Lecture 4

Cases of practical interest: the nonlinear BAE problem and the nonlinear TAE problem from recent hybrid simulation results

Fulvio Zonca

http://www.afs.enea.it/zonca

Associazione Euratom-ENEA sulla Fusione, C.R. Frascati, C.P. 65 - 00044 - Frascati, Italy.

Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou 310027, P.R.C.

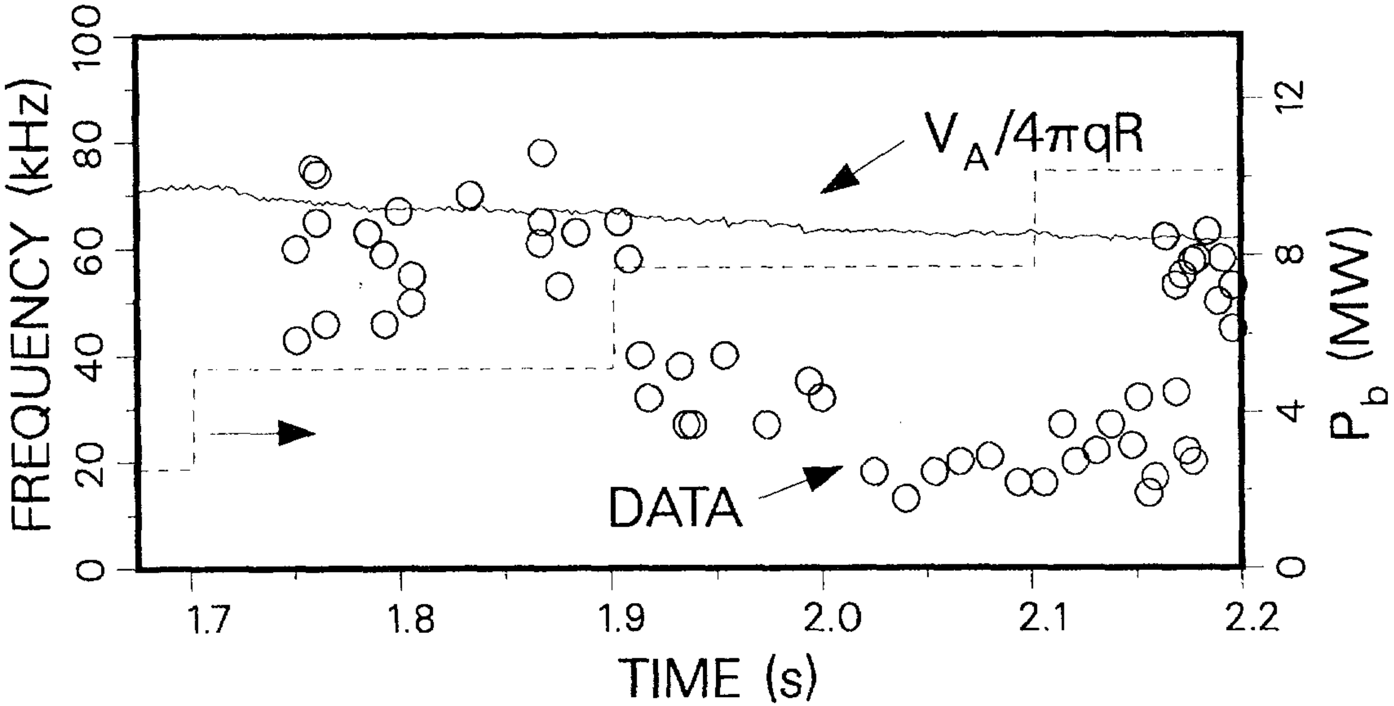
May 7.th, 2012

IFTS Intensive Course on Advanced Plasma Physics-Spring 2012,
Nonlinear charged particle dynamics in tokamaks: theory and applications
2–12 May 2012, IFTS – ZJU, Hangzhou

(I) Background

- BAE first observed in DIII-D with fast ions, and then TFTR plasmas. They can cause large energetic ion losses.

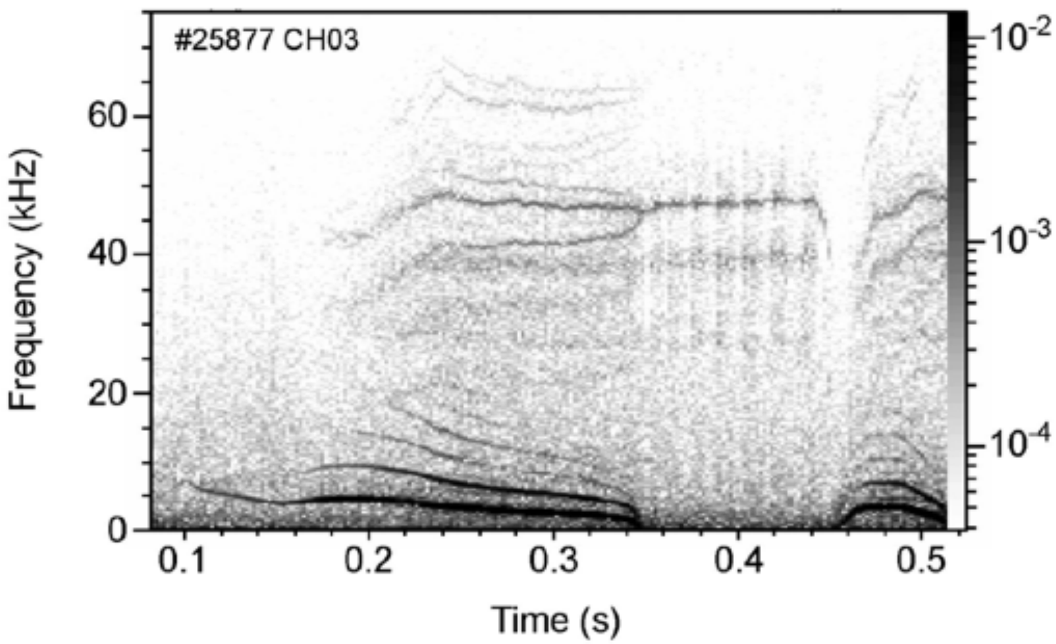
W. W. Heidbrink, PRL. Lett.71 (1993) 895.
R. Nazikian, PoP, 3 (1996) 593.

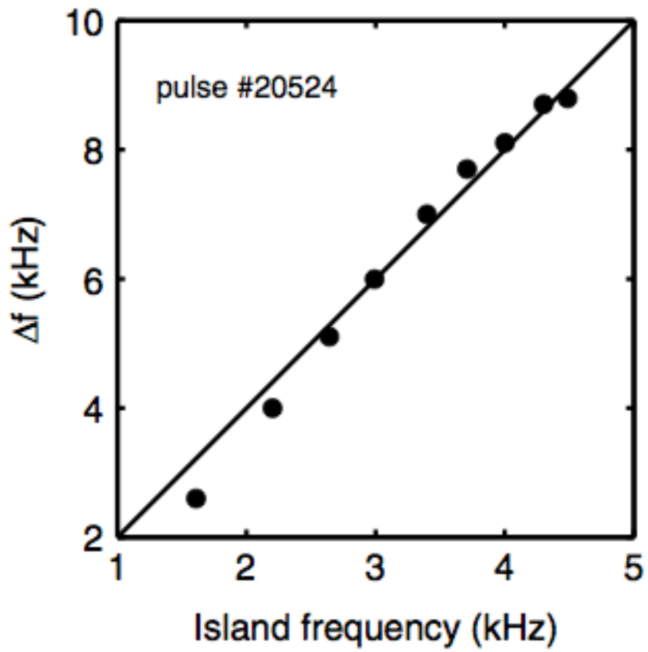


- BAE also observed during strong tearing mode (TM) activity in FTU and TEXTOR, in purely Ohmic discharges, thus in the absence of fast ions .

  P. Buratti, NF,45 (2005) 1446.

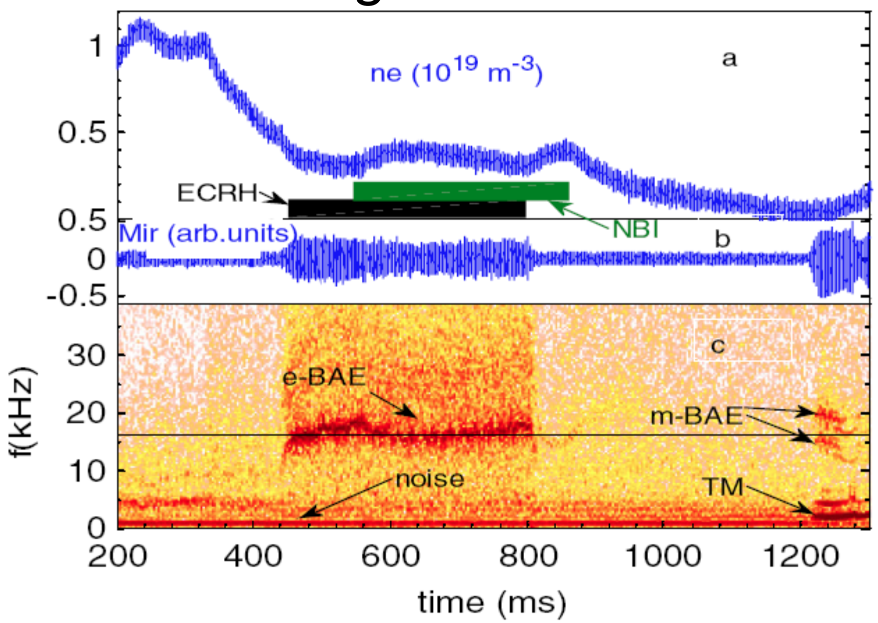
  O. Zimmermann,32nd EPS, 2005,P4.059.



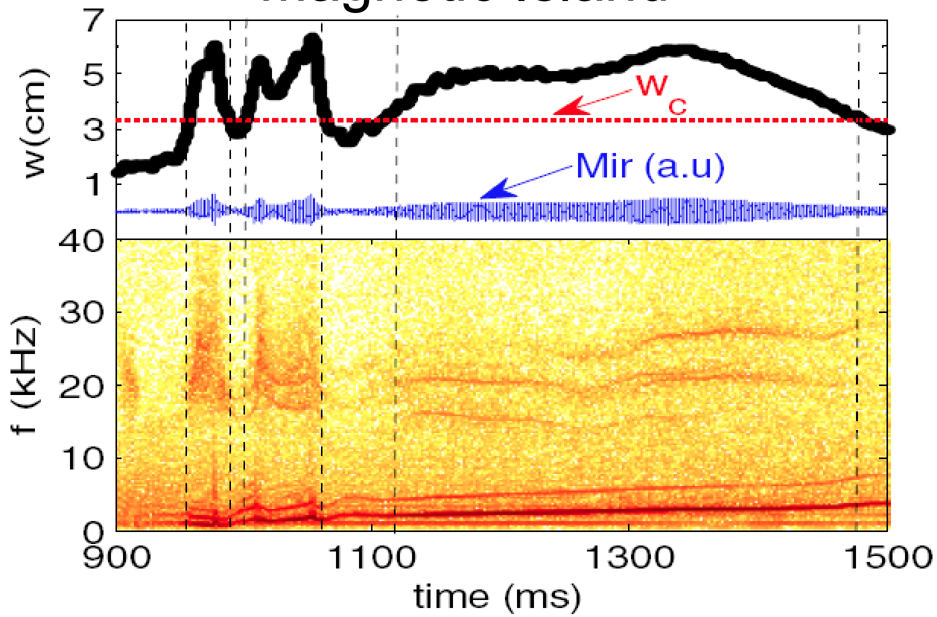


- BAE have also been reported during a sawtooth cycle in ASDEX-UG and TORE-SUPRA plasmas with fast ions.
  C.Nguyen, PPCF, 51 (2009) 095002.
  P. Lauber, PPCF, (2009) 124009.
- BAE are excited by energetic electrons and large magnetic islands on HL-2A (*courtesy of Chen Wei and HL-2A Team*).

Energetic electrons



Magnetic Island



(II) Linear dynamics

- BAE is a special case of the general *fishbone-like* dispersion relation (GFLDR), (Chen and Zonca 06-07) (Spring 09 Lecture 4):

$$i\Lambda + \delta W_f + \delta W_k = 0$$

where δW_f and δW_k play the role of fluid (core plasma) and kinetic (fast ion) contribution to the potential energy, while Λ represents a generalized inertia term.

- The generalized fishbone-like dispersion relation can be derived by asymptotic matching the regular (ideal MHD) mode structure with the general (known) form of the shear Alfven wave (SAW) field in the singular (inertial) region, as the spatial location of the shear Alfven resonance, $\omega^2=k^2_{||}v^2_A$, is approached.

- $\boxed{\Lambda^2(\omega/\omega_A|\omega_{ti},\omega_{*i})=0}$ ⇨ $\boxed{\omega=\omega_{cap}}$ is the continuum accumulation point frequency.

(II.1) Hybrid MHD GK simulations

- HMGC: Hybrid Magnetohydrodynamic Gyrokinetic Code

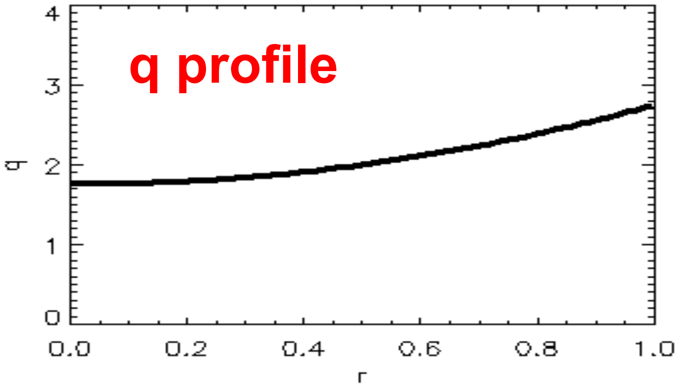
$$\boxed{\rho_b\left(\frac{\partial}{\partial t} + \delta \mathbf{v_b}\cdot\nabla\right)\mathbf{v_b} = -\nabla P_b - (\nabla\cdot\mathbf{P_E})_\perp + \frac{\mathbf{J}\times\mathbf{B}}{c}.}$$

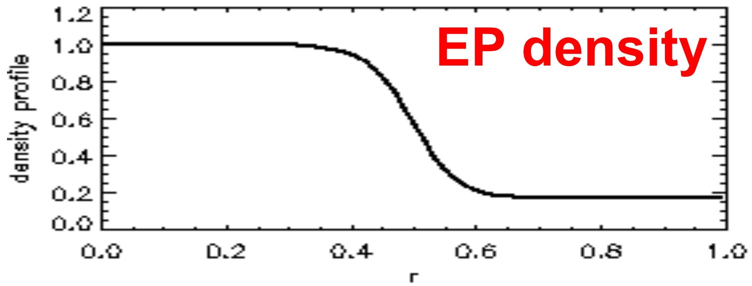
The model equation is based on the pressure coupling equation with a fluid description of bulk plasma (cold plasma) dynamics and a gyrokinetic "closure" for the pressure tensor of EP.

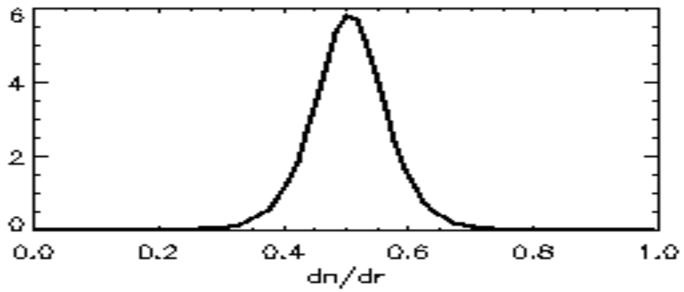
$$\omega_{*pE} \gg \omega_A \geq \omega \geq \omega_{*pi} \gg \omega_{ti}$$

- (X)HMGC: an eXtended version of HMGC by taking into account (Wang et al 11)
  ① thermal ion compressibility (kinetic thermal ion effect)
  ② diamagnetic effects
  ③ finite parallel electric field

(III) BAE Nonlinear dynamics







Slowing down distribution

$$
f_{\mathrm{EQ,sd}} = \frac{n_{H0}}{E_{crit,0}^{3/2}} \frac{3}{4\pi} \left(\frac{m_H}{2} \right)^{3/2} \hat{f}_{\mathrm{EQ,sd}} \, ,
$$

$$
\hat{f}_{\mathrm{EQ,sd}} = \frac{\hat{n}(\psi)}{\tau(\psi)^{3/2}} \frac{\Theta(\alpha, \alpha_0, \Delta)}{\left[(E/E_{crit})^{3/2} + 1 \right] \ln \left[1 + (E_0/E_{crit})^{3/2} \right]} \, ,
$$

$$
\boxed{
\begin{aligned}
\Theta(\alpha, \alpha_0, \Delta) &\equiv \frac{4}{\Delta\sqrt{\pi}} \frac{\exp \left[- \left(\frac{\cos\alpha - \cos\alpha_0}{\Delta} \right)^2 \right]}{\mathrm{erf}\left(\frac{1 - \cos\alpha_0}{\Delta} \right) + \mathrm{erf}\left(\frac{1 + \cos\alpha_0}{\Delta} \right)} \, , \\
\cos\alpha &\equiv \frac{u}{\sqrt{2E/m_H}} \, ,
\end{aligned}
}
$$

$$
\Delta \longrightarrow \infty \quad = \quad \Theta(\alpha) = 1
$$

$$
\Delta \longrightarrow 0 \quad = \quad \Theta(\alpha) = 2\delta(\cos\alpha - \cos\alpha_0)
$$

General remarks on NL SAW-EP dynamics

Shear Alfvén wave (SAW) – energetic particle (EP) physics:

Existence of two regimes:

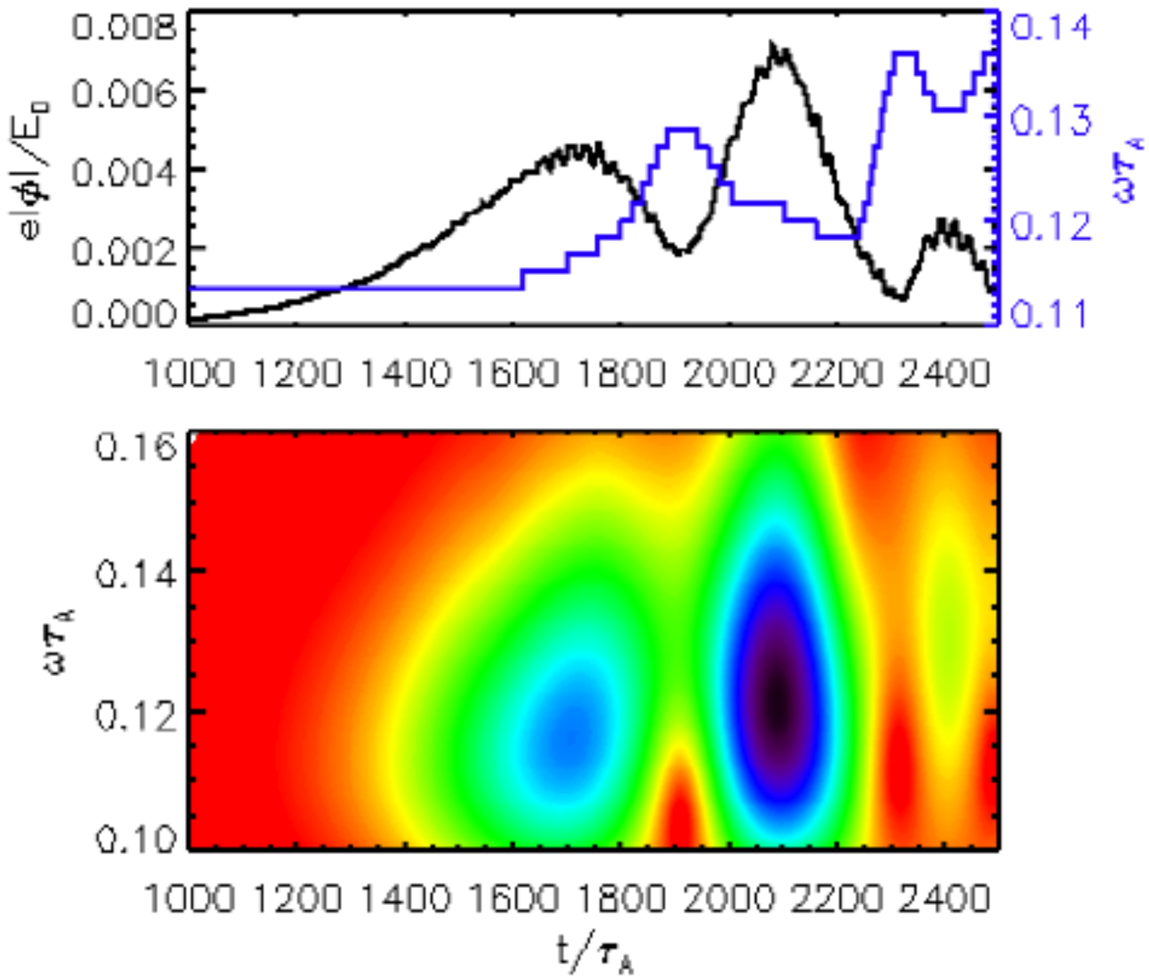
- Weakly driven system: near marginal stability;
  EPMs are not excited;
  same as a uniform system;
  saturation due to wave-particle trapping
  Local and independent of the radial mode structure

- Strongly driven system: - EPMs are excited at EP characteristic frequency;
  - EP transport occurs in avalanches (Spring 10 Lecture 5)

- ***Transition regime***: of particular interest in this Lecture (demonstrate the increasingly important role of nonuniformity and geometry)

[X.Wang et al submitted to PRL]

➢ Upward frequency sweeping



Note that:

(1) BAE localized around (r/a=0.5)
(2) q=2,
(3) n=2 and m=4 dominant harmonic
(4) $\omega_{cap}\approx 0.127/\tau_A$ calculated from kinetic theory
(5) $\omega_{tmax}\approx 0.2/\tau_A$ calculated at birth energy
(6) $\omega_0\approx 0.11/\tau_A$
(7) $\gamma_L\approx 0.006/\tau_A$

Upward frequency sweeping and GFLDR

Generalized fishbone like dispersion relation (GFLDR):

$$i\Lambda = \delta W_f + \delta W_k$$

Nonlinear frequency shift expressed as:

$$\frac{\Delta\omega}{\omega_A}\left[\frac{\omega_0}{\sqrt{\omega_{cap}^2-\omega_0^2}} - \omega_A \frac{\partial}{\partial\omega_0}\mathrm{Re}\left(\delta W_{kL}\right) - i\omega_A \frac{\partial}{\partial\omega_0}\mathrm{Im}\left(\delta W_{kL}\right)\right] =$$

$$\mathrm{Re}\left(\delta W_{kNL}\right) + i\,\mathrm{Im}\left(\delta W_{kNL}\right)$$

Note that:

(1) mode birth in the gap ($\delta W_k < 0$)

(2) $i\Lambda = -\sqrt{\omega_{cap}^2 - \omega^2}\,/\,\omega_A$

(3) linear part can be expanded about ω_0

(4) Re $\delta W_{kNL} > 0$ can not be expanded, because it can vary rapidly with $\omega \Rightarrow$ truly nonlinear response (Spring 10 Lecture 5)

$$\frac{\Delta\omega}{\omega_A}\left[\frac{\omega_0}{\sqrt{\omega_{cap}^2-\omega_0^2}}-\omega_A\frac{\partial}{\partial\omega_0}\mathrm{Re}\left(\delta W_{kL}\right)-i\omega_A\frac{\partial}{\partial\omega_0}\mathrm{Im}\left(\delta W_{kL}\right)\right]=$$

$$\mathrm{Re}\left(\delta W_{kNL}\right)+i\,\mathrm{Im}\left(\delta W_{kNL}\right)$$

$$\mathrm{L.H.S.}=\frac{\Delta\omega}{\omega_A}ae^{-i\alpha},\qquad\left(0<\alpha<\pi/2\right)$$

$$\mathrm{R.H.S.}=be^{-i\beta},\qquad\left(0<\beta<\pi/2\right)$$

Im $\delta W_{kNL}<0$
(Spring 10 Lecture 5)

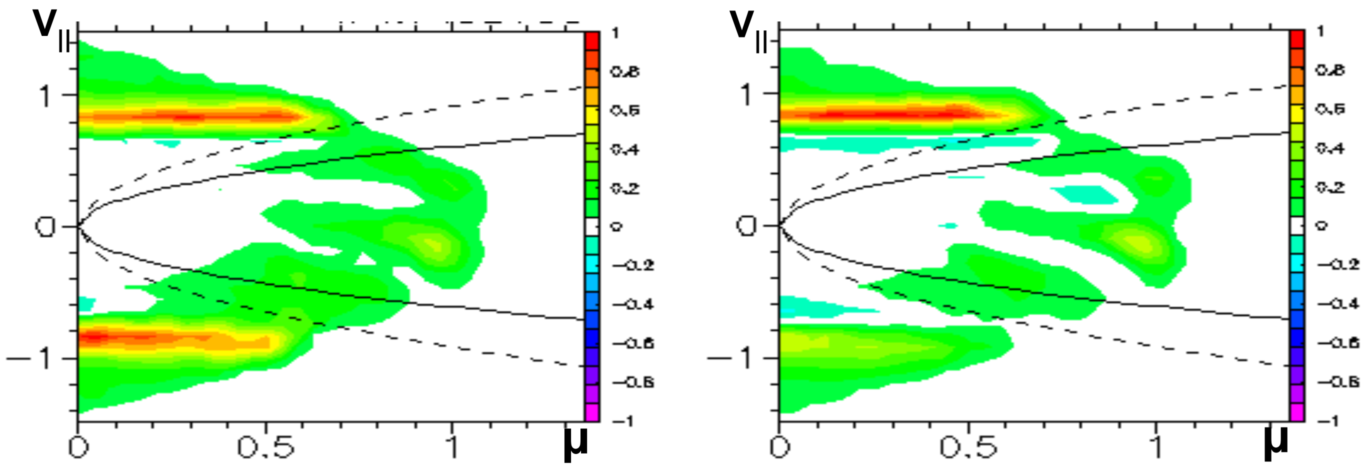
- Upward frequency sweeping: consistent with numerical simulation results

$$\frac{\Delta\omega}{\omega_A}=\frac{b}{a}e^{i(\alpha-\beta)}$$

[X.Wang et al submitted to PRL]

➢ Symmetry breaking in $v_{||}$

Wave-particle power exchange



(1) Left panel (t/τ_A)=600, linear growth phase, co- and counter-passing particles are almost symmetric in the phase space.
(2) Right panel (t/τ_A)=1700, saturation phase,
(3) Evidence of symmetry breaking and resonance detuning

(III.1) Resonance detuning

Passing particle resonance condition is (Lecture 2):

$$\omega = \omega_t + \left(n\bar{q} - m\right)\sigma\omega_t \quad ,$$

where $\sigma = \mathrm{sgn}(v_\parallel)$, $\bar{q} = (2\pi)^{-1}\int q\,d\theta$ the orbit average of safety factor and the transit frequency is defined as $\omega_t = 2\pi / \int d\theta / \dot{\theta}$ in Clebsch toroidal flux coordinates (Ψ,θ,ξ).

Resonance condition: stationarity in the wave-particle phase

$$\dot{\Theta} = 0$$

Resonance detuning: (Lecture 2) including frequency sweeping

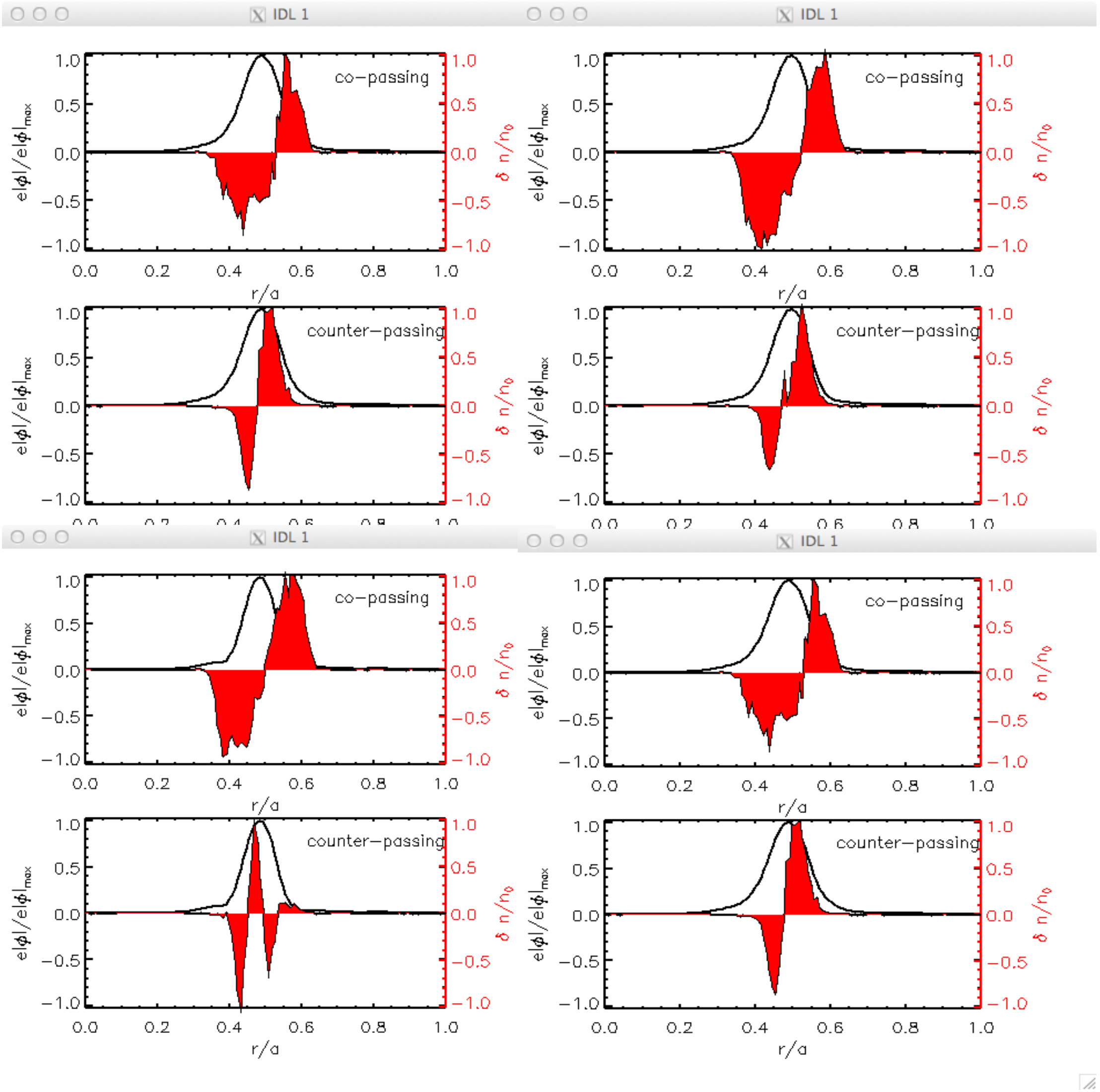
$$\Delta\dot{\Theta} \cong \left(n\bar{q}'\sigma + \frac{\omega_0}{\omega_t^2}\partial_r\omega_t \right)\omega_t\Delta r - \left(\omega - \omega_0\right),$$

$$\Delta\dot{\Theta} \cong \left(n\overline{q}'\sigma + \frac{\omega_0}{\omega_t^2}\partial_r\omega_t \right)\omega_t \Delta r - (\omega - \omega_0),$$

- Resonance detuning due to radial particle displacement
- Symmetry breaking: for upward frequency chirping, more easily maintain the resonance condition for co-passing than counter-passing particles
- Phase-locking: for particles that balance radial resonance detuning with frequency sweeping; particles that most efficiently drive the mode and contribute to transport

[X.Wang et al submitted to PRL]

➢ Energetic particle radial displacement

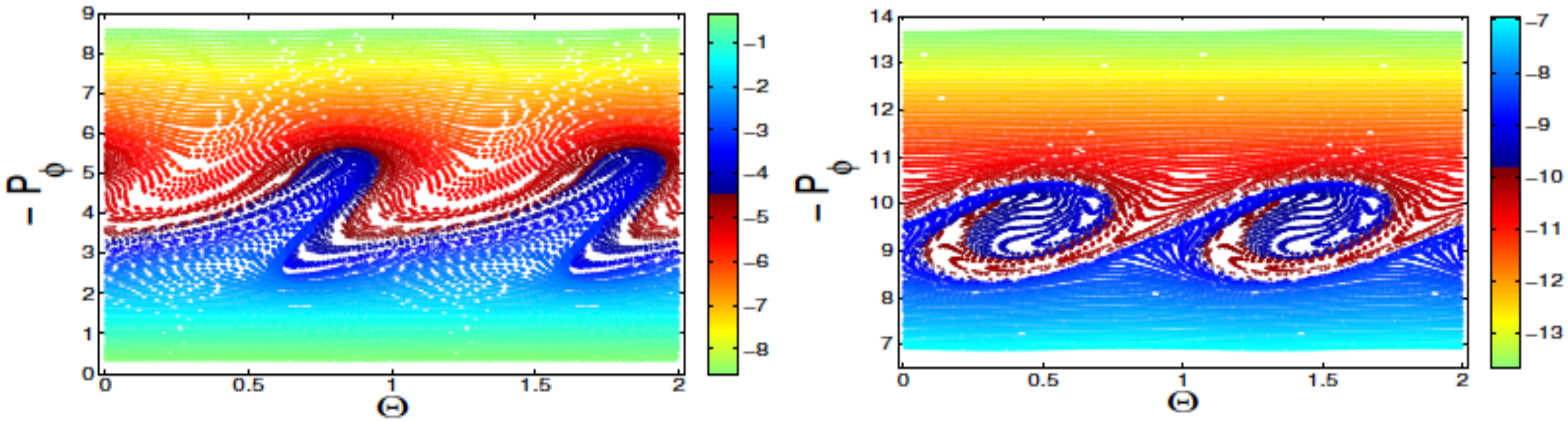


Flux averaged EP fluctuating distribution, integrated on a narrow $\Delta v_{||}$ near resonance.

Wave interaction with co-passing particles is lasting longer and covering a broader radial domain, of the order of the radial mode-width.

[X.Wang et al submitted to PRL]

➢ Phase-space structures

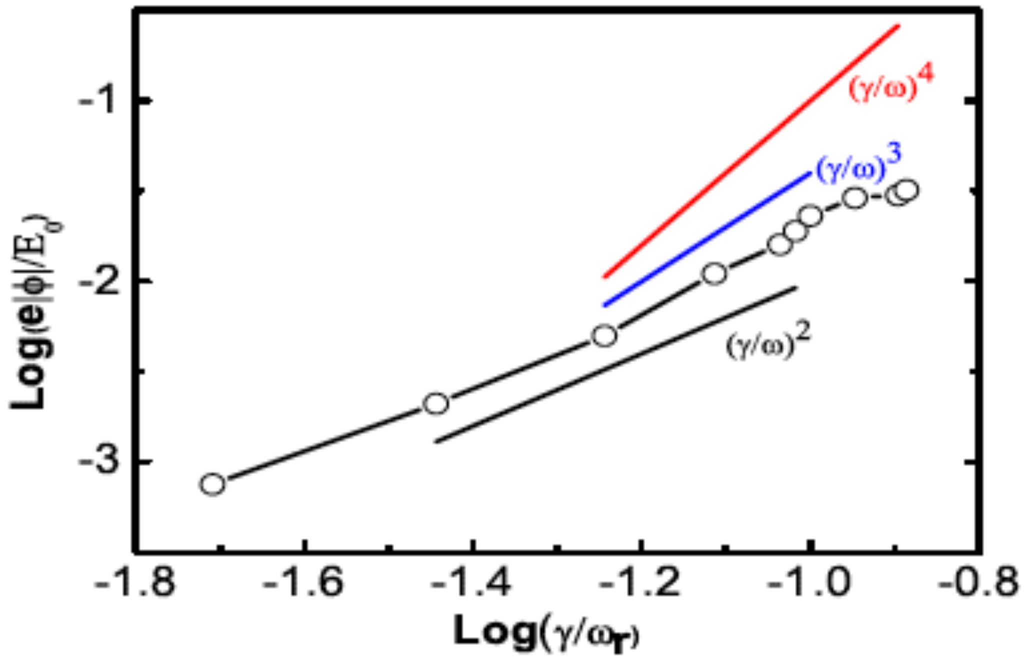


- Co-passing: wave-particle phase is essentially constant.
- Resonance condition is effectively maintained through saturation.
- Frequency chirping is connected with **phase-locking**, for particles to be most efficiently transported outward.

- Counter-passing: rapid wave-particle **phase mixing**
- Reduced drive

[X.Wang et al submitted to PRL]

➢ Scaling of saturation amplitude with linear growth rate



Further evidence of radial resonance detuning and importance of equilibrium nonuniformity and radial mode structures (Lecture 5).

(IV) TAE Nonlinear dynamics

[S. Briguglio et al
To be submitted to PRL]

• Hybrid code HMGC uses Fourier decomposition ∝ exp (-inφ+imθ) [φ = ζ for high aspect ratio, circular shifted surfaces.

• With respect to the (opposite) notation of Lecture 3, this corresponds to taking negative mode numbers for modes rotating in the ion diamagnetic frequency (positive) ⇔ positive mode numbers correspond to negative frequency.

• From Lecture 3

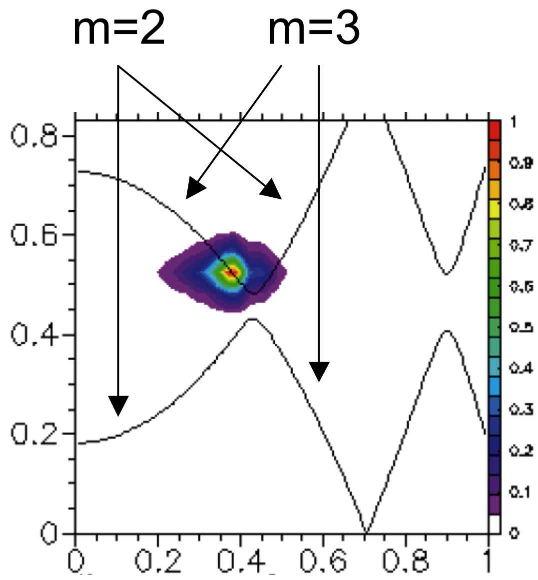
$$\left\{\begin{aligned} \dot{\Theta} &= n(d\bar{q}/d\bar{r})\omega_t \Delta r \\ \Delta\dot{r} &= 2n\bar{q}\left[\left(\frac{\omega}{\omega_t} + m - n\bar{q}\right)\mathcal{A} + \left(\frac{\omega}{\omega_t} + 1 + m - n\bar{q}\right)\mathcal{B}\right]\sin\Theta \\ &\simeq -2n\bar{q}\left[\mathcal{A} - \left(\frac{\omega}{\omega_t} + 1 + m - n\bar{q}\right)\mathcal{B}\right]\sin\Theta \ , \end{aligned}\right.$$

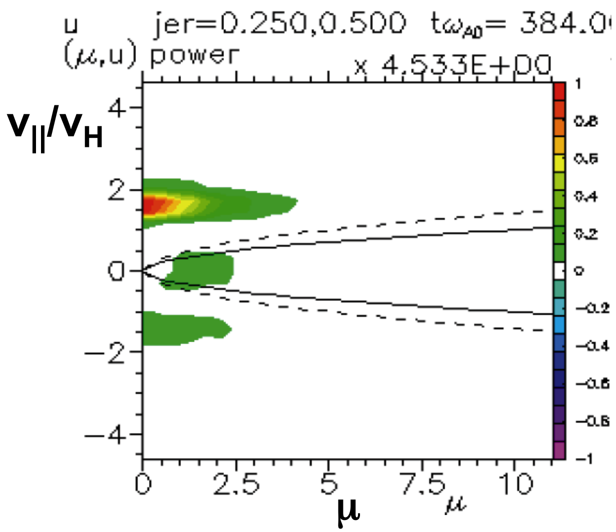
$$\mathcal{A} \simeq (c/B_0)(\delta\phi_{m,n}/\bar{r}) \qquad \mathcal{B} \simeq (c/B_0)(\delta\phi_{m+1,n}/\bar{r})$$

n=2 TAE case

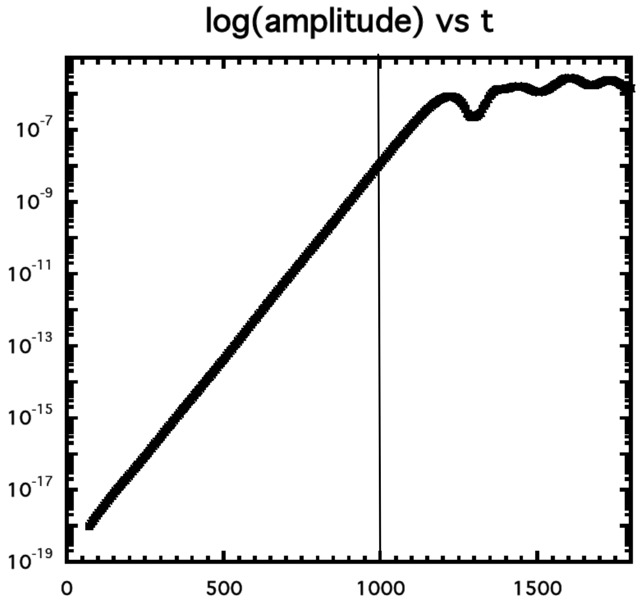
- The mode is born in the continuum.
- The real frequency is $\omega_r \approx 0.52\ /\tau_A$
- Growth rate $\gamma/\omega_r = 0.00235$
- $v_{||}$ asymmetry

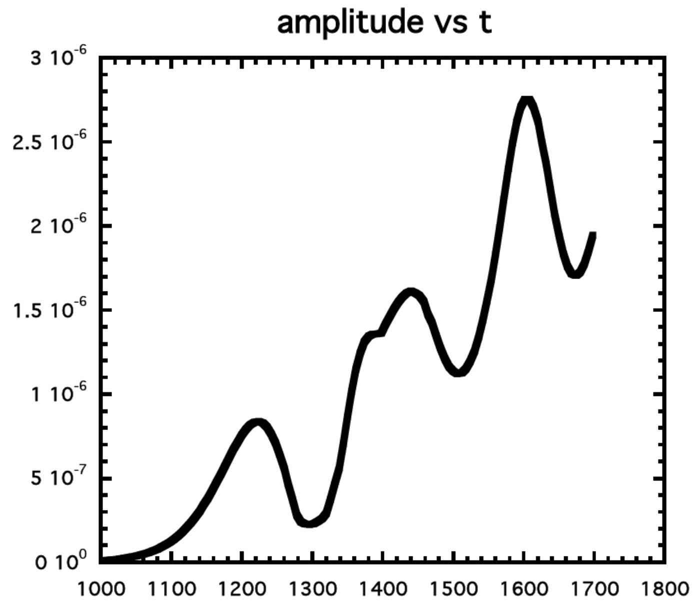
• m=3 mode is dominant (minimizes line bending) and interacts with $v_{||}>0$.
• This causes $v_{||}$ asymmetry also in linear stage (m=2 interacts with $v_{||}<0$).
• Consistent with analysis in Lecture 3

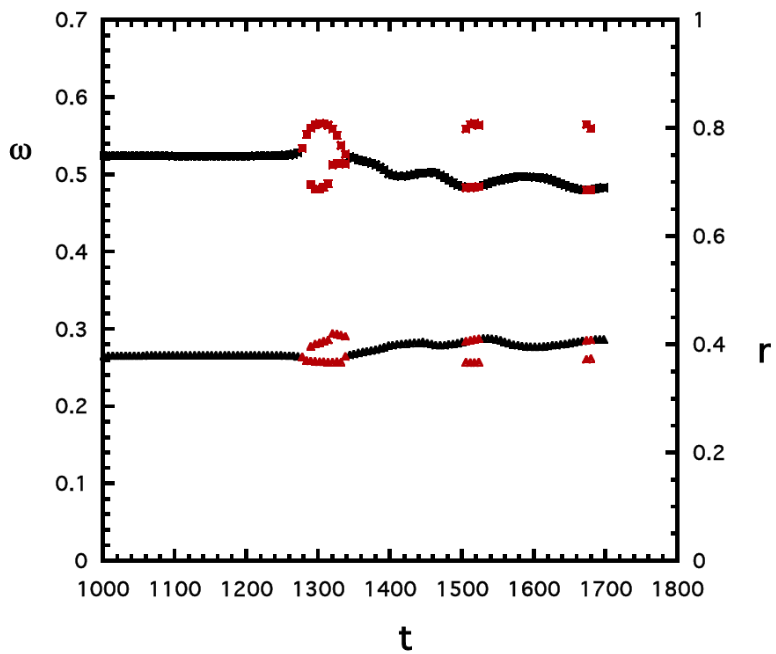




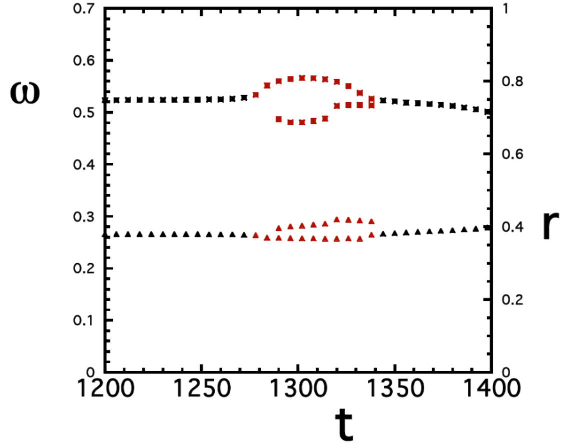
Nonlinear behavior





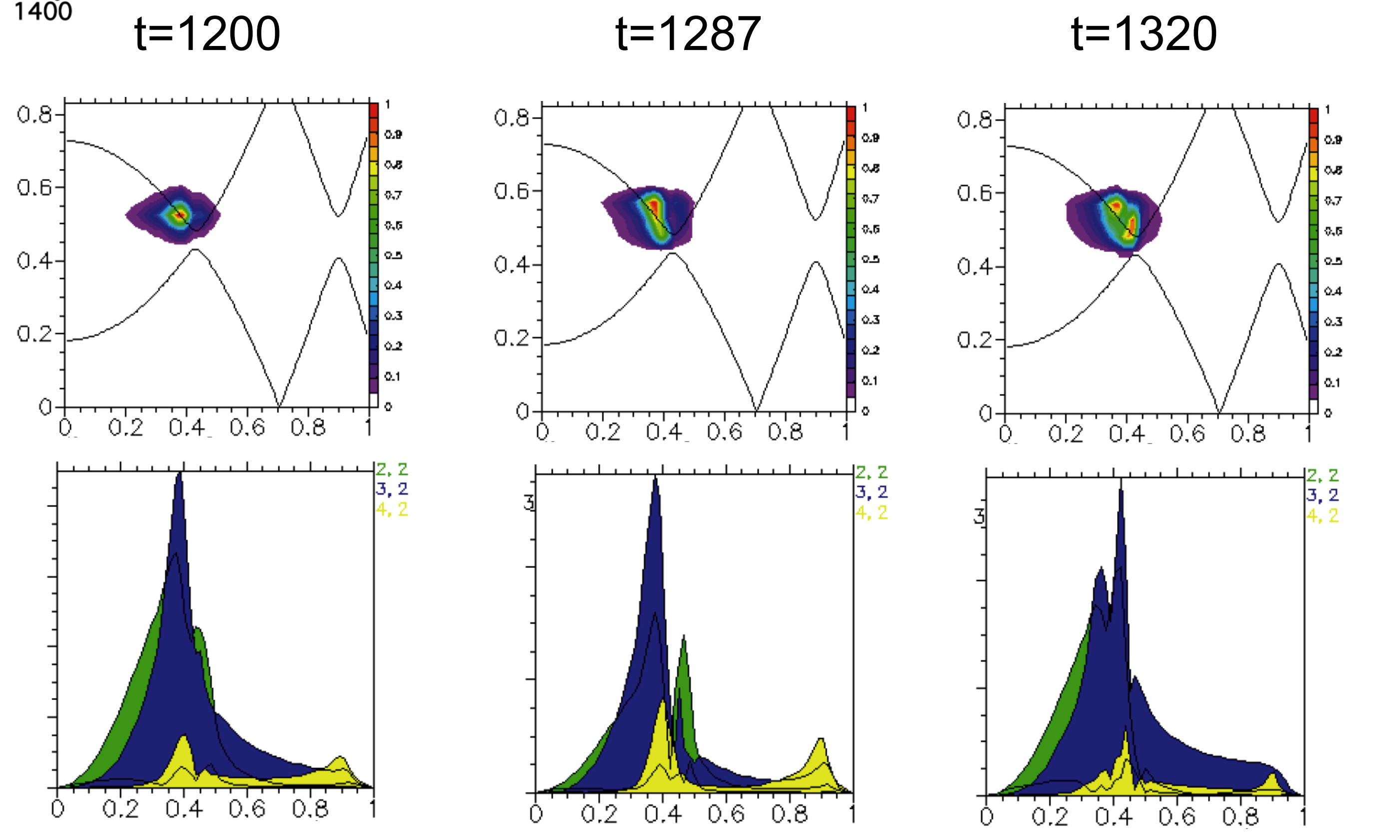


Frequency and radial splitting corresponding to amplitude minima: double resonance (Lecture 3 see also p.18)

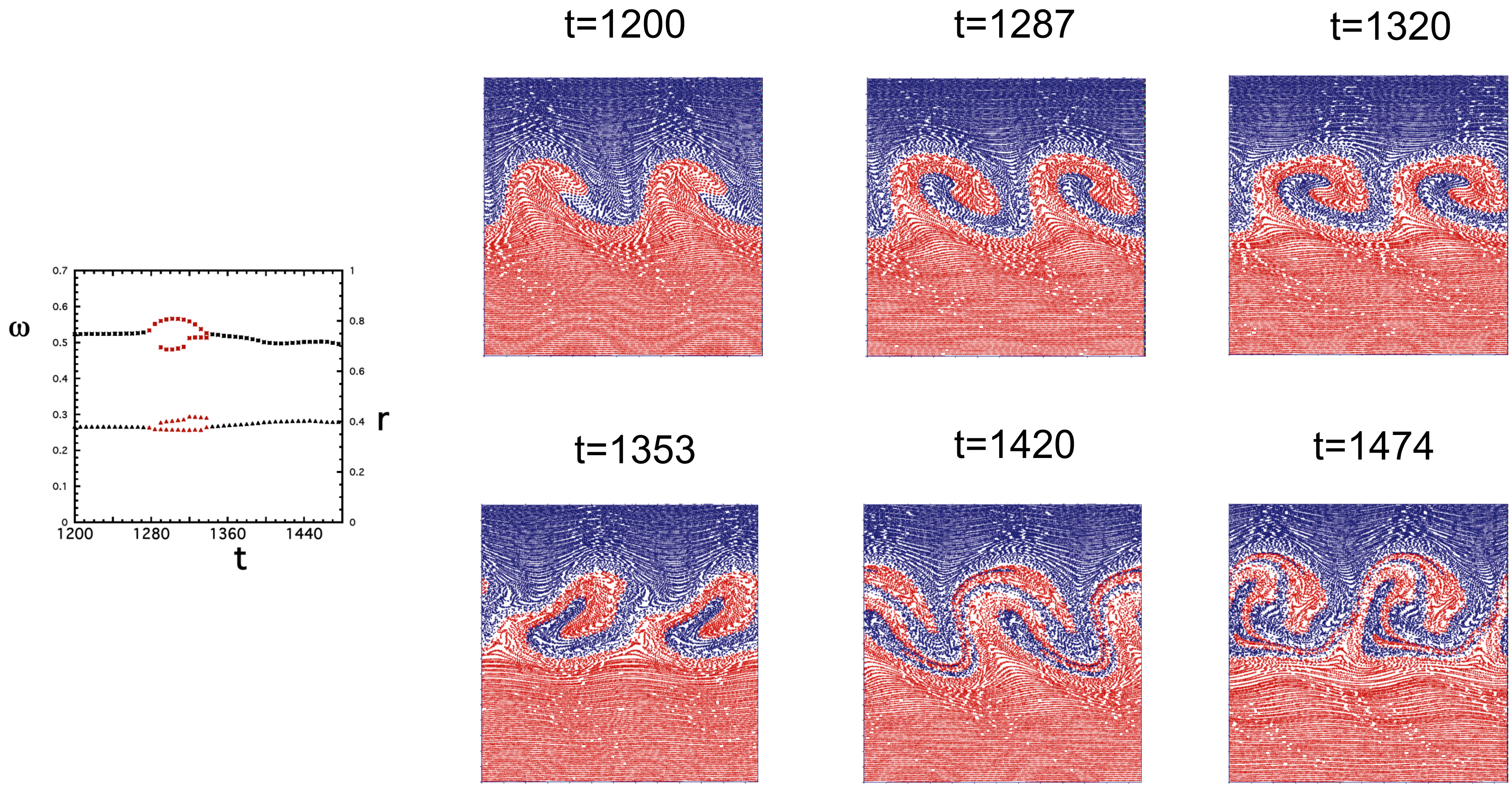


frequency splitting

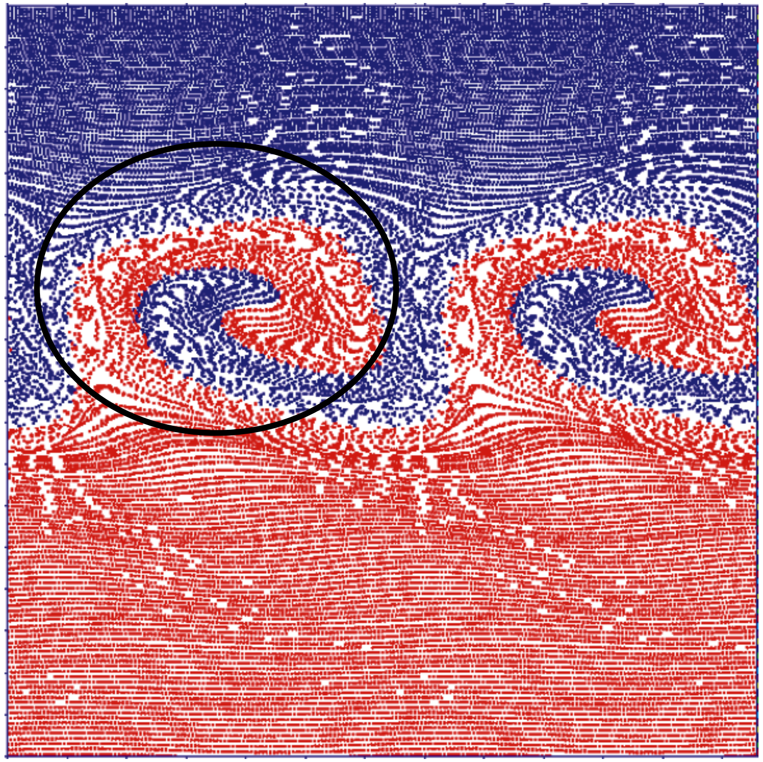
mode structure splitting



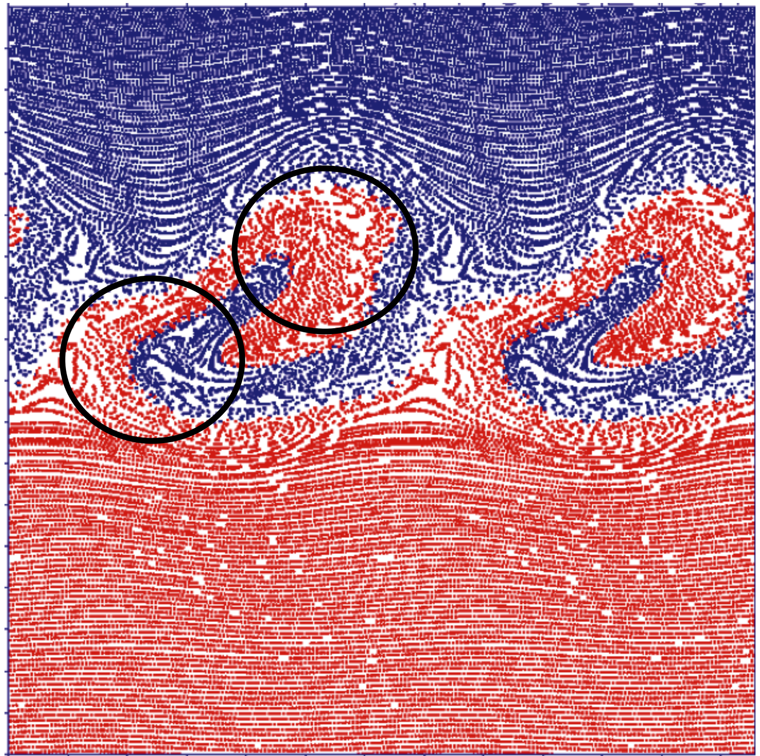
Phase space structure separation as predicted by NL equations derived in Lecture 3



Phase space structure separation: key ingredient is nonadiabatic evolution of structures



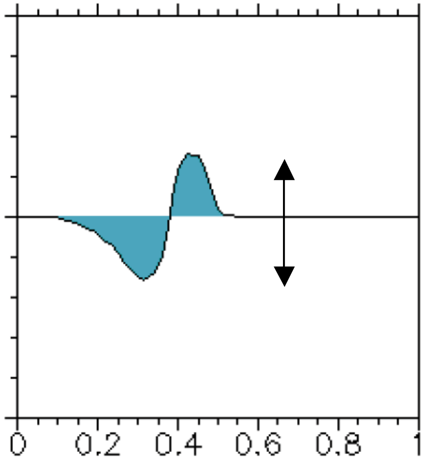
one resonance



two resonances

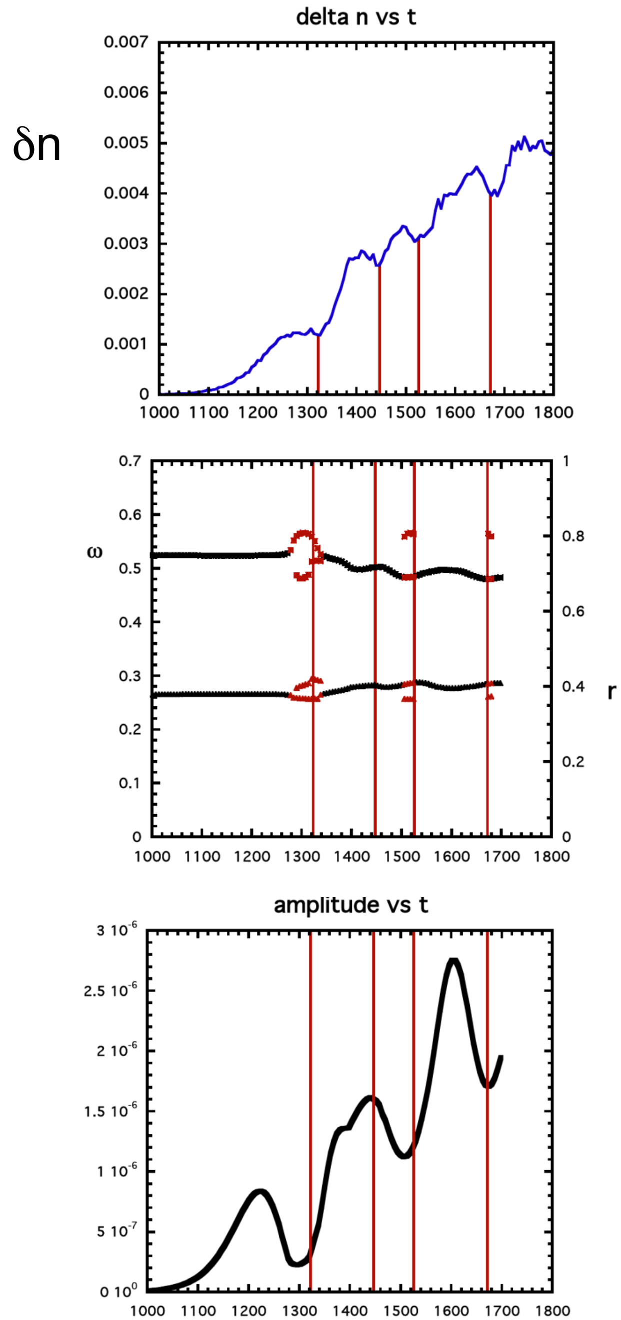
Stop&go resonant particle transport

$$\int_{copassing} dv_{||} \int d\mu \delta F_{00}$$



δn

r



Red lines correspond to beginning of phase-space structure separation

Observations

- Resonant particle radial excursion comparable with mode width
- No proper wave-particle trapping: particles can cross the separatrix in phase space (non adiabatic behavior)
- Stop&go increase of resonant particle radial transport correlated to phase-space structure separation and mode amplitude time evolution
- Frequency and mode structure splitting corresponding to mode amplitude relaxation

References and reading material

F. Zonca and L. Chen, Plasma Physics and Controlled Fusion **48**, 537 (2006).

L. Chen and F. Zonca, Nuclear Fusion **47**, 886 (2007).

X. Wang, S. Briguglio, L. Chen, C. Di Troia, G. Fogaccia, G. Vlad and F. Zonca, Physics of Plasmas **18**, 052504 (2011).