Lecture 6

Advanced topics in Quasi-linear theory. Thermal plasma effects and the turbulent trapping model.

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Revisiting quasi-linear theory

- From Lecture 5, the quasi-linear evolution equation for F_0 is

$$\begin{aligned}\frac{\partial}{\partial t}F_0 &= \frac{e^2}{m^2}\sum_{k_0} k_0^2 \frac{\partial}{\partial v}\left[\pi\delta(\omega_{k_0 r} - k_0 v)|\delta\phi_{k_0}(t)|^2\frac{\partial}{\partial v}F_0\right. \\ &\left. -\frac{1}{2}\frac{\partial}{\partial\omega_{k_0 r}}\mathcal{P}\left(\frac{1}{\omega_{k_0 r} - k_0 v}\right)\partial_t|\delta\phi_{k_0}(t)|^2\frac{\partial}{\partial v}F_0\right]\end{aligned}$$

- This is complemented by the evolution equation for the fluctuation intensity

$$\omega_p^{-1}\partial_t|\delta\phi_{k_0}(t)|^2 = \frac{\omega_p^2}{n k_0}\int dv \pi\delta(\omega_{k_0 r} - k_0 v)|\delta\phi_{k_0}(t)|^2\frac{\partial}{\partial v}F_0$$

- These equations satisfy particle, momentum and energy conservation (Lecture 5).

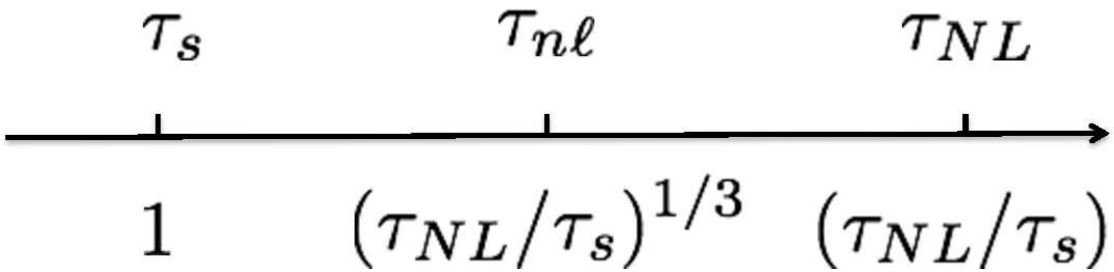
- Validity limits are imposed by the derivation from Dyson equation (Lecture 5), and the assumption of broad spectrum of overlapping modes: $|\gamma_{k_0}| \ll |k_0 \Delta v|$ and

$$\frac{1}{\tau_{NL}} \sim |\omega| \sim \left| \frac{D_v}{(\Delta v)^2} \right| \ll |k_0 \Delta v| \quad ; \quad \Rightarrow \quad \left| \frac{e}{m} \delta\phi_{k_0}(\tau) \right|^{1/2} \ll |\Delta v| \quad .$$

$$\tau_{NL} \gg \omega_B^{-1} \gtrsim |\gamma_{k_0}|^{-1} \gg |k_0 \Delta v|^{-1}$$

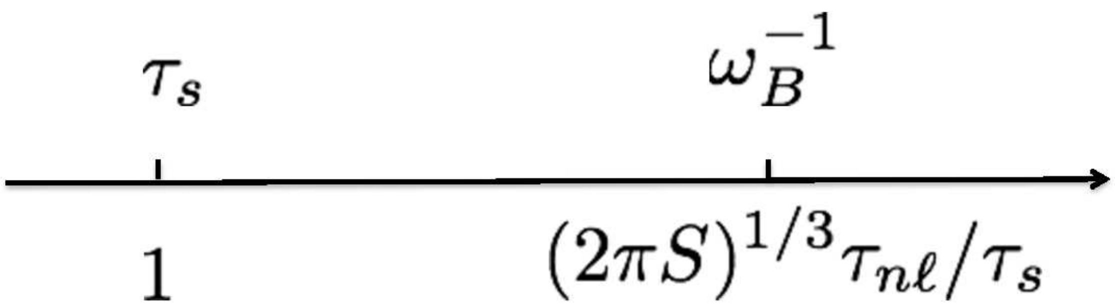
- Recall from Lecture 5: two fundamental time scales enter the problem:
  - time scale for a particle to diffuse over $\sim k_0^{-1}$: $(D_v k_0^2/3)^{-1/3} \equiv \tau_{n\ell}$
  - autocorrelation time of fluctuations: $|k_0 \Delta v|^{-1} \equiv \tau_s$, necessary for relative phase of two waves to randomize due to different phase velocities
  - a third important time scale is connected with $|\gamma_{k_0}|^{-1}$

E: Comment on the importance of fully accounting for $|\gamma_{k_0}|^{-1}$ among the fundamental time scales. What important element may be missing otherwise?



- The Chirikov parameter is introduced to quantify resonance overlap [Chirikov 1959]

$$S = \frac{\Delta\omega_r}{\Delta\omega_{\rm sep}} \simeq \frac{\omega_B}{\Delta\omega_{\rm sep}} > 1$$



- Original formulation of quasi-linear theory assumes $\tau_s \ll \tau_{n\ell}$, $\tau_s \ll |\gamma_{k_0}|^{-1}$ [Vedenov et al. 1961; Drummond and Pines 1962]. $\Rightarrow$ No assumptions on relative ordering of $\tau_{n\ell}$ and $|\gamma_{k_0}|^{-1}$.

- The additional condition for validity of quasi-linear theory, expressed as $|\gamma_{k_0}|\tau_{n\ell} \equiv R^{-1/3} \gg 1$ was introduced by [Laval and Pesme, 1983,1984].

E: Convince yourself that this additional condition coincides with imposing sufficiently weak fluctuation intensity.

- A simple estimate of D_v (p.7 Lecture 5) gives

$$D_v \sim \frac{2\pi}{k^2} S\omega_B^3 \, , \qquad \Rightarrow \qquad \omega_B \sim (2\pi S)^{-1/3} \tau_{n\ell}^{-1}$$

- Noting that $|\gamma_{k_0}| \gtrsim \omega_B$ [O'Neil et al 1971,1972] (Spring 2015 Lecture 3)

$$|\gamma_{k_0}|\tau_{n\ell} \gtrsim (2\pi S)^{-1/3} \, , \qquad R \equiv (|\gamma_{k_0}|\tau_{n\ell})^{-3} \sim 2\pi S(\omega_B^3/|\gamma_{k_0}|^3)$$

The role of mode-mode couplings

- The possible break-down of quasilinear theory is due to the role of mode-mode couplings (see Part II of this Intensive Course), which have been neglected in deriving the equations on p. 2.
- This may be understood from the "weak beam" bump-on-tail instability discussed by [O'Neil et al 1971,1972] (Spring 2015 Lecture 3), which states that thermal plasma nonlinearity enters at an order $(n_B/n)^{1/3}$ higher than EP nonlinearity.
- It was noted by [Laval and Pesme, 1983,1984] that, for $S \gg 1$ one may have $R \gg 1$ and, in that case, mode coupling terms become important. Mode coupling terms break the assumption that electric field fluctuation behaves as a Gaussian process along the free particle orbits when computing $\langle \Delta v^2 \rangle$.

E: Show that, for weak fluctuation level, one can have $S > 1$ (resonance overlap) and $R \ll 1$ (weak nonlinearity) at the same time. In this case quasi-linear theory still applies.

Turbulent trapping model

- A particle of velocity v interacts resonantly with a wave-packet of group velocity v_g when the wave-packet width Δk_{NL} is

$$\Delta k_{NL} = \tau_{n\ell}^{-1}/|v - v_g|$$

- This corresponds to an interaction length and interaction time (not to be confused with the diffusive relaxation nonlinear time introduced above, τ_{NL})

$$L_{NL} = \ell \Delta k_{NL}^{-1} \ , \qquad \tau_I^{NL} = L_{NL}/|v - v_g| \sim \ell \tau_{n\ell}$$

- One can have that the motion of two particles in x_1 and x_2 remain strongly correlated over distances larger that $\lambda \sim k^{-1}$

$$|x_1 - x_2 - n\lambda| \lesssim L_{NL}$$

- That correlation length can be longer than λ is sign of existence of mode-mode coupling terms: resonant $2n$ wave coupling mediated by the resonant particles.

- When this happens, [Laval and Pesme, 1983,1984] have shown that

$$\gamma_k/\gamma_k^{ql} = D_v/D_v^{ql} = 1 + 2(A_{2p} - 1) \simeq 2.2$$

- This effect is connected with the modification of the Fokker-Planck equation to account self-consistently of the polarization field due to the particle motion, inducing an acceleration (cf. Lecture 5) $\langle \Delta v/\Delta t \rangle_{pol}$

$$\partial_t F = \partial_v \left(\langle \Delta v \times \Delta v/\Delta t \rangle \, \partial_v F \right) - \partial_v \left(\langle \Delta v/\Delta t \rangle_{pol} \, F \right)$$

- S. I. Tsunoda, F. Doveil and J. Malmberg [1987,1991], performed experiments with an electron beam in a traveling wave tube to check these predictions: "quasilinear predictions look right, while quasilinear assumptions are proved to be completely wrong. Indeed no renormalization is measured, but mode-mode coupling is not negligible at all" [Escande 2016].

E: Convince yourself that the polarization term is a friction.

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