} \bar{F}_0} + \overline{e^{iQ_z} \frac{F(\psi)}{B_0} \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} \bar{F}_0},
 \end{aligned}$$

one can derive the evolution equation for  $\overline{\delta g_{Bz}} \Big|_F$  as well [JPCS submitted]



- Evolution equation for the **micro spatiotemporal scale equilibrium** variation

$$\begin{aligned}
 \partial_t \overline{\delta g_{Bz}} \Big|_F &= - \overline{e^{iQ_z} \frac{e}{m} \partial_t \left[ \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} \Big|_{\bar{\psi}} \bar{F}_0 \right]} \Big|_F + \overline{e^{iQ_z} \frac{F(\psi)}{B_0} \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} \partial_t \bar{F}_0} \Big|_F \\
 &+ \overline{e^{iQ_z} [C_g + \mathcal{S}]} \Big|_{zF} - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\psi}_z \delta F_z} \right]_F - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\mathcal{E}}_z \delta F_z} \right]_F \\
 &- \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\psi} \delta F} \right]_{zF} - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[ \overline{\tau_b e^{iQ_z} \delta \dot{\mathcal{E}} \delta F} \right]_{zF} .
 \end{aligned}$$

- Analogously, one can derive the evolution equations for  $\delta \tilde{g}_{Bz}$  (and, thus,  $\delta \tilde{F}_{Bz}$ ), omitted here for brevity [M.V. Falessi I-14 Varenna 2020].
- In order to self-consistently close the system of equations with the ZFS equations and the NLSE-like equation for the nonlinear envelope evolution of symmetry breaking fluctuations we need, finally, the **equation for the non-adiabatic particle response,  $\delta g$** .

$$\begin{aligned}
\left(\partial_t + \dot{\mathbf{X}}_0 \cdot \nabla\right) \delta g &= -\frac{e}{m} \partial_t \left[ \langle \delta L_g \rangle \frac{\partial}{\partial \mathcal{E}} \Big|_{\bar{\psi}} \bar{F}_0 \right] + \frac{F(\psi)}{B_0} \langle \delta A_{\parallel g} \rangle \frac{\partial}{\partial \bar{\psi}} \partial_t \bar{F}_0 \\
&\quad - c \partial_\zeta \langle \delta L_g \rangle \frac{\partial}{\partial \bar{\psi}} \bar{F}_0 + [C_g + \mathcal{S}] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \theta} \left[ \mathcal{J}D \delta \dot{\theta} \delta F_z \right] \\
&\quad - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \psi} \left[ \mathcal{J}D \delta \dot{\psi} \delta F_z \right] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \mathcal{E}} \left[ \mathcal{J}D \delta \dot{\mathcal{E}} \delta F_z \right] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \theta} \left[ \mathcal{J}D \delta \dot{\theta}_z \delta F \right] \\
&\quad - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \psi} \left[ \mathcal{J}D \delta \dot{\psi}_z \delta F \right] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \mathcal{E}} \left[ \mathcal{J}D \delta \dot{\mathcal{E}}_z \delta F \right] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \theta} \left[ \mathcal{J}D \delta \dot{\theta} \delta F \right] \\
&\quad - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \psi} \left[ \mathcal{J}D \delta \dot{\psi} \delta F \right] - \frac{1}{\mathcal{J}D} \frac{\partial}{\partial \mathcal{E}} \left[ \mathcal{J}D \delta \dot{\mathcal{E}} \delta F \right],
\end{aligned}$$



# Nonlinear evolution of resonance structures

- Rich non-linear behaviors due to non-perturbative W-P interactions: [C&Z RMP16]
  - ⊙ resonances may evolve nonlinearly (chirping)
  - ⊙ phase-space structures may form on spatiotemporal meso-scales (phase-locking, bunching ...) and yield **secular transport** and/or **avalanches** [Zonca et al NF05]; [C&Z RMP16]
  - ⊙ favorable conditions for this phenomenology
    - continuous spectrum of modes that can be resonantly excited
    - **Non-perturbative W-P interactions** modify lowest order dispersion properties
  
- Within the present theoretical framework, these physics are **accounted for self-consistently**
  - ⇒ renormalized expression of reference state distribution by emission and re-absorption of symmetry breaking perturbations
  - ⇒ **Dyson-like equation for PSZS** [Zonca et al NJP15], [C&Z RMP16]



# Dyson-like equation for PSZS

- Dropping for simplicity radial modulations by ZFS, and writing formal solution for the particle response keeping relevant NL terms and considering precession resonance only

$$\partial_t \delta \bar{G}_z \sim -i \sum_{\mathbf{k}} \frac{nc}{d\psi/dr} \frac{\partial}{\partial r} \left[ \frac{e^{iQ_z} \langle \delta L_g \rangle_{-k} e^{-iQ_k}}{(\bar{\omega}_{dk} - \omega_k - i\partial_t + i\Delta \dots)} \frac{(nc)/(d\psi/dr)}{e^{iQ_k} \langle \delta L_g \rangle_k e^{-iQ_z}} \frac{\partial}{\partial r} \delta \bar{G}_z \right]$$

- Importance of fluctuation spectrum in determining NL PSZS evolution:
  - broad spectrum  $\Rightarrow$  QL diffusion [Al'tshul' & Karpman 65]
  - narrow (quasi-coherent) spectrum: ballistic transport of phase-locked particles and convective amplification of radially propagating wave-packets (NLSE)  $\Rightarrow$  Non-adiabatic chirping [C&Z RMP16]
- Simplest example of the Dyson Schrödinger transport Model (DSM) for EP transport in burning plasma that can be constructed in general from the NLSE-like nonlinear envelope evolution equation and the corresponding evolution equation for PSZS.



# Summary

- This work provides a general description of the self-consistent energetic particle phase space transport in burning plasmas, based on nonlinear gyrokinetic theory. [cf. F. Zonca P-15; M. V. Falessi I-14 Varenna20]
- The self consistency is ensured by
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