Lecture 1

General introduction and background: the concept of phase space zonal structures

Fulvio Zonca

http://www.afs.enea.it/zonca

ENEA C.R. Frascati, C.P. 65 - 00044 - Frascati, Italy.

Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou 310027, P.R.C.

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Nonlinear dynamics of phase-space zonal structures and energetic particle physics in fusion plasmas
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Lecture Plan and Teaching Material

Abstract: This Lecture series addresses the importance of phase-space zonal structures for the investigation of nonlinear Alfvénic fluctuation behaviors in fusion plasmas and related transport phenomena. A general introduction is devoted to explaining the concept of phase space zonal structures and fluctuation induced transport processes, which generally occur on the same spatial and temporal scales. The role of wave-particle resonances is analyzed, with emphasis on the importance of equilibrium magnetic field geometry and nonuniformity in the onset of nonlocal behaviors. The concepts of resonance detuning, radial decoupling and phase-locking are introduced and explained within this framework. In the uniform plasma limit, the nonlinear dynamics of beam-plasma interaction is adopted as paradigm for understanding saturation by wave-particle trapping and to illustrate the calculation of renormalized particle distribution function as solution of the corresponding Dyson equation. The standard quasi-linear plasma description is also derived from these results. For nonuniform toroidal plasmas of fusion interest, the general governing equations for the nonlinear dynamics of phase-space zonal structures are derived and discussed, including renormalized particle response and Dyson equation. Actual solutions for phase-space zonal structures

in cases of practical interest are discussed in detail for the cases of fishbone fluctuations and Energetic Particle Modes, for which the self-consistent interplay between mode structure nonlinear dynamics and energetic particle transport is of crucial importance. Prerequisite of the course is a preliminary knowledge of general plasma physics, fluid description of magnetized plasmas and kinetic theory, including linear gyrokinetic equations, which are briefly reviewed in this Lecture series.

Main areas that will be explored are:

i) General introduction and background: the concept of phase space zonal structures. (Lecture 1)

ii) The role of wave-particle resonances: onset of nonlocal behaviors. (Lecture 2)

iii) The uniform plasma limit: the nonlinear beam-plasma system. (Lecture 3)

iv) Nonlinear dynamics of phase-space zonal structures: governing equations, renormalized particle response and Dyson equation. (Lecture 4)

v) Nonlinear description of fishbones: nonlinear evolution (mode-particle pumping) and reduced (predator-prey) equations. (Lecture 5)

vi) Energetic Particle Modes: solitons in active media and modulation of the radial envelope. (Lecture 6)

Sources: Portions of these lecture notes are based on Lectures I gave for the *IFTS Intensive Courses on Advanced Plasma Physics-Spring* 2011 and 2012 at the Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou, China. These Lecture Notes are all available online.

Lecture Notes: Available in electronic form following the hyperlinks above. At the end of each lecture, a list of specific reading material is given explicitly. Should you have difficulty in finding literatures and papers, please contact me at fulvio.zonca@enea.it.

Exercises: In the Lectures, both Exercises (E) and Research Projects (RP) of various difficulty levels are suggested.

Alfvén waves and energetic particles

- The important role of shear Alfvén waves (SAW) and their interaction with energetic particles/charged fusion products (EP) in burning fusion plasmas is widely acknowledged [Chen NF07, Fasoli NF07, Heidbrink POP08].

- Historically, this important role was realized first in the pioneering works by Rosenbluth and Rutherford [PRL75] and Mikhailovskii [Sov.Phys.JETP75].

- Alfvénic oscillations can be excited by energetic particles as well as thermal plasma components [Zonca POP99, Nazikian PRL06] $\rightarrow$ broad spectrum of wavelengths, frequencies and growth rates.

- Drift Alfvén waves (DAW) may have the features of broad band turbulence with $\gamma/\omega \sim 1$ or nearly periodic (quasi-coherent) nonlinear fluctuations with $\gamma/\omega \ll 1$ [Chen NF07].

- These two components of the SAW/DAW spectrum have different effects on energetic particle transports: in this *Lecture Series*, the focus is on the nearly periodic (quasi-coherent) nonlinear fluctuations with $\gamma/\omega \ll 1$.

Phase-space holes and clumps

- When considering transport processes due to nearly periodic fluctuations with $\gamma/\omega \ll 1$, wave-particle resonances play a crucial role both in wave excitations as well as transport processes [Chen JGR99].

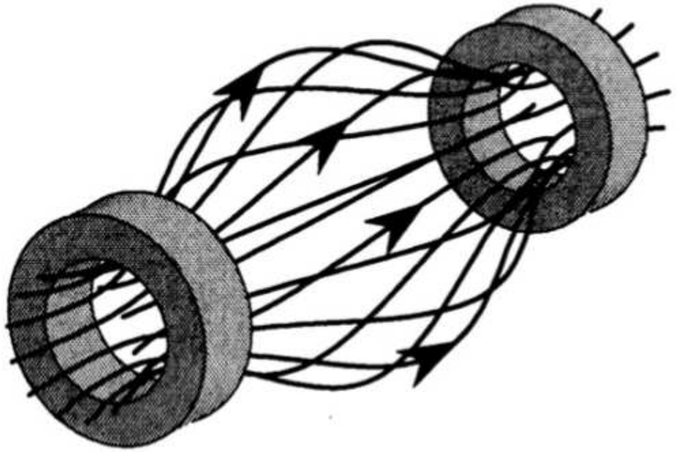
- Phase-space holes (lack of density w.r.t. surrounding phase-space) and clumps (excess of density w.r.t. surrounding phase-space): extensively investigated after pioneering work by Bernstein, Greene and Kruskal (BGK) [Phys.Rev57] $\Rightarrow$ nonlinear dynamics of 1D uniform Vlasov plasmas.

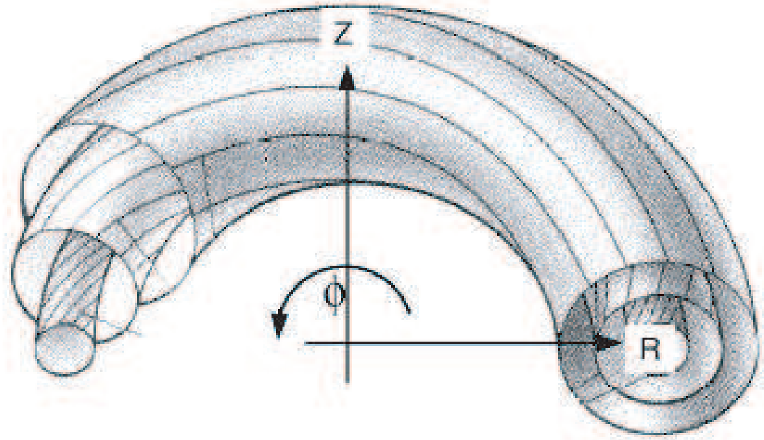
- Nonlinear dynamics of phase-space holes and clumps in the presence of sources and collisions: widely adopted by Berk, Breizman and coworkers (review by [Breizman PPCF11]) $\Rightarrow$ 1D uniform beam-plasma system as paradigm for nonlinear behaviors of Alfvén Eigenmodes near marginal stability [Berk PFB90] (refer to Spring 2010 Lecture Notes).

The concept of phase space zonal structures

- Phase-space holes and clumps are particular cases of phase-space zonal structures, where time scale separation applies between their long characteristic dynamic evolution and the much shorter wave-particle trapping time.
- More generally, phase space zonal structures ($n = m = 0 \Rightarrow \boldsymbol{k} \cdot \boldsymbol{b} = 0$ everywhere) $\Rightarrow$ generation of nonlinear equilibria, which generally evolve on the same time scale of the nonlinear fluctuations [Chen NF07b].
- The concept of phase-space zonal structures is very general and encompasses the modification of particle distribution functions as consequence of fluctuation induced transport processes $\Rightarrow$ corrugations of equilibrium radial profiles [Zonca NF05, PPCF06; Chen NF07, NF07b].
- Phase-space zonal structures also encompass zonal flows and fields/currents [Diamond PPCF05].

The beam-plasma system vs. EP-SAW interactions in tokamaks

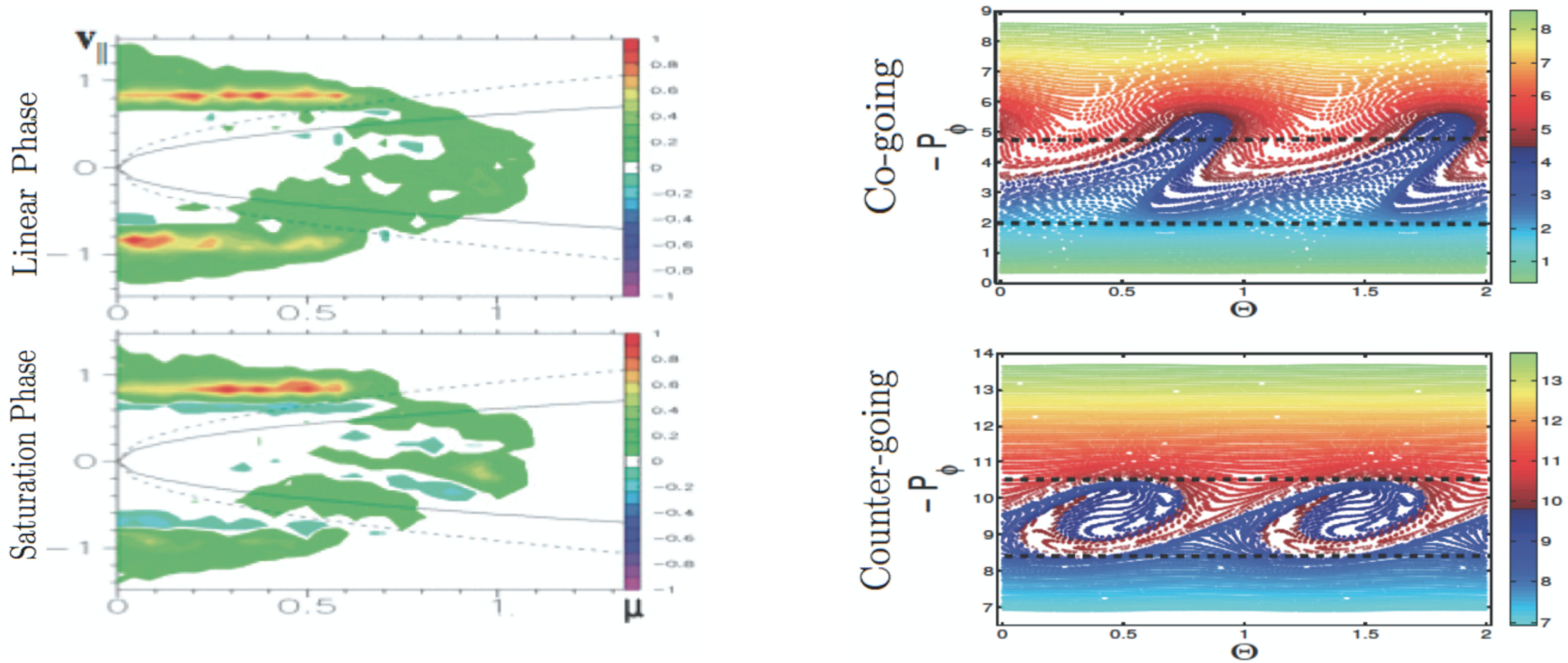




- Similarities can be drawn but strong differences and peculiarities emerge depending on the strength of the drive [Zonca NF05]:
  - Advantages of using a simple 1-D system for complex dynamics studies [Berk PFB90]; [Review by Breizman PPCF11] (Lecture 3)
  - Roles of mode structures, non-uniformity and geometry in determining nonlinear behaviors [Zonca NF05, PPCF06]; [Chen NF07, RMP14] for $1 \gg \gamma/|\omega| \gtrsim 10^{-3} \div 10^{-2}$ (depending on resonances) (Lecture 2)

Nonlinear dynamics of BAEs

- Hybrid MHD-Gyrokinetic simulations of Beta induced Alfvén Eigenmodes (BAE) [Wang PRE12] with XHMGC [$\dot{\omega} > 0$, $\dot{\omega}_{\rm res} \propto {\rm sgn} v_{\parallel}$]



- Evidence of phase locking and radial decoupling being more important than resonance detuning in nonlinear mode dynamics (cf. also later) ⇒ confirmed by nonlinear GKE simulations with GTC [Zhang PRL12] (Lecture 2).

Non-adiabatic dynamics and phase-locking

- Consider conditions of practical interest in which significant resonant energetic particle redistributions take place over the radial region where fluctuations are initially localized.

- These conditions are in one-on-one correspondence with fast frequency chirping events (not related with thermal plasma equilibrium changes) where [Chen RMP14]
  - the frequency variation is of the order of the frequency shift from the closest SAW resonance $\Rightarrow$ $\mathcal{O}(1)$ change of mode structure
  - the EP drive variation is $\mathcal{O}(1)$ and EP nonlinear excursion $\sim \lambda_\perp$

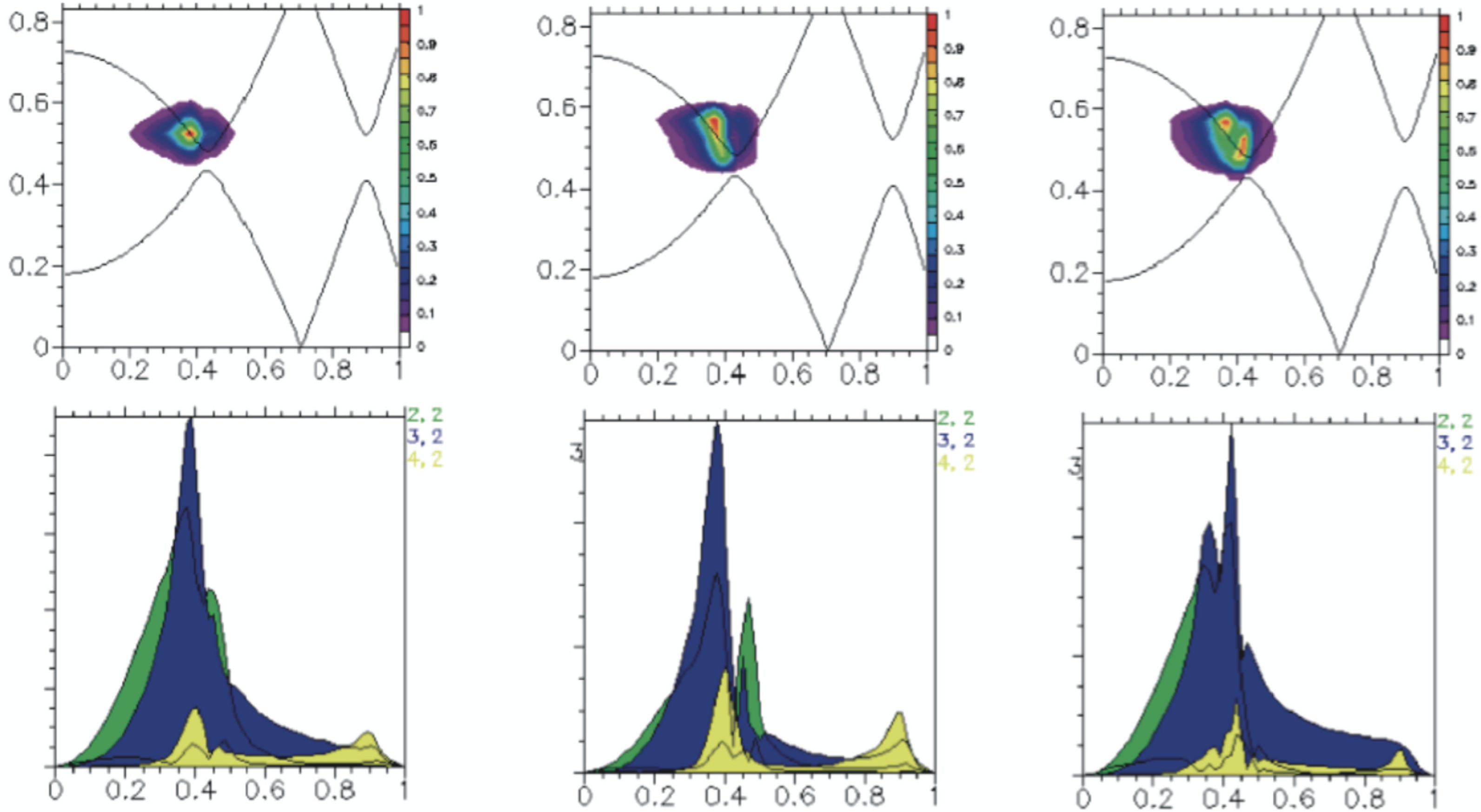
- Energetic particle dynamics is non-perturbative and mode structures, via nonlinearly modified dispersiveness, dynamically respond on the same time scale of the evolving phase space zonal structures.
  - nonlinear dispersiveness $\Rightarrow$ frequency sweeping (non-adiabatic)
  - phase-locked particles maximize wave-particle interactions/transport

Adiabatic vs. non-adiabatic nonlinear dynamics

- Frequency sweeping of Alfvénic modes is a very important phenomenon, as recognized since the early experimental observations of these phenomena [Heidbrink PPCF95].
- Wave-particle energy exchange can be enhanced by resonance sweeping, if BGK mode sweeping rate can be externally controlled.
- Berk and Breizman [IFS.Rep95] showed that this enhancement is higher for adiabatic ($\dot{\omega} \ll \omega_B^2$) than for non-adiabatic ($\dot{\omega} \lesssim \omega_B^2$) frequency chirping.
- In the presence of sufficiently strong instability, frequency chirping is determined by maximization of wave-particle power transfer [Zonca NF05] and related transports [Chen JGR99] not of wave-particle energy exchange.
- Wave-particle power transfer is maximized for phase-locked particles [Wang PRE12], which are convected outward $\Rightarrow$ mode particle pumping, introduced for explaining energetic particle transport due to fishbones [White PF83] $\Rightarrow$ non-adiabatic dynamics ($\dot{\omega} \sim \omega_B^2$).

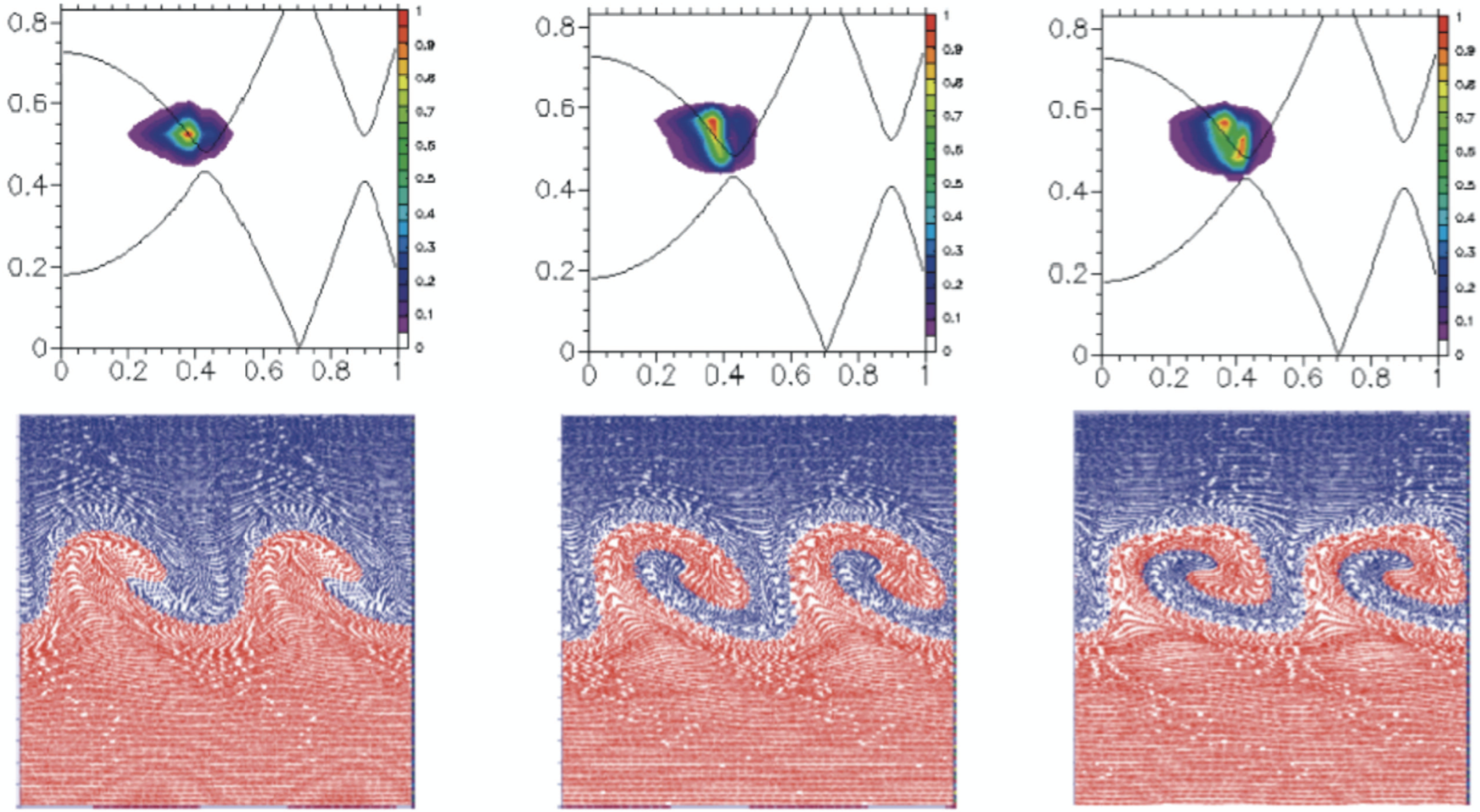
Nonlinear dynamics of (localized) EPMs

- Numerical Experiment: Hybrid MHD-Gyrokinetic simulations of Energetic Particle Modes (EPM) with XHMGC [Briguglio EPL13]



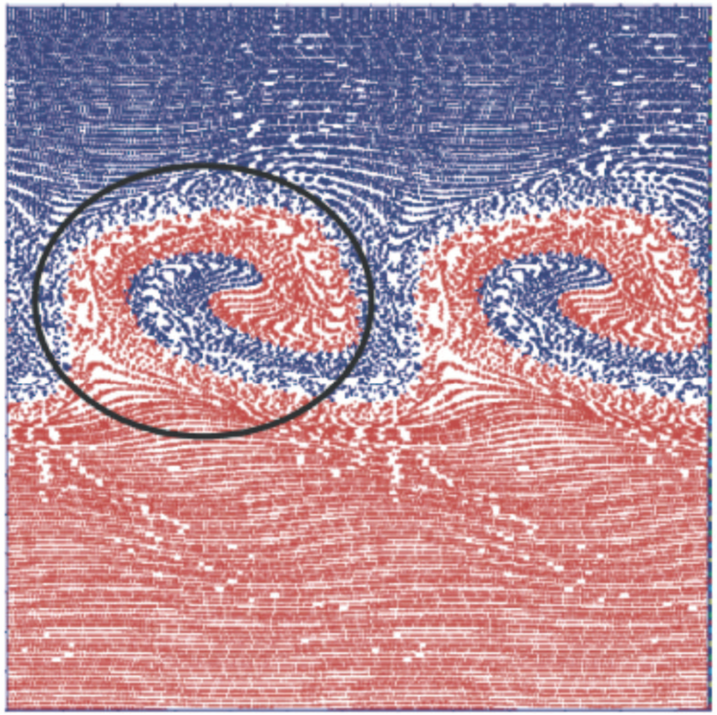
- Evidence of mode/frequency splitting-merging.

- Evidence of mode/frequency splitting-merging ⇔ phase locking and $\omega_B t \sim 1$ ⇒ non-perturbative ⊕ non-adiabatic dynamics [Briguglio EPL13].

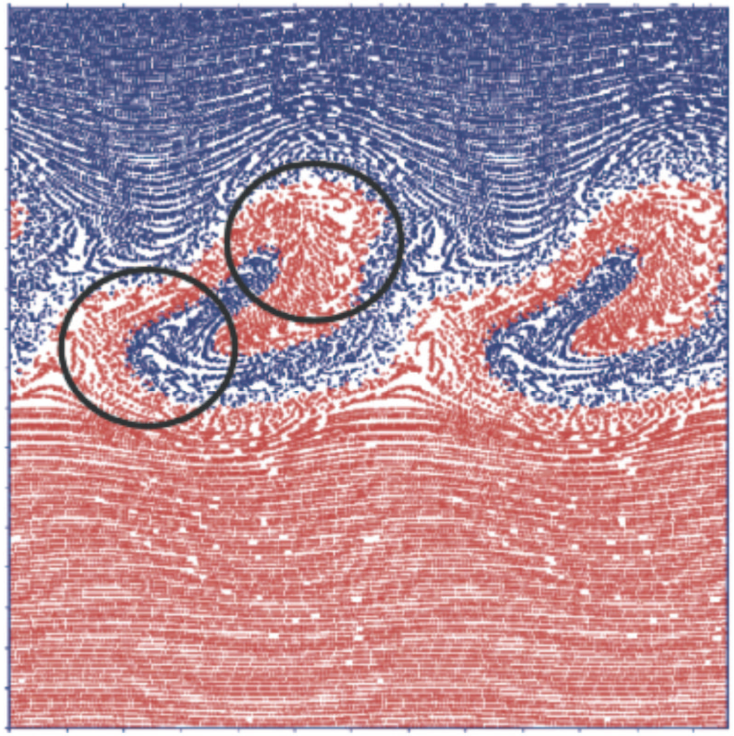


- Crucial role played by the readiness of EPM wave-packets (continuous spectrum) to respond to nonlinearly modified phase-space zonal structures ⇒ Crucial roles of non-uniformities and geometries [Chen RMP14].

- Evidence of structure formation in particle phase space on $\omega_B t \sim 1$ [Briguglio EPL13] $\Rightarrow$ non-perturbative $\oplus$ non-adiabatic dynamics.



$\omega_A t = 1320$



$\omega_A t = 1353$

Description of phase-space zonal structures

(Lecture 4)

- Based on $\gamma/\omega \ll 1 \Rightarrow$ Particle motion is slightly modified by fluctuations in one poloidal bounce/transit time $\Rightarrow$ Fluctuation induced transport is a cumulative effect on resonant particles over many bounce/transit times.

$$f(r,\theta,\xi) = \sum_{m,n,\ell} e^{i(n\bar{\omega}_d+\ell\omega_b)\tau+i\Theta_{n,m,\ell}}\mathcal{P}_{m,n,\ell}\circ f_{m,n}(\bar{r}+\Delta r) \qquad \text{[Chen RMP14]}$$

(r,θ,ξ) toroidal flux (Clebsch) coordinates

$$\begin{tikzcd} & E \arrow[d, "p"] \\ X \arrow[ur, "\tilde{f}"] \arrow[r, "f"'] & B \end{tikzcd}$$

- $\mathcal{P}_{m,n,\ell}$ are projection operators for fluctuating (m,n) Fourier harmonic with phase along the unperturbed particle motion; $\ell =$ bounce harmonics.
- They include the phase factor $\exp\left[i\left(n\bar{q}(\bar{r})-m\right)\omega_b\tau\right]$ for circulating particles.
- $\tilde{f}$ is a lifting of f
- $\Theta_{n,m,\ell}$ is the nonlinear wave-particle phase $\Rightarrow$ resonance detuning.
- $\mathcal{P}_{m,n,\ell}\circ f_{m,n}(\bar{r}+\Delta r)$ accounts for radial decoupling of resonant particles from finite radial mode structure [Briguglio EPL13].

Equations for phase-space zonal structures
(Lecture 4)

- Nonlinear GK description of phase-space zonal structures: adiabatic + non-adiabatic responses ($\delta L_g \equiv \delta\phi_g - (v_\parallel/c)\delta A_{\parallel g}$, $\langle ... \rangle$ = gyroaverage)

$$\delta f_z = \sum_m \left\{ \mathcal{P}_{m,0,0} \circ \left[J_0(\lambda)\delta g \right]_{m,0} \right\} - \left[J_0(\lambda) \left(\frac{e}{m}\frac{1}{B_0}\frac{\partial \bar{F}_0}{\partial \mu} \langle \delta L_g \rangle \right) \right]_{0,0}$$
$$+ \frac{e}{m}\left[\frac{\partial \bar{F}_0}{\partial \mathcal{E}}\delta\phi + \frac{1}{B_0}\frac{\partial \bar{F}_0}{\partial \mu}\delta L \right]_{0,0}$$

- Assuming that $|k_\parallel| \ll |\boldsymbol{k}_\perp|$ [Zonca NF05; Chen RMP14]

$$\frac{\partial \delta g_z}{\partial t} = -\mathcal{P}_{0,0,0} \circ \left(\frac{e}{m}\frac{\partial}{\partial t}\langle \delta L_g \rangle_z \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right)_{0,0} + i\sum_m \mathcal{P}_{m,0,0} \circ \frac{c}{d\psi/dr}\frac{\partial}{\partial r}\sum_n n\left(\delta g_n \langle \delta L_g \rangle_{-n} \right)_{m,0}$$

- Accounts for zonal flows/fields as well as corrugation of radial profiles.

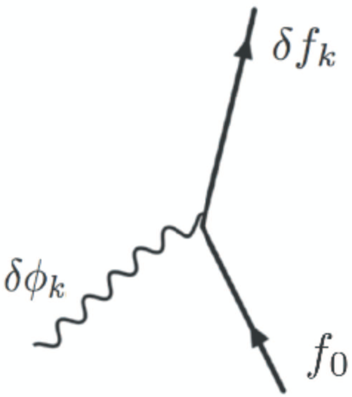
- Assuming single n, the evolution equation for δg_n $(n \neq 0)$ is

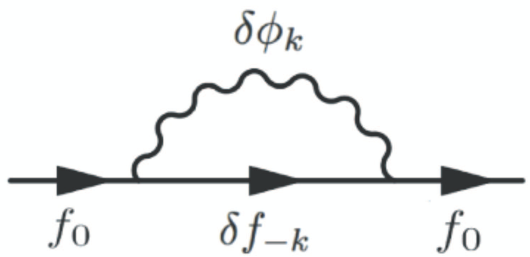
$$\left(\frac{\partial}{\partial t} - \frac{inc}{d\psi/dr} \langle \delta L_g \rangle_z \frac{\partial}{\partial r} + v_\parallel \nabla_\parallel + \boldsymbol{v}_d \cdot \boldsymbol{\nabla}_\perp \right) \delta g_n = i \frac{e}{m} \left(Q\bar{F}_0 - \frac{nB_0}{\Omega d\psi/dr} \mathcal{P}_{0,0,0} \circ \frac{\partial \delta g_z}{\partial r} \right) \langle \delta L_g \rangle_n \quad .$$

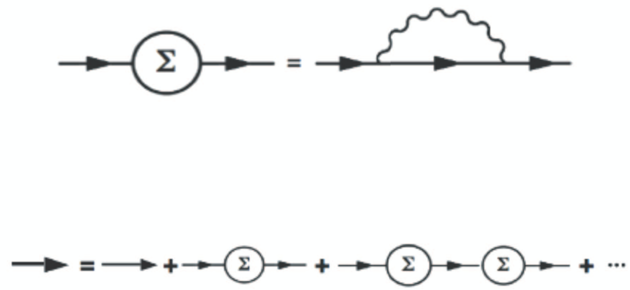
$$Q\bar{F}_0 = \frac{\partial \bar{F}_0}{\partial \mathcal{E}} i \frac{\partial}{\partial t} + \frac{\boldsymbol{b} \times \boldsymbol{\nabla} \bar{F}_0}{\Omega} \cdot (-i\boldsymbol{\nabla})$$

- These equations describe the dominant (shortest time-scale) dynamics of nearly periodic (quasi-coherent) nonlinear fluctuations with $\gamma/\omega \ll 1$ [Al'tshul' Sov.Phys.JETP66] (Lecture 3).

- Evolution equation for renormalized particle distribution function (δg_z) $\Rightarrow$ Dyson Equation (Lecture 4): Plasma instability $\Rightarrow$ spontaneous emission of symmetry-breaking fluctuations







$\Rightarrow$ Route to Transport!

- These equations for δg_z and δg_n are closed by the $(\delta\phi_n, \delta A_{\parallel n})$ DAW field equations for the dynamic evolution of Alfvénic fluctuations and by the equations for the zonal flows/fields and $(\delta\phi_z, \delta A_{\parallel z})$.

- Studied so far in simplified limits:

  - Neglecting wave-particle resonances $\Rightarrow$ dominant zonal flows/fields [Chen POP00; Chen NF01; Guo PRL09; Kosuga POP12]

  - Neglecting effect of zonal flows/fields $\Rightarrow$ dominant EP wave-particle resonances [Zonca Th.Fus.Pl.00; NF05; PPCF06]

- In this *Lecture Series*, focus on dominant EP wave-particle resonances.

Nonlinear DAW field equations

- Adopt the theoretical framework of the general fishbone like dispersion relation (GFLDR) [Chen NF07b, Zonca PPCF06] and wave-packet propagation decomposition in toroidal geometry [Lu POP12; Chen RMP14].

$$\frac{\partial D_n}{\partial \omega_0}\left(i\frac{\partial}{\partial t}\right) A_{n0}(r,t) + \frac{\partial D_n}{\partial \theta_{k0}}\left(-\frac{i}{nq'}\frac{\partial}{\partial r} - \theta_{k0}\right) A_{n0}(r,t) + \frac{1}{2}\frac{\partial^2 D_n}{\partial \theta_{k0}^2}\left[\left(-\frac{i}{nq'}\frac{\partial}{\partial r} - \theta_{k0}\right)^2 - \frac{i}{nq'}\frac{\partial \theta_{k0}}{\partial r}\right] A_{n0}(r,t) = \left(\delta\bar{W}_f^{NL} + \delta\bar{W}_k^{NL}\right)_n - i\Lambda_n^{NL} + S_n^{ext}(r,t) \qquad k_r \equiv nq'\theta_{k0}$$

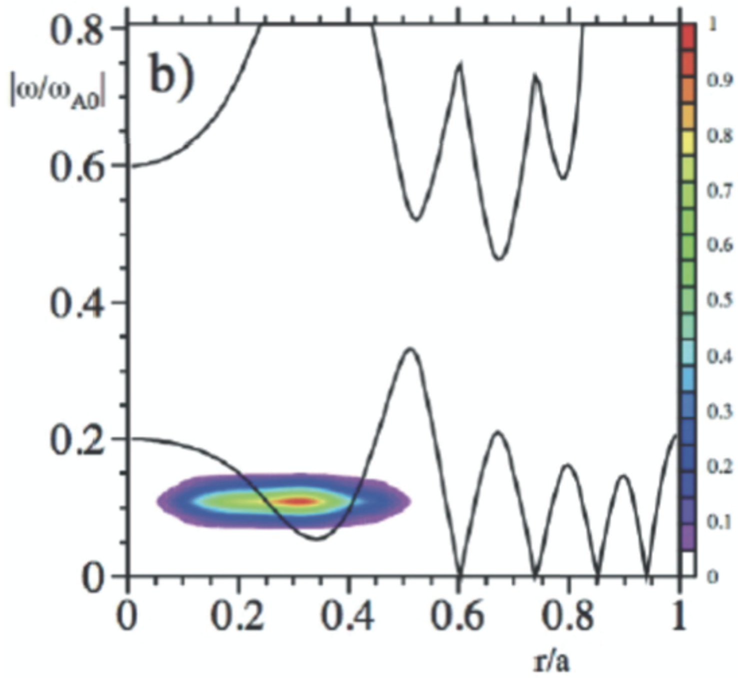
- Slow spatiotemporal evolution. According to the GFLDR:

$$D_n = i\Lambda_n - \left(\delta\bar{W}_f + \delta\bar{W}_k\right)_n$$

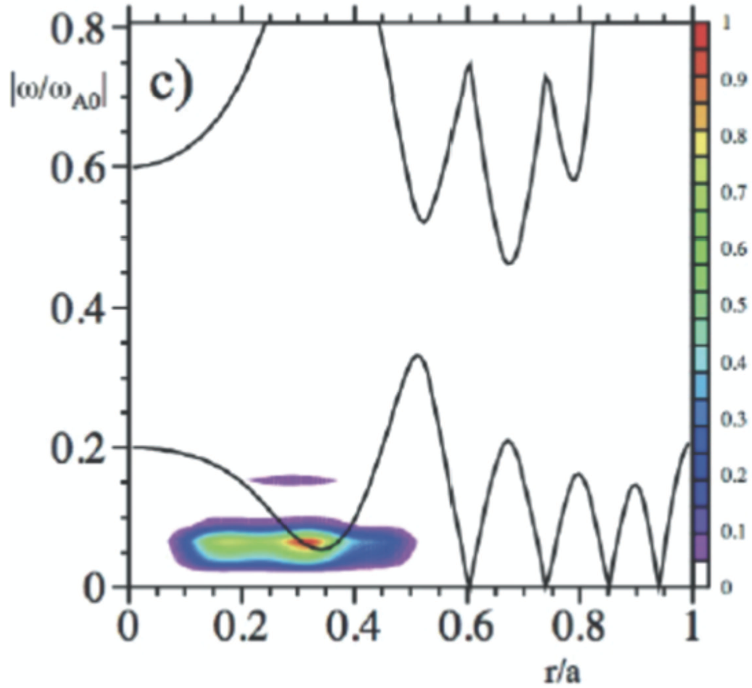
  - Λ_n: generalized inertia response
  - $\delta\bar{W}_{fn}$: potential energy due to fluid plasma response
  - $\delta\bar{W}_{kn}$: potential energy due to kinetic plasma response

Electron-Fishbone Hybrid MHD-Gyrokinetic simulations

- Frequency chirping and EP radial redistribution [Vlad et al., 2013]

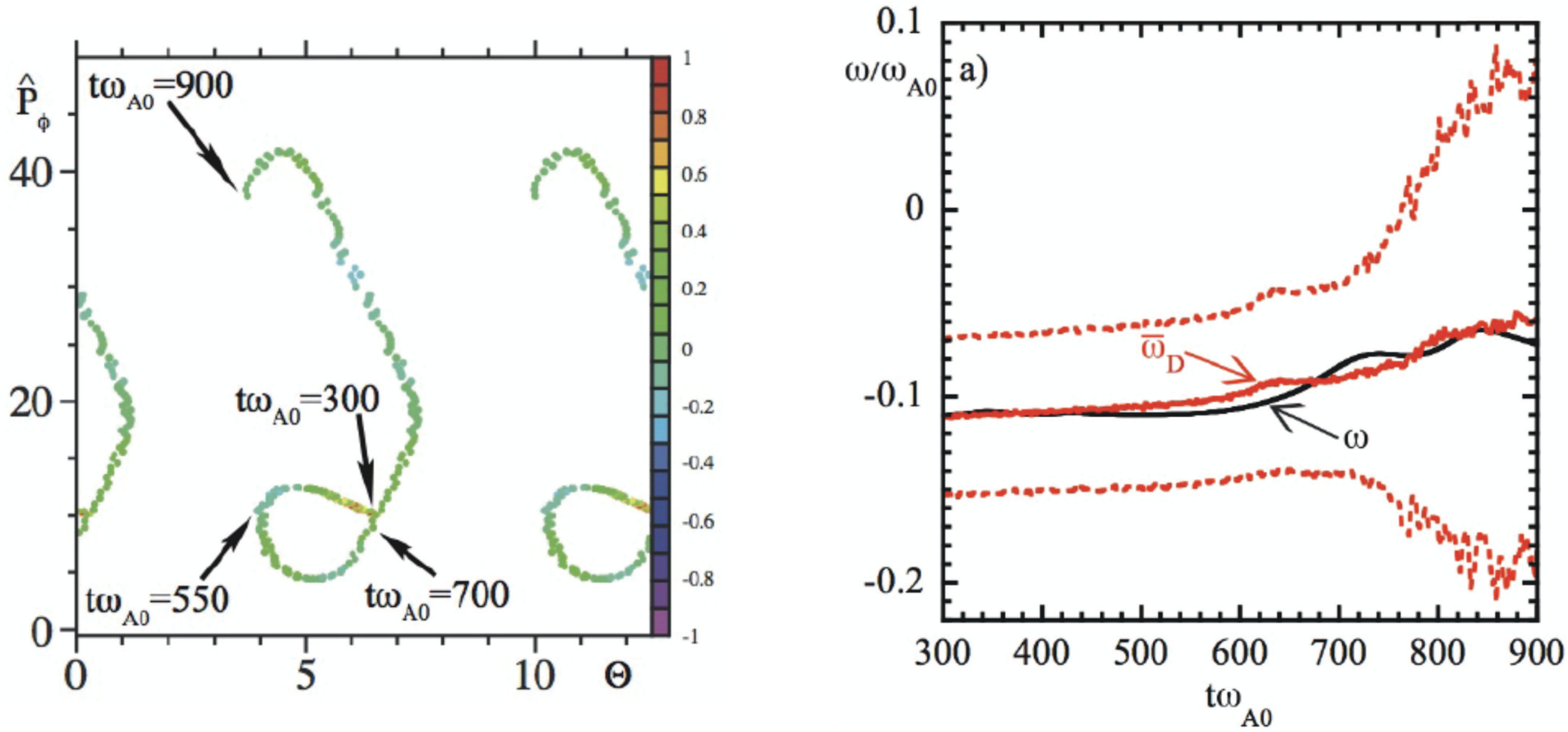


$T = 300$ (linear)



$T = 900$ (saturation)

□ Secular radial motion and phase locking [Vlad et al., 2013]



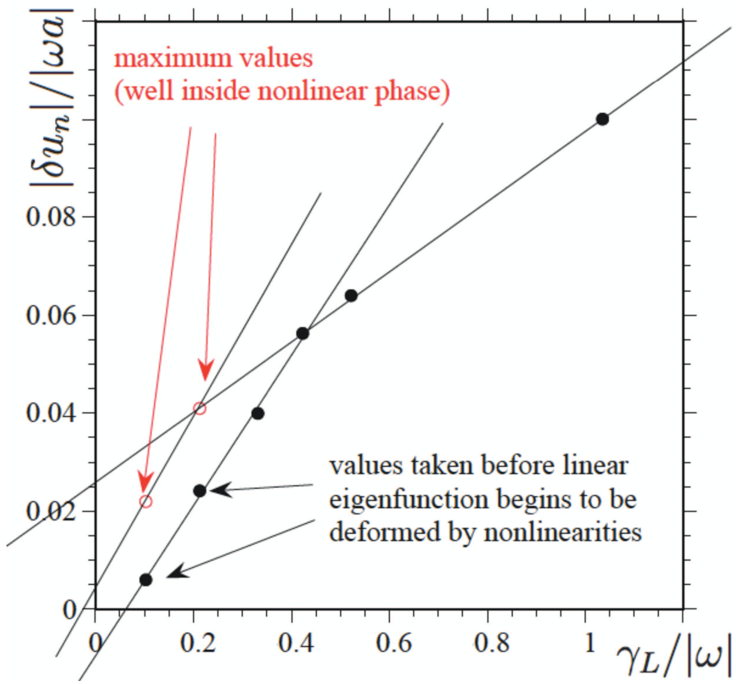
- "Fishbone" Nonlinear Theory [Zonca 2007] based on the "Fishbone" dispersion relation (Lecture 5)

$$i\Lambda(\omega) = \delta W_f + \delta W_k\,(\omega|F_{0EP})$$

- Resonant EPs convect outward with radial speed $|\delta u_n|$ $\Rightarrow$ Nonlinear saturation occurs when $|\delta u_n|/\gamma_L \sim r_s$

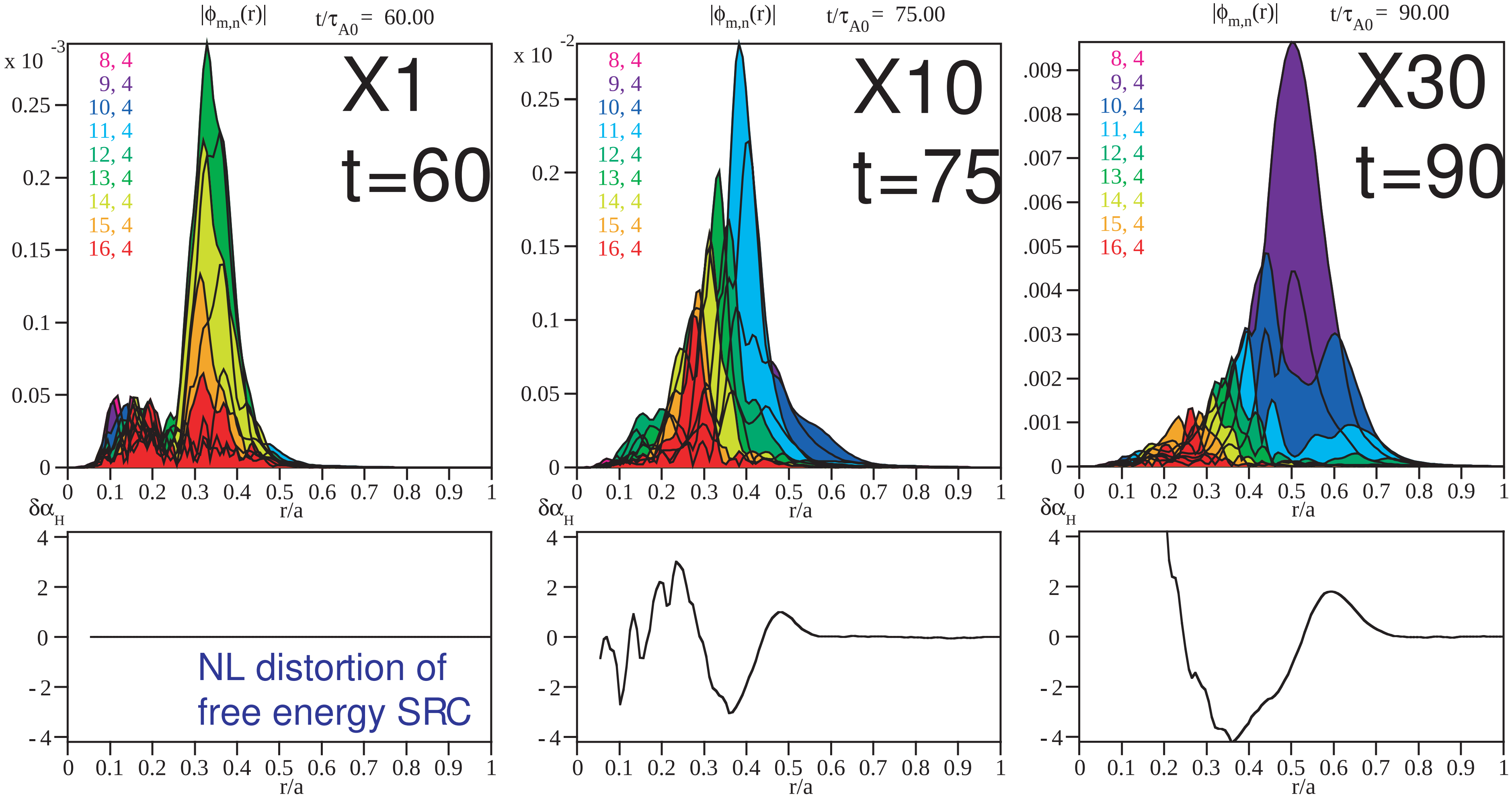
  [$r_s \sim$ mode structure width $\rightarrow$ Wave-EP interaction domain]

- [Vlad et al., 2013] simulation results



Zonca et al. IAEA, (2002)

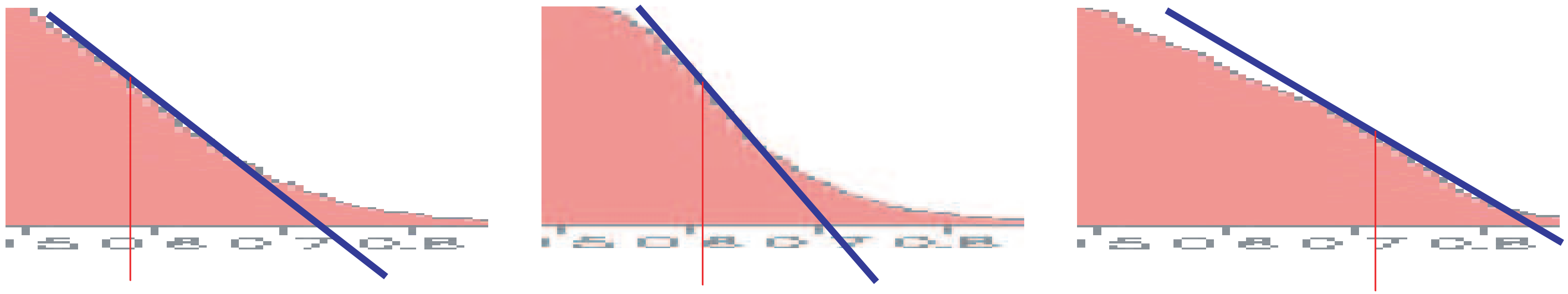
Avalanches and NL EPM dynamics

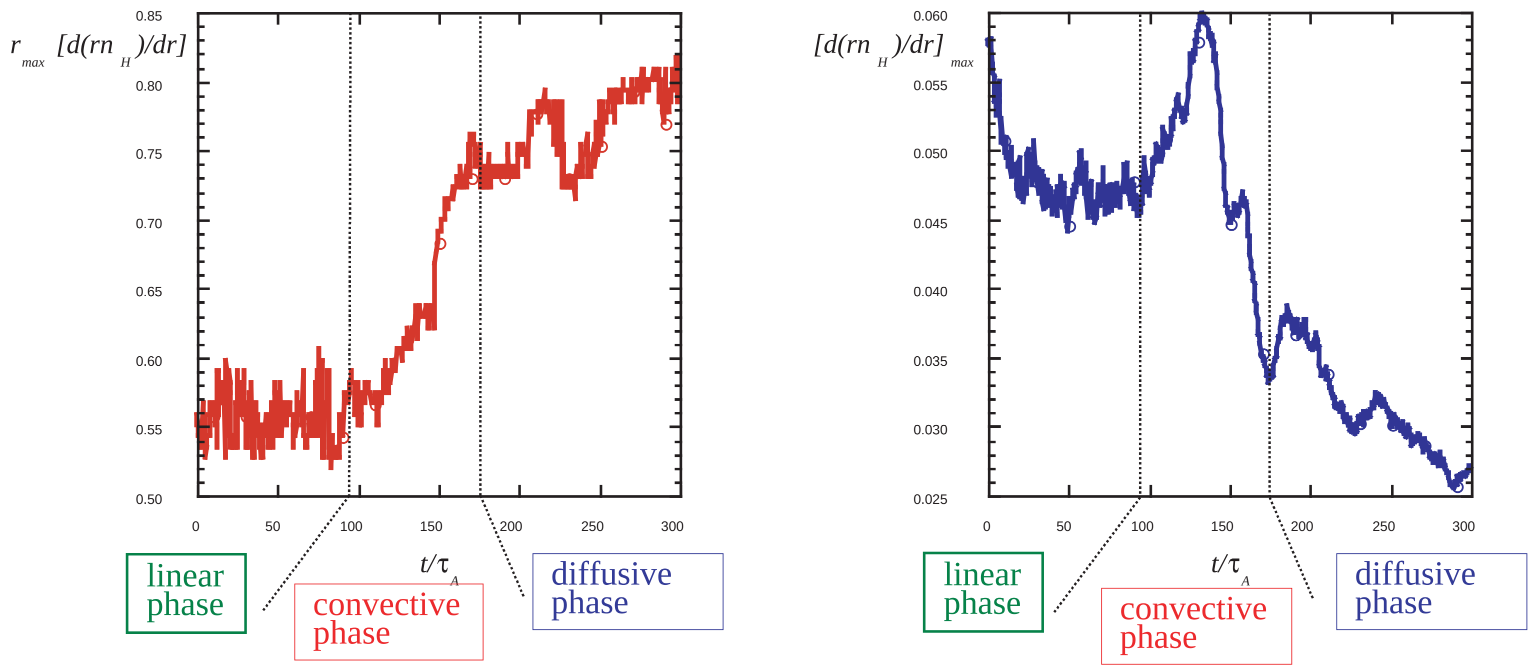


- Importance of toroidal geometry on wave-packet propagation and shape

Vlad et al. IAEA-TCM, (2003)

Propagation of the unstable front



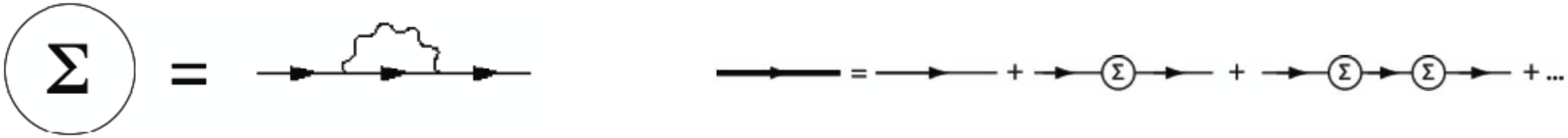


- ☐ Gradient steepening and relaxation: spreading ... similar to turbulence

Nonlinear Dynamics of a single-n coherent EPM

- NL dynamics of a single-n coherent EPM: excited by precession resonance in the small orbit width limit (Lecture 6).

- Summation of the Dyson series for nearly periodic fluctuations with $|\dot{\omega}| \ll \gamma|\omega|$ ...



- ... yields the following expression for the Laplace-transformed $\bar{F}_0(t)$

$$\hat{F}_0(\omega) = \frac{i}{\omega}\mathrm{St}\hat{F}_0(\omega) + \frac{i}{\omega}\hat{S}(\omega) + \frac{i}{2\pi\omega}\bar{F}_0(0) + \frac{e}{m}\frac{nc}{\omega(d\psi/dr)}\frac{\partial}{\partial r}\left\{\left[\frac{Q^*_{k,\omega_k(\tau)}}{\omega^*_k(\tau)}\right.\right.$$

$$\left.\left.\times\frac{\hat{F}_0\left(\omega - 2i\gamma(\tau)\right)}{\omega - \omega_k(\tau) + \bar{\omega}_{dk}} + \frac{Q_{k,\omega_k(\tau)}}{\omega_k(\tau)}\frac{\hat{F}_0\left(\omega - 2i\gamma(\tau)\right)}{\omega + \omega^*_k(\tau) - \bar{\omega}_{dk}}\right]\hat{\omega}_{dk}\left|\delta\bar{\phi}_{k0}(r,\tau)\right|^2\right\} .$$

$$\delta W_{k,T} = \frac{3\pi\epsilon^{1/2}}{8\sqrt{2}|s|}\alpha_E \left[1 + \frac{\omega}{\bar{\omega}_{dF}} \ln\left(\frac{\bar{\omega}_{dF}}{\omega} - 1\right) + i\pi\frac{\omega}{\bar{\omega}_{dF}} \right.$$
$$\left. + i\pi\frac{\omega}{\bar{\omega}_{dF}} k_\theta^2 \rho_E^2 \frac{T_E}{m_E} \frac{1}{\alpha_E |A|^2} \partial_t^{-1} |A|^2 \partial_r^2 \partial_t^{-1} \left(\alpha_E |A|^2\right)\right] .$$

- Note: this NL interaction term is not quasi-linear. QL theory implies the existence of many overlapping resonances yielding diffusion in the velocity space. This term describes coherent NL interactions
- Note that the NL time scale, $\tau_{NL} \approx |\partial_t^{-1}| \approx |A|^{-1}$ (mode particle pumping [White PF83; Chen PRL84]).
- The operator ∂_t^{-1} must be interpreted in symbolic sense.
- This analysis naturally yields to nonlinear Schrödinger equation with integro-differential nonlinear terms.

On the nonlinear Schrödinger equation

- The EPM soliton formation and convective wave-packet amplification yield (Lecture 6)

$$\partial_\xi^2 U = \lambda_0 U - 2iU|U|^2 \ .$$

- This is similar but not the same as the equation a nonlinear oscillator in the so-called "Sagdeev potential" $V = (-U^2 + U^4)/2$, which generates the equation of motion

$$\partial_\xi^2 U = U - 2U^3 \ ,$$

  and gives $U = \mathrm{sech}(\xi)$.

- This is the solution that appears in the problem of ITG turbulence spreading via soliton formation [Guo PRL09].

- More generally, this form appears in soliton-like solutions of nonlinear Schrödinger equations; e.g.
  - the Gross-Pitaevsky equation describing the ground state of a quantum system of identical bosons [Gross 61; Pitaevsky 61]
  - the envelope of modulated water wave groups [Zakharov 68]
  - the propagation of the short optical pulse of a FEL in the superradiant regime [Bonifacio 90]

- The complex nature of EPM equation is novel and connected with the peculiar role of wave-particle resonances, which dominate the nonlinear dynamics of EPMs via resonant wave-particle power exchange, whose maximization yields two effects:
  - the mode radial localization, similar to the analogous mechanism discussed for the linear EPM mode structure
  - the strengthening of mode drive, connected with the steepening of pressure gradient, convectively propagating with the EPM wave-packet

Summary

- Phase-space zonal structures play fundamental roles in EP transport induced by DAW with $|\gamma/\omega| \ll 1$ in burning plasmas.
- Phase-locking plays a major role in the underlying dynamic behaviors, causing the nonlinear particle displacement at saturation to become of the order of the radial mode width and the nonlinear dynamic itself to be non-adiabatic ($|\gamma/\omega| \lesssim 10^{-2}$ for typical parameters).
- These physics are affected by system geometry and equilibrium nonuniformity: radial decoupling becomes more important than resonance detuning.
- Depending on the strength of mode drive (EPM/Fishbone are excited), the self-consistent non-linear interplay between EP transport and mode structures is a crucial element for understanding EP secular loss.
- These dynamics are described by a nonlinear Schrödinger equation with new twists, due to the peculiar role of wave-particle resonances. Generally, NLSE with integro-differential nonlinear terms.

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