Energetic particles and multi-scale dynamics in fusion plasmas

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2015 Plasma Phys. Control. Fusion 57 014024
(http://iopscience.iop.org/0741-3335/57/1/014024)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 79.42.224.194
This content was downloaded on 29/11/2014 at 09:30

Please note that terms and conditions apply.
Energetic particles and multi-scale dynamics in fusion plasmas

F Zonca\(^1,2\), L Chen\(^2,3\), S Briguglio\(^1\), G Fogaccia\(^1\), A V Milovanov\(^1\), Z Qiu\(^2\), G Vlad\(^1\) and X Wang\(^1\)

\(^1\) ENEA C.R. Frascati, Via Enrico Fermi 45, CP 65-00044 Frascati, Italy
\(^2\) Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou 310027, People’s Republic of China
\(^3\) Department of Physics and Astronomy, University of California, Irvine, CA 92697-4575, USA
\(^4\) Max-Planck-Institut für Plasmaphysik, Boltzmannstraße 2, Garching D-85748, Germany

E-mail: fulvio.zonca@enea.it

Received 28 July 2014, revised 22 September 2014
Accepted for publication 23 September 2014
Published 28 November 2014

Abstract

The role of energetic particles (EPs) in fusion plasmas is unique as they could act as mediators of cross-scale couplings. More specifically, EPs can drive instabilities on the macro- and meso-scales and intermediate between the microscopic thermal ion Larmor radius and the macroscopic plasma equilibrium scale lengths. On one hand, EP driven shear Alfvén waves (SAWs) could provide a nonlinear feedback onto the macro-scale system via the interplay of plasma equilibrium and fusion reactivity profiles. On the other hand, EP-driven instabilities could also excite singular radial mode structures at SAW continuum resonances, which, by mode conversion, yield microscopic fluctuations that may propagate and be absorbed elsewhere, inducing nonlocal behaviors. The above observations thus suggest that a theoretical approach based on advanced kinetic treatment of both EPs and thermal plasma is more appropriate for burning fusion plasmas. Energetic particles, furthermore, may linearly and nonlinearly (via SAWs) excite zonal structures, acting, thereby, as generators of nonlinear equilibria that generally evolve on the same time scale of the underlying fluctuations. These issues are presented within a general theoretical framework, discussing evidence from both numerical simulation results and experimental observations. Analogies of fusion plasma dynamics with problems in condensed matter physics, nonlinear dynamics, and accelerator physics are also emphasized.

Keywords: Alfvén waves, fast particle effects, fusion plasmas, BGK modes, nonlinear phenomena, phase locking, solitons

(Some figures may appear in colour only in the online journal)

1. Introduction

There is increasing interest in fusion plasmas as complex systems [1–4], where many interacting degrees of freedom yield a variety of interesting nonlinear behaviors characterized by a broad range of spatiotemporal scales [5–8].

Adopting the conceptual framework of [2, 3], where magnetized fusion plasmas are assumed to be close to marginal stability, thereby supporting the Ansatz that their dynamics is governed by self-organized criticality (SOC) [9], single transport events (avalanches) may exhibit characteristic aspects of sandpile physics involving SOC [4]. The application of this approach, aimed at characterizing and interpreting strong nonlinear phenomena in fusion as well as astrophysical plasmas, was reviewed in [10, 11] and is beyond the scope of the present work. Here, we emphasize that fusion plasmas may be (and generally are) strongly driven systems, where an ideal SOC state can be destabilized above a certain critical level of external forcing, and that the dynamics may become dominated by bursting periodic relaxation events, for which
the name ‘fishbone-like instability of SOC’ has been suggested [12, 13] by analogy with the nonlinear dynamics of ‘fishbone’ fluctuations in fusion plasmas [14].

Multi-scale dynamics suggests the necessity to go beyond theoretical formulations of fluctuation induced transport based on local closures [15–19] and to replace the corresponding description of turbulent flows connected to thermodynamic forces through nonlocal modifications of Fick’s law [20–22]. In particular, meso-scale dynamics plays a crucial role in turbulent transport for two fundamental reasons: (i) drift-wave turbulence (DWT) itself can spontaneously generate patterns [zonal flows and currents/fields, generally zonal structures (ZS)] [23–26] on scales that are typically larger than the perpendicular (w.r.t. the equilibrium magnetic field $B_0$) wavelength of the underlying fluctuations, $\lambda_p$, but still shorter than the equilibrium nonuniformity scale length; and (ii) the interplay of DWT and ZS may result in turbulence spreading [27–29] and the self-consistent evolution of equilibrium radial profiles in the form of avalanches [30]. These phenomena have been observed in recent global gyrokinetic simulations [21, 30–32].

Less attention has been devoted to multi-scale dynamics and complex behaviors connected with energetic particles and fusion products (EPs) [33–37], despite the fact that EPs are expected to dominate the power balance in burning fusion plasmas. This is the focus of the present work, with the aim of emphasizing general aspects of nonlinear behaviors with their similarities and differences with respect to those discussed above, and of highlighting issues of common interest of fusion science with neighboring fields of physics. In section 2, we first introduce and discuss the spatiotemporal structures of shear Alfvén waves (SAWs) and, more generally, of DWT. Section 3 then addresses the excitation of SAW fluctuations by resonant wave-EP interactions in nonuniform plasmas and the role of the SAW continuous spectrum [38] by which EPs could act as mediators of cross-scale couplings. For these analyses, we adopt the unified theoretical framework of the general fishbone like dispersion relation (GFLDR), recently presented, discussed, and reviewed in [39, 40]. Furthermore, we discuss the importance of the so-called ‘Alfvénic state’, where SAWs can exist in uniform, incompressible MHD plasmas independently of their amplitude due to the cancellation of Reynolds and Maxwell stresses and the incompressible plasma motion produced by SAWs [41–43]. Section 4 introduces nonlinear dynamics of ZS excited by SAWs [44–46] and/or DWT [47, 48] in the presence of EPs. In this case, interesting analogies may be drawn with the competition of nonlinearity versus randomness in the discrete Anderson nonlinear Schrödinger equation (DANSE) (see, e.g. [49–52]). In section 5, we illustrate nonlinear dynamics of energetic particle modes (EPMs) [53], excited within the SAW continuous spectrum as discrete fluctuations at the frequency maximizing wave-EP power exchange above the threshold condition set by continuum damping. EPMs are an example of autoresonance [54] and superradiance [55] in fusion plasmas [38, 56] and suggest similarities with the amplification of a short optical pulse in a free electron laser (FEL) [57]. Finally, section 6 provides a summary and conclusions.

2. Spatiotemporal structures of SAW and DWT

We adopt straight magnetic field line toroidal flux coordinates $(r, \theta, \zeta)$, with $r$ as the radial ‘magnetic flux’ variable and $\theta$ and $\zeta$ as periodic angular coordinates along poloidal and toroidal directions, respectively. Thus, denoting $\mathbf{b} = B_{\perp}/B_0$, the safety factor $q(r)$

$$
q(r) = b \cdot \nabla \zeta / b \cdot \nabla \theta,
$$

(1)
describing the pitch of $B_{\perp}$ field lines, is a flux function. SAWs as well as DWT, represented by $\delta \phi(r, \theta, \zeta) = \sum_{n, m} \exp(i n \zeta - \omega t) \delta \phi_{nm}(r, \theta)$, can be decomposed as [58]

$$
\delta \phi(r, \theta, \zeta) = 2\pi \sum_{l, n \in \mathbb{Z}} \exp\left(-i n q(\theta - 2\pi l)\right) \delta \phi_l(n \theta, 2\pi l) \\
= \sum_{m, n \in \mathbb{Z}} \exp\left(-i m \theta\right) \int \exp\left(-i q(m \zeta - 2\pi n)\right) \delta \phi_{mn}(r, \theta) d\theta \\
= \sum_{m, n \in \mathbb{Z}} \exp\left(-i m \theta\right) \int \exp\left(-i q(m \zeta - 2\pi n)\right) \delta \phi_{mn}(r, \theta) d\theta, \quad (2)
$$

where the mapping $\delta \phi_{mn}(r, \theta); \delta \phi_l(n \theta, 2\pi l) \Rightarrow \delta \phi_{mn}(r, \theta)$ [38–40] of the scalar potential $\delta \phi(r, \theta, \zeta)$ to $\delta \phi_{mn}(r, \theta)$ holds for arbitrary mode numbers and follows from the general properties of the Poisson summation formula [58]. Here, time dependence is suppressed to simplify notation, and the same mapping can be adopted for the parallel vector potential $\delta A_l$ and any other fluctuating field, including the perturbed particle distribution $\delta f$, whose value at equilibrium can be denoted by $f_0$. In equation (2), $\theta$ plays the role of an extended poloidal angle following $B_{\perp}$ field lines. When the radial length scale of $\delta \phi_{mn}(r, \theta)$ is longer than $|nq(\theta)|^{-1}$, equation (2) reduces to the well-known ballooning representation (see e.g. [59, 60]).

Collective SAW fluctuations excited by EPs generally have a dense spectrum of modes with characteristic frequencies and spatial locations [34, 61]. Thus, for SAWs as well as DWT, the mode structure decomposition of equation (2) can be simplified, introducing

$$
\delta \phi_{mn}(r, \theta) = A_n(r) \delta \phi_{0mn}(r, \theta) \approx A_n(r) \delta \phi_{00m}(\theta). \quad (3)
$$

Equations (2) and (3) show that mode structures can be represented by three degrees of freedom: the toroidal mode number $n$, the radial envelope $A_n(r)$, and the parallel (to $\mathbf{b}$) mode structure $\delta \phi_{00m}(\theta)$, with only a slow radial variation on the equilibrium scale length $L$. Correspondingly, nonlinear interactions can take the following three forms: mode coupling between two $n$s, modulation of the radial envelope, and distortion of the parallel mode structure [62, 63]. These three degrees of freedom also have their own characteristic spatio-temporal scales. The toroidal mode number $n$ of most unstable modes is determined by the condition that $\lambda_p$ is of the order of the characteristic orbit width ($q \omega_{\text{pi}}$, the Larmor radius) of destabilizing particle species (EPs for SAWs and thermal plasma particles for DWT/SAWs). The corresponding time scale is set by three-wave coupling across sections, i.e. it tends to infinity in the linear limit, where $n$ is fixed for each separate fluctuation because of the toroidal equilibrium symmetry. The radial envelope $A_n(r)$, meanwhile, varies on ‘meso-scales’
$L_A (\rho_L \ll L_A \ll L)$ that are different for SAWs excited by EPs and DWT, and its formation time scale, $\tau_A$, is typically $\tau_A \sim \tau_L^{-1}$, with $\tau_L$ as the fluctuation linear growth rate. Finally, the parallel mode structure $\delta \Phi_{0n}^L (r, \theta)$ has a typical parallel scale length $\sim q_{LO}$ (the connection length of a torus of major radius $R_0$), while its radial scale is $\sim L$ and its characteristic time is $\sim |\omega_0|^{-1}$ (the inverse mode frequency).

For the sake of simplicity, we assume that the nonlinear time, $\tau_{NL}$, is such that $\tau_{NL} \sim \tau_A \sim \tau_L^{-1} \gg |\omega_0|^{-1}$, which is still a rather general condition of practical interest [38–40, 64–66]. In this way, the parallel mode structure $\delta \Phi_{0n}^L (r, \theta)$ remains essentially unmodified on the shortest time scale (longer time scale dynamics is not considered here), while nonlinear dynamics can be understood as coupling of the remaining relevant degrees of freedom [n and $A_{n}(\theta)$] on time scale $\tau_{NL} \sim \tau_A \sim \tau_L^{-1}$. A possible way to visualize this ‘reduced’ nonlinear dynamics is conceiving the plasma as made of particles and quasi-particles. Unlike particles, which are conserved in number, quasi-particles are ‘rigid’ field-aligned elongated structures carrying energy and momentum that can be emitted and reabsorbed due to the nonuniform particle distribution. Their nonlinear interaction takes place via spectral transfers in $n$-space and change in number density [$\propto A_{n}(\theta)$], generally due to nonlocal processes.

3. SAWs and EPs as mediators of cross-scale couplings

Further to the spatiotemporal structures of SAWs and DWT, discussed in section 2, SAWs in nonuniform toroidal plasmas are characterized by a continuous spectrum with gaps, due to periodic poloidal variation of the Alfvén speed, $v_A$, along $B_0$ field lines and/or finite plasma compressibility. In fact, SAW group velocity is directed along $B_0$ and thus SAWs behave as (electron) wave-packets in a one-dimensional (1D) periodic lattice. In general, frequency gaps are formed when the Bragg reflection condition is met, i.e. when two degenerate and counter-propagating SAWs can form a standing wave for $\omega^2 = \omega_n^2 \pm \omega_{n0}^2$, $n \in \mathbb{N}$ [34]. Continuing this analogy, nonuniform plasma equilibrium plays the role of ‘defects’, causing discrete modes to exist inside the frequency gaps. Different plasma equilibria and related profiles correspond to various types of Alfvén Eigenmodes (AEs) inside different SAW continuum frequency gaps [34] (the Alfvén ‘zoology’ [67]), the first example of which are Toroidal AEs (TAEs) [68].

The importance of AEs lies in the fact that they are readily excited by resonant interactions with EPs since they are essentially unaffected by SAW continuum damping [68–71] and may have strong impact on EP transports and confinement [72, 73]. However, when resonant EP drive excesses the threshold condition set by continuum damping, EPMs [53] are excited within the SAW continuum spectrum as discrete fluctuations. For EPMs, EPs play the role of a free energy source as well as ‘defects’, i.e. EPMs are excited at the resonant EP characteristic frequency [53] and are radially localized so as to maximize wave-EP power exchange [74, 75]. This property of EPMs has important consequences on nonlinear mode dynamics and EP particle transport [76–80] (see section 5).

Linear and nonlinear dynamics of AEs and EPMs excited by EPs in fusion plasmas can be described within the unified theoretical framework of the general fishbone like dispersion relation (GFLDR) [38–40], which reads

$$\left[ iA_n - (\delta \Phi_{0}^{L} + \delta \Phi_{n}^{L} ) \right] A_n (r) = D_n (r, \theta_A, \omega_{n}) A_n (r) = 0. \quad (4)$$

Here, we have assumed the eikonal form $A_n(\theta_{n}) \propto \exp \int n q' \theta_{n}(r) \, dr$ [81, 82] and $D_n (r, \theta_A, \omega_{n})$ plays the role of a local WKB dispersion function, obtained by projection of nonlinear gyro-kinetic quasineutrality condition and vorticity equation on the parallel mode structures $\delta \Phi_{0n}^L (r, \theta)$. In fact, equation (4) is written for a single $n$, but, in general, it should be intended as the $n$-th row of a matrix equation for the complex amplitudes (…, $A_{n}$, …) involving nonlinear mode couplings [33, 38]. It is a kinetic energy principle, which reduces to well-known forms of kinetic MHD energy principle in the appropriate limits [83–87]. Here, it is specialized to localized mode structures, but it can also be written for global fluctuations. The $A_n$ term represents a generalized ‘inertia’ due to the plasma response in the kinetic/singular layer at the radial location(s) where the SAW continuum spectrum is resonantly excited [70, 71]; and the Alfvén fluctuation spectrum is characterized by $|k| \gg |k_\parallel|$. Meanwhile, $\delta \Phi_{0n}^L$ and $\delta \Phi_{n}^{L}$ represent the contributions from the regular regions with $|k| \sim |k_\parallel|$ and can be interpreted as fluid and kinetic ‘potential energies’ [14].

General expressions of $A_n$, $\delta \Phi_{0n}^L$ and $\delta \Phi_{n}^{L}$ are given in [39, 40]. Here, we only note that the various types of AEs are obtained for $\Re A_{n}^2 < 0$, while EPMs are described by $\Re A_{n}^2 > 0$. Consistent with this, the general (kinetic) structure of the SAW continuum spectrum, for $q^2 R_0^2 |\omega|^2 \ll v_A^2$, is given by

$$A_n^2 = k^2 q^2 R_0^2. \quad (5)$$

Adopting the time scale ordering $\tau_{NL} \sim \tau_A \sim \tau_L^{-1} \gg |\omega_0|^{-1}$, discussed in section 2, we can examine the spatiotemporal evolution of SAW wave packets expanding the solutions of equation (4) about the linear characteristics

$$D_n^L (r, \theta_A (r), \omega_{n0}) = 0, \quad (6)$$

where $D_n^L$ is the linearized dispersion function introduced in equation (4). Thus, letting $A_n(\theta_{n}) = \exp(\text{i} \int n q' \theta_{n}(r) \, dr)$, the initial value problem for $A_n(\theta_{n})$ becomes [39, 58]

$$\frac{\partial D_n^L}{\partial \theta_{n0}} \left( i \frac{\partial}{\partial t} \right) A_{n0} + \frac{\partial D_n^L}{\partial n q' \, \frac{\partial}{\partial r} - \theta_{n0}} A_{n0} + \frac{1}{2} \frac{\partial^2 D_n^L}{\partial n q' \, \frac{\partial}{\partial r}} \left[ \left( i \frac{\partial}{\partial r} - \theta_{n0} \right)^2 A_{n0} - \frac{i}{n q' \, \frac{\partial}{\partial r}} A_{n0} \right] = S_{n}(r, t). \quad (7)$$

The $S_{n}(r, t)$ term on the right hand side represents a generic source term, including external forcing, $S_{n}^{\text{ext}}(r, t)$, and/or nonlinear interactions [38, 39, 58]

$$S_{n}(r, t) = S_{n}^{\text{ext}}(r, t) - e^{i \omega_{n} t} \left[ iA_{n}^{NL} - \left( \delta \Phi_{0n}^{NL} + \delta \Phi_{n}^{NL} \right) \right] e^{-i \omega_{n} t} A_{n0}. \quad (8)$$

Thus, equation (7) has the general form of a nonlinear Schrödinger equation [33, 38] with integro-differential
nonlinear terms [39]. For global modes, for which \( \partial_0 D_n = \partial_0 D_0 = \partial_0^2 D_n = 0 \), equation (7) still holds as an evolution equation, provided that \( A_{00} \) is interpreted as mode amplitude at a fixed radial position and \( D_n \) is suitably redefined as global dispersion function [39].

The spatiotemporal structures for SAWs excited by EPs described equation (7) are those generally discussed in section 2 and demonstrate the peculiar role of EPs as mediators between different spatiotemporal scales. In fact, as emphasized in section 1, EPs as fusion products provide the dominant power density source in burning plasmas and, when produced by external heating and current drive systems, they are used to tailor plasma profiles [35]. Thus, they provide a stirring of the system on the macro-scales (L) and are responsible for collective excitation of macroscopic fluctuations with \( n = O(1) \). Furthermore, EPs eventually determine long time scale plasma evolution due to the mutual feedback of fusion reactivity and thermal plasma profiles (see section 6). Energetic particles also provide a free energy source for SAW and drift-Alfvén wave (DAW) [88–92] excitations on the micro- and meso-scales (see section 2), where \( 1 \ll |\nu q|^2 \sim r(\nu L_E) \ll (r(\nu L_L)) \) and \( \rho_L \) are energetic and thermal ion Larmor radii, respectively. It is worthwhile noting that the micro-scale for SAWs/DAWs excited by EPs (\( \lambda_\perp \sim \rho_L \)) is typically the same as that belonging to meso-scale nonlinear structures generated by DWT, e.g. avalanches (see sections 1 and 4) and that, for the same reason, meso-scale nonlinear structures generated by EPs (\( L_A \)) may extend into the macro-scales (L). However, the existence of the SAW continuous spectrum, whose linear and nonlinear structures are accounted for by \( \Lambda_n \) in equation (7), naturally brings an even shorter spatial scale into the problem under consideration, due to mode conversion of SAW/DAW to short wavelength kinetic Alfvén waves (KAWs) with \( \lambda_\perp \sim \rho_L \) at the resonances with the SAW continuum [89]. The importance of short spatial scales is further emphasized by the cancellation of Reynolds and Maxwell stresses in the ‘Alfvénic state’ [41–43], where long wavelength SAWs can exist in uniform, incompressible MHD plasmas independently of their amplitude. Nonlinear behaviors of SAWs/DAWs are thus intimately connected with the mechanisms that ‘break the Alfvénic state’ [45], i.e. plasma compressibility, the violation of ideal MHD constraint (\( \delta E || = 0 \)), and nonuniformity due to plasma equilibrium profiles and magnetic field geometry (see section 4). Because of finite compressibility, a SAW can decay into a sideband SAW and an ion sound wave (ISW) [93]. At sufficiently short wavelength, this MHD process is taken over by the parametric decay of a KAW into a sideband KAW and ISW [89]. The branching ratio of kinetic to MHD decay cross sections is \( \sim (\Omega)|\omega_0|^2 (k_\perp \rho_L)^4 \), with \( \Omega \) as the ion cyclotron frequency. Thus, the kinetic process is dominant for \( |k_\perp \rho_L|^2 > |\omega_0|/\Omega \sim O(10^{-2}) \) [89, 94]. However, most importantly, the transition from MHD to kinetic regimes corresponds to a crucial change in the consequence of SAW/KAW parametric decay on transport [94]. In fact, in the MHD regime [93], the perpendicular sideband wave vector is parallel to that of the original SAW, \( k_\perp \parallel \mathbf{k}_{0,1} \), whereas \( k_\perp \perp \mathbf{k}_{0,1} \) in the kinetic regime [89, 94]. Thus, assuming an initial spectrum of EP driven ‘mode converted KAWs’ (\( \mathbf{k}_{0,1} \approx \mathbf{k}_{0,0} \mathbf{V} \mathbf{r} \)), the MHD parametric decay does not break poloidal angular momentum, \( P_\theta \), conserving, causing negligible cross-field transport, while the kinetic parametric decay tends to isotropize the perpendicular KAW spectrum, thereby causing \( P_\theta \) breaking and significant transport, as observed in numerical simulations [95]. All these elements confirm the unique roles played by EPs as mediators between different spatiotemporal scales in fusion plasmas, also noted in [96] when discussing the properties of the geodesic acoustic mode (GAM) [97] spectrum in the presence of a spatially broad EP beam (see section 4).

The processes discussed so far can be schematically illustrated in figure 1 (see section 2). Figure 1(a) shows fluctuation induced EP transport due to the emission and reabsorption of two similar SAW modes (a) and the same quasi-particle (b).
transported from reabsorption, energy and momentum of the quasi-particle is without further appreciable energy exchange [104].

The EP transport process of figure 1(a) is what is usually observed in experimental conditions, e.g. during ‘TAE avalanches’ in the National Spherical Torus Experiment (NSTX) [98, 99], where rapid EP losses occur in bursts of non-adiabatic frequency sweeping modes [100, 101], which are consistent with the general features of EPMs, following the activity of quasi-periodic TAE fluctuations with limited frequency chirping. The same processes, in a two-stage combination, underlie the idea of ‘alpha channeling’ [102], i.e. ‘the diversion of energy from energetic alpha particles to waves’ [103]. In the first stage, radio-frequency waves are used to extract energy from EPs with negligible radial displacement [103]. In the first stage, radio-frequency waves are used to extract energy from EPs with negligible radial displacement ($r_1 \approx r_2$) [104] and selectively heat the thermal plasma, while in the second stage, after EPs are sufficiently ‘cooled’, low-frequency macroscopic waves are adopted (stimulated emission) to remove the particles from the plasma ($|r_2 - r_1| \sim L$) without further appreciable energy exchange [104].

Figure 1(b) illustrates EP transport processes due to emission and reabsorption of the same quasi-particle $\delta \hat{\phi}_r$. After reabsorption, energy and momentum of the quasi-particle is transported from $r_1$ to $r_2$ and thus occurs on the typical time scale associated with quasi-particle propagation rather than particle transport. Moreover, while propagating between $r_1$ and $r_2$, $\delta \hat{\phi}_r$ can exchange energy and momentum with the EP distribution (by change in the number density, $A_n$) and, more generally, with the plasma equilibrium (corresponding more articulated diagrams are omitted here). Transport processes of figure 1(b) may be connected with KAW that are generally not absorbed near the mode conversion layer in high temperature plasmas [105–107]. These properties further show that mode structures and stability of SAWs are truly kinetic and global in character and that, consequently, transport in fusion plasmas can be significantly affected. Evidence of such KAW ‘action at a distance’ is given by bursting Alfvén instabilities in Wendelstein 7-AS [108, 109], where frequency chirping is attributed to radial KAW propagation and time varying Doppler shift. At the same time, modulation of plasma energy by the Alfvén wave bursts is explained as heat convection due to wave excited at $r_1$ and absorbed at $r_2$ (see figure 4 of [109], reproduced from the original of [108]). This mechanism of ‘spatial channeling’, proposed in [110], is different from that of ‘alpha channeling’ [102] discussed above and has also been invoked for explaining similar evidence in NSTX [109].

4. Nonlinear dynamics of zonal structures

Zonal structures [23–26] are patterns spontaneously generated by DWT as well as SAWs/DAWs on meso-scales ($L_A$; see section 2), which are typically larger than $\lambda_A$ of the underlying fluctuations, but still shorter than the equilibrium length scale $L$ (see section 1). Due to finite magnetic shear, which suppresses convective cells in toroidal fusion plasmas, the prevalent ZS have $n = m = 0$ and can be interpreted as radial corrugations of nonuniform plasma equilibrium [111]. Spontaneously generated ZS scatter original fluctuations into shorter radial wavelength stable domain, thus acting as self-regulation mechanism of plasma instabilities. Continuing the schematic picture proposed in section 2 for DWT and SAWs/DAWs, ZS are generated by ‘decay’ of the $\delta \hat{\phi}_r$ quasi-particle via modulational instability [64], by which ZS ‘modulate’ the number density $A_n(r)$ in the radial direction. The spontaneous emission of ZS, denoted for brevity with $\delta \phi_0$ (having omitted the parallel vector potential $\delta A_{||}$), is depicted in figure 2(a), where it generates a ‘multiplet’ $\delta \hat{\phi}_n$ of the original quasi-particle made of $\delta \hat{\phi}_0$ and its radial ‘modulations’ due to the ZS action on $A_n(r)$.

Due to the properties of the Alfvénic State [41–43] discussed in section 3, the SAW continuous spectrum makes negligible contributions to nonlinear ZS generation. However, finite amplitude waves of the discrete AE spectrum and EPMs may spontaneously excite ZS via modulational instability of the radial envelope $A_n(r)$, due to the breaking of the Alfvénic State via geometry and/or finite plasma/EP compressibility effects [44–46, 113], in addition to the short wavelength kinetic effects discussed in section 3 [45, 89, 94]. Thus, a theoretical approach based on advanced kinetic treatment of both EPs and thermal plasma, accounting for realistic equilibrium geometries and nonuniformities, is necessary for a proper...
analysis of cross-scale couplings in burning fusion plasmas. Meanwhile, ZS generated by EP driven instabilities reflect the different spatiotemporal structures characterizing the SAW/DAW fluctuation spectrum with respect to DWT, as discussed in section 3. A further distinctive element of ZS generated in the presence of EPs is that ZS may not be linearly stable and, hence, the SAWs/DAWs interaction with ZS and DWT should be formulated taking that into account. This is the case in, for example, EP driven GAMs (EGAMs) [114, 115], where the picture of parametric GAM excitation by DWT [116–118] acquires additional twists due to the active/reactive EP response [119]. In fact, recent global nonlinear gyrokinetic simulations of the interplay between EGAMs and DWT [47, 48] have demonstrated that this interaction may be more complicated than expected and that EGAMs could enhance turbulent transport and couple to meso-scale avalanches, with obvious impact on nonlocal properties of the system. The self-consistent nonlinear interplay of SAWs/DAWs, DWT, and ZS in the presence of a non-perturbative EP population is one of the most challenging and open problems in magnetic fusion research. Figure 2(a) also suggests that interactions of ZS with DWT and SAWs/DAWs have two components: a coherent part [64, 116, 120, 121], due to the interaction with the self-generated ZS, and a random contribution [25, 26], due to interaction with ZS produced by other incoherent components of the fluctuation spectrum. Assuming non-dispersive waves and the local character of the associated nonlinear interactions in n-space, equation (7) can be cast in the mathematical form of a discrete Anderson nonlinear Schrödinger equation (DANSE) with randomness, e.g. [50, 51, 122],
\[
\frac{\hbar}{i} \frac{\partial}{\partial t} \psi_n = H_I \psi_n + \beta |\psi_n|^2 \psi_n,
\]
as shown in [38]. Localization of longitudinal and transverse electron plasma waves by disorder in uniform plasmas was investigated first in [123]. Here, the focus is on the competition between nonlinearity and randomness. In equation (9), \(H_I\) is linear Hamiltonian in the tight binding approximation [124] and accounts for random transitions between nearest-neighbor states through a hopping matrix with disorder. In magnetized fusion plasmas, it corresponds to random shearing by ZS incoherent coupling. Meanwhile, \(\beta\) is the strength of nonlinearity and describes self-consistent ZS interaction. The competition of nonlinearity versus randomness in equation (9) yields different behaviors depending on the sign and the value of \(\beta\). For repulsive nonlinearity (\(\beta > 0\), localization by disorder is destroyed and unlimited spreading is observed for \(\beta > \beta_{\text{max}} \approx 1/\ln 2 \approx 1.4427\). This result, which is mathematically exact, is obtained through a topological approximation of the nonlinear Anderson problem in equation (9) using a mapping procedure on a Cayley tree (Bethe lattice) and percolation theory in Hilbert space [122, 125]. For attractive nonlinearity (\(\beta < 0\), instead, soliton solutions are generally obtained [126] as in the typical case of ZS interacting with DW turbulence. Figure 2(b), taking plasma nonuniformity into account, illustrates turbulence spreading via soliton formation due to self-consistent DWT-ZS interaction [112]. There, \(r\) is expressed in units of \(\rho_i\), i.e. \(\rho_i L\) computed at the electron temperature, while \(t\) is in units of the inverse diamagnetic frequency \(\omega_0^\perp\).

5. Phase space zonal structures, phase locking and convective amplification

The nonlinear evolution of the EP distribution function, \(F_0\), illustrated in figure 1(a) (see section 3), is characterized by the formation of ‘phase space zonal structures’ [38, 127–129], i.e. \(\delta_F\), which are the EP phase space counterpart of the ZS discussed in section 4 [111]. When the fluctuation spectrum consists of a predominant quasi-particle \(\delta_F\) (nearby periodic fluctuation), the evolution of \(F_0\) is given by all possible emission and reabsorption processes, obtained by closing the quasi-particle paths in figure 1(a) into a loop and accounting for all possible loops as in the Dyson series shown in the upper part of figure 3(a), which correspond to the solution of the Dyson equation [38, 127, 128, 130]. This is the case, for example, of an externally forced-driven mode or of a single (most) unstable mode, spontaneously ‘emitted’ by \(F_0\), thus, the nonlinear interaction of \(\delta_F\) with \(F_0\) can be schematically depicted as in the lower part of figure 3(a). Focusing on one isolated resonance, the EP response is obtained from the universal description of a non-autonomous system with one degree of freedom [131]. Sufficiently close to marginal stability such that the fluctuation induced (nonlinear) EP displacement, \(\Delta r_{NL}\), satisfies \(\Delta r_{NL} \ll \lambda_L\), the system can be described by the ‘bump-on-tail’ paradigm, widely used by Berk, Breizman, and coworkers (see the review [132]). This paradigm adopts a 1D uniform beam-plasma system with sources and collisions for analyzing nonlinear behaviors of AEs near marginal stability [133] and has been used for interpretation of various experimental evidences of SAW collective instabilities excited by EPs in tokamak plasmas [132–136]. As \(\Delta r_{NL} \ll \lambda_L\), however, self-consistent SAW/DAW nonlinear interactions with EPs, in general, are significantly influenced by nonuniformities in realistic plasmas [38, 80, 127, 128]. In fact, the EP response fully reflects the 3D geometries although it is still described as a non-autonomous system with one degree of freedom, as demonstrated also by numerical simulations [129, 137].

Near marginal stability and with perturbative EP dynamics, AEs evolve adiabatically and, when their frequency ‘chirps’, \(\omega_0 \ll \omega_R\) and \(\omega_0 \ll \omega_\theta\) [138, 139], with \(\omega_R\) the wave-particle trapping frequency. Thus, there exists a conserved phase space action connected with wave-particle trapping and hole-clump pairs are formed [134–136, 138, 139] whose frequency slowly evolves by balancing the rate of energy extraction of the moving holes/clumps in the phase space with the fixed background dissipation. These processes are similar to the EP transport in ‘buckets’ introduced in [140]. They are also similar to ‘autoresonance’ [54, 141], by which a nonlinear pendulum can be driven to large amplitude, evolving in time to instantaneously match its frequency with that of an external drive with sufficiently slow downward frequency sweeping. Here, it is worthwhile noting that original theories on adiabatic frequency sweeping of holes/clumps [134–136, 138] have been significantly extended in [139, 142], taking into account the local TAE radial mode structure near one
is the fluctuation induced EP velocity and \( \epsilon_2 \) is the continuum damping increased [38, 127, 128]. The mode is close to marginal stability, while, at the same time, \( D_n \) reflects EP transport, consistent with nonlinear change in mode structure change and nonlinear frequency shift/chirping.

The phase \( \phi_W(x, \xi) \) (dashed line) is shown as a reference for \( \xi(x, \xi) \) (dash–dotted line) of the EPM soliton; sech(\( \xi \)) and phase.

For increasing EP source strength, non-perturbative EP effects are visible on both AEs and EPMs [74, 76, 145, 146] and have been naturally developed in nonlinear dynamics of phase space holes and clumps, as anticipated in other works [75, 80, 144]. These works, nonetheless, suggest that non-adiabatic chirping is naturally developed in nonlinear dynamics of phase space holes and clumps, as anticipated in other works [75, 80, 144]. Furthermore, [142] extended model equations predict the possible penetration of downward frequency sweeping clumps connected with chirping TAEs into the lower SAW continuum. There, as the mode structure evolves into that of an EPM [39, 40, 53, 74, 145] (see section 3 and further discussions hereafter), a non-perturbative treatment of EP nonlinear dynamics is, however, necessary in general. This can be understood, since the condition \( \Delta r_{NL} \lesssim \lambda_\perp \) can be readily reached for EPs, unless the mode is close to marginal stability, while, at the same time, continuum damping is further increasing [38, 127, 128].

For strong drive and non-perturbative EPs, meanwhile, mode structure change and nonlinear frequency shift/chirping reflect EP transport, consistent with nonlinear change in \( D_n \) (GFLDR; see section 3) on the right hand side of equation (7). For increasing EP source strength, non-perturbative EP effects are visible on both AEs and EPMs [74, 76, 145, 146] and have been recently documented in a comparison of experimental measurements of TAE mode structures in DIII-D with numerical simulation results [147]. The effects of non-perturbative EP dynamics are particularly important for EPMs, which (see section 3) are excited at the resonant EP characteristic frequency [53] and are radially localized near the maximum of the resonant EP drive [74, 75]. Hence, they maximize wave-EP power exchange by preserving the resonance condition throughout the nonlinear evolution. That is, by ‘phase locking’ [38, 127–129, 137, 148, 149] EPM frequency adapts to local resonance condition, which can be shown to imply non-adiabatic frequency chirping, i.e.

\[
\omega_n \approx \delta \tilde{X}_L \nu_{\text{res}} \sim \omega_{NL}, \tag{10}
\]

where \( \delta \tilde{X}_L \) is the fluctuation induced EP velocity and \( \omega_{\text{res}} \) is the space-dependent resonance frequency. Equation (10) may be viewed as an expression of the ‘intrinsic autoresonant character’ of nonlinear EPM dynamics, unlike the usual ‘autoresonance’ case, where the drive frequency is controlled externally [54, 141]. Note that ‘phase locking’ is generally used in the description of ‘autoresonant’ nonlinear evolution of phase space holes/clumps to describe the instantaneous matching of the nonlinear oscillator frequency with that of the external drive [54, 141]. ‘Phase locking’ is also used in [142] to describe the slow (adiabatic) evolution of phase space holes/clumps that, by moving to a lower energy state, compensate the energy dissipation due to background damping. This confirms the analogy of theories on adiabatic frequency sweeping of holes/clumps [134–136, 138, 139, 142] with ‘autoresonance’ [54, 141], discussed above in this section. In both cases, the shortest nonlinear time scale is set by wave-particle trapping, with the conservation of the corresponding phase space action on the longer time scale. Equation (10), meanwhile, illuminates the qualitatively different nature of ‘phase locking’ for sufficiently strong instability drive and non-perturbative EP response. In this case, the wave-particle phase evolves slowly on the shortest nonlinear time scale and wave-particle trapping is de facto suppressed [38, 80, 128, 129, 137, 149].

Because of phase locking, the finite interaction length of EP with EPM is extended; thus, \( \Delta r_{NL} \gtrsim \lambda_\perp \) and the EPM radial envelope evolves as a convectively amplified soliton in an active medium [38, 79, 80, 127]. Assuming the EPM mode structure is radially localized about \( n_0(t) \) with
\[ |\theta_b| \ll 1, \text{ the EPM frequency and growth rate at lowest order are obtained from equation (6) as } D_k^2 (r_0, \omega_0) = 0, \text{ i.e. the mode locks onto the local resonance frequency [53, 74, 75], consistent with equation (10). Then, by letting } A_d (\xi, t) = U (\xi) \exp [-i \omega t] \Delta \alpha (t') d' t' \equiv W (\xi) \exp [i \varphi (\xi) - i \omega (t') d' t'], \text{ with } \Delta \alpha (t) \text{ the complex nonlinear frequency shift, } \xi \equiv k_{\alpha 0} (r - r_0 (t)) \equiv k_{\alpha 0} (r - r_0 (0) - \int_0^t v_k (t') d't'). \]

Here, phase locking has been assumed, i.e. \(|\theta_b | - \delta \vec{X} \cdot \vec{V} \omega_{\alpha 0} r_k^2 \ll 1\). Furthermore, given \(v_k \times \vec{B}_r\), the EP radial \(E \times B\) drift at the peak EPM amplitude, we have \(v_k = \lambda_{\alpha E} \times \vec{B}_r\), with \(\lambda_{\alpha}\) a parameter to be determined and \(k_{\alpha 0} = k_{\alpha 0}^0 = k_{\alpha 0} = k_{\alpha 0}^0 (\omega_0)\) a constant depending on local plasma equilibrium parameters [38, 127]. Thus, the product \(k_{\alpha 0} v_k\) is fixed for a given EPM amplitude and local plasma equilibrium, but equation (7) allows us to compute a one-parameter \((\lambda_{\alpha})\) family of EPM wave-packet solitons. These are shown in figure 3(b), illustrating the solution of equation (11) for \(\lambda_{\alpha} \approx -0.47 + i 1.33\), corresponding to the ground state of the complex nonlinear oscillator. Meanwhile, the EPM frequency, as a solution of equations (6) and (7), is obtained from the condition

\[ D_k^2 (r_0, \omega_0) + P_k (r_0, \omega_0; \ldots) \lambda_{\alpha} \lambda_{\alpha}^2 = 0, \]

with \(P_k (r_0, \omega_0; \ldots)\) a known function also depending on local plasma equilibrium parameters [38, 127, 128]. Thus, \(\omega_0 = \Omega (r_0, \lambda_{\alpha} / \lambda_{\alpha}^2, \ldots)\), with \(\Re \lambda_{\alpha} \) and \(\Im \lambda_{\alpha} \) accounting for, respectively, the (downward) EPM nonlinear frequency shift and the strengthening of mode drive, due to self-consistent EPM-EP interactions. Furthermore, the fastest growing EPM wave packet is obtained for \(\lambda_{\alpha}\) satisfying \(\partial^2 \lambda_{\alpha} / \partial \lambda_{\alpha} = 0\). The convective EPM wave packet amplification is locked onto the radial EP gradient steepening and, hence, it is an avalanche phenomenon [33, 34, 38, 80]. However, unlike SOC avalanches, which can be considered as chain reactions of local events with no characteristic scale, the EPM avalanche scale length is set by phase locking. Convective amplification continues until it is quenched by plasma nonuniformities [38, 127, 128], accounted for by the dependence on \(r_0\), e.g. increased continuum damping, followed by radial EP gradient relaxation [79, 80]. Because of phase locking, EPMs coherently emit EPM quasi-particles as in figure 3(a) over a finite cooperation length and then lose the resonance condition before they begin gaining energy back. This suggest similarities with the amplification of a short optical pulse in a free electron laser (FEL) [57] as, similar to a superradiant FEL pulse, the intensity of EPM wave packet grows as \([r_0 (t) - r_0 (0)]^2\) [80].

6. Conclusions and discussion

In the recent years, significant progress has been made in understanding the individual nonlinear processes that are important in fusion plasmas; the corresponding spatiotemporal structures are reasonably well understood. One of the most significant open problems is how the interplay of these individual processes determine long time scale behaviors. Mutual positive feedback between theory, simulation, and experiment are necessary for further progress.

In this work, we have presented and discussed the multi-scale dynamics and complex behaviors connected with EPs in burning plasmas. Wave–wave and wave-particle nonlinear interactions with EPs are the two routes that yield saturation of SAW/DAW instabilities and cause a variety of fluctuation induced transport processes. Here, we have introduced a conceptual framework that treats such processes as general interactions of quasi-particle and particles and sheds light onto the mechanisms that may cause non-local behaviors. In particular, due to the existence of the SAW continuous spectrum and the properties of the Alfvénic state, the role of EPs in fusion plasmas is unique as they could act as mediators of cross-scale couplings, ranging from the system size macro-scales to the micro-scales of the thermal ion Larmor radius. Important examples of these behaviors are the impact on plasma transport of parametric decays of EP driven ‘mode converted KAWs’; ‘alpha channeling’ as ‘diversion of energy from energetic alpha particles to waves’; and the more recently proposed ‘spatial channeling’, by which an ‘action at a distance’ is possible, with KAW transferring energy and/or momentum between two spatially separated regions. Furthermore, similar to DWT and Alfvén waves, EPs may generate zonal flows and fields and their resonant structure counterpart in the EP phase space. It is suggestive that these meso-scale dynamics may influence the long time scale behaviors of fusion plasmas and the cross-scale couplings between DWT and EP driven SAW/DAW. However, addressing these issues requires accounting for additional physics not discussed in this work, e.g. collisions and finite magnetic compression effects. Here, we emphasize again that effects of plasma nonuniformities and realistic equilibrium magnetic field geometries are crucially important for predicting SAW/DAW nonlinear dynamics and ensuing EP transport in conditions of practical interest. This occurs because the nonlinear displacement of resonant EPs becomes comparable with the perpendicular fluctuation wavelength for sufficiently strong drive. The interplay of SAW/DAW instability mode structures with non-perturbative EP dynamic response also underlies rapid (non-adiabatic) frequency sweeping of phase space zonal structures. Because of phase locking, i.e. the preservation of wave-particle resonance through the nonlinear evolution phase, fast frequency chirping is characterized and accompanied by secular radial motion of a ‘steepened’ pressure profile, consistent with the changing SAW/DAW radial structures. Such a meso-scale ‘avalanche’ could, depending on the plasma nonuniformity and equilibrium magnetic field geometry, redistribute EPs up to the system macro-scales.

The general problems posed by investigations of burning plasma physics often have broader applications than just fusion science. Here, we have given examples of how fusion plasmas may yield a wide class of nonlinear problems, which may be readily extended to neighboring fields, e.g. condensed
matter and accelerator physics, nonlinear dynamics, and plasma astrophysics.

Acknowledgments

This work was supported by the European Union’s Horizon 2020 Research and Innovation Program under grant agreement number 633053 as Enabling Research Project CIP-WP1-ER-01/ENEA_Frascati-01 and by the US DoE, ITER-CN number 633053 as Enabling Research Project CfP-WP14. We are also grateful to Z Guo for granting his permission of reproducing one figure from his original work [112].

References

[34] Chen L and Zonca F 2007 Nucl. Fusion 47 S272
[38] Chen L and Zonca F 2014 Physics of Alfven waves and energetic particles in burning plasmas Rev. Mod. Phys. submitted
[40] Zonca F and Chen L 2014 Phys. Plasmas 21 072121
[41] Alfvén H 1942 Nature 150 405
[51] Iomin A 2010 Phys. Rev. E 81 017601
[53] Chen L 1994 Phys. Plasmas 1 1519
[58] Lu Z X, Zonca F and Cardinalli A 2012 Phys. Plasmas 19 042104
[63] Lin Z, Chen L and Zonca F 2005 Phys. Plasmas 12 056125
[64] Chen L, Lin Z and White R B 2000 Phys. Plasmas 7 3129