Spontaneous excitation of convective cells by kinetic Alfvén waves

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Abstract – Spontaneous excitation of convective cells by kinetic Alfvén waves in a uniform plasma is investigated analytically employing the nonlinear gyrokinetic equations. Self-consistent theoretical analysis demonstrates the novel results that excitation via modulational instability can only occur when the finite ion Larmor radius effects are properly included, and, furthermore, both the electrostatic and magnetostatic convective cells are excited simultaneously. Theoretical predictions are verified with direct numerical simulations; showing excellent agreement in the modulational growth rate and field structures. Significant implications of the present results to the cross-field transport in space and fusion plasmas are also briefly discussed.

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Introduction. – Kinetic Alfvén waves (KAW) \cite{1} and convective cells (CC) are prevalent and fundamental electromagnetic waves and structures in magnetized plasmas. Spontaneous excitation of CC by KAW has been of interest for many years \cite{2,3} due to its important implications to transport across the confining magnetic field \cite{4}, and to the dynamics of the upper auroral ionosphere \cite{5,6}. In recent years, it has received renewed interest since zonal flow \cite{7–9} and zonal current/field \cite{10–14} or, generally, zonal structures \cite{15–17}, which could regulate plasma turbulence and, thereby, the associated transport, have direct correspondence, respectively, with electrostatic CC (ESCC) and magnetostatic CC (MSCC) \cite{17,18}. Thus, CC excitation by KAW can be considered as a paradigmatic example of zonal structure generation by turbulence in magnetized plasmas, and of structure formation effect on fluctuation-induced cross-field transport.

The process by which KAW may generate CC is modulational instability; that is, the reinforcement by nonlinearity of the deviation from wave periodic behavior, which may lead to spectral sidebands and possibly to breaking of the periodic fluctuation into modulated pulses. This process, originally investigated for surface waves in deep water \cite{19}, has been extensively studied in fluid and plasma physics literature, \textit{e.g.} \cite{10–12,20–26}, as the 4-wave parametric interaction of a large pump wave with up- and down-shifted sidebands due to low-frequency modulation.

In the plasma physics literature, another approach has been adopted to analyze the generation of zonal structures by plasma turbulence, based on the wave kinetic equation and the underlying systematic spatial scale separation of turbulent fluctuations and large-scale zonal structures \cite{13,14,27–31}. A thorough discussion of different scalings of modulational instability growth rates with fluctuation amplitude and wave number are given in refs. \cite{12,26} and are beyond the scope of the present work. Here, we merely emphasize that modulational instability is maximized for short-wavelength zonal structures at lengthscales of the order of those typical of plasma turbulence. Thus, systematic spatial scale separation cannot, in general, be assumed and, hereafter, we focus on CC excitation by KAW via 4-wave parametric interactions.

In this letter, we employ the nonlinear gyrokinetic equation (NLGKE) \cite{32} and investigate analytically the spontaneous excitation of CC by KAW. Note that NLGKE, while assuming $|\omega/\Omega_i| \ll 1$, remains valid for arbitrary $|k_{\perp}\rho_i|$. Here, $k_{\perp}$ is the wave number perpendicular to
the confining magnetic field $\mathbf{B}_0$, and $\rho_i = (T_i/m_i)^{1/2}/\Omega_i$, is the ion Larmor radius, with temperature expressed in energy units and $\Omega_i = eB_0/(m_i c)$ the ion cyclotron frequency. Our analysis demonstrates the novel results that spontaneous excitation of CC can only occur when one keeps the crucial finite ion Larmor radius (FILR) effects, i.e., in the $[k_{\perp}\rho_i] \sim \mathcal{O}(1)$ regime [17,18]. In addition, our analysis demonstrates that, in general, ESCC and MSCC are intrinsically coupled. Previous theoretical analyses [33–36], adopting either small FILR ($[k_{\perp}\rho_i] \ll 1$) and/or decoupling of ESCC and MSCC could, thus, lead to conflicting or erroneous predictions on the stability of modal coupling interactions as well as the associated field structures [37–41] (cf. ref. [42] for a relatively recent discussion of these issues). The importance of the $[k_{\perp}\rho_i] \sim \mathcal{O}(1)$ ordering was noted in refs. [38–40] via extending the fluid description of KAW and CC to short wavelengths. Our analysis, however, shows that, due to the finite KAW parallel electric field and frequency mismatch between KAW and sidebands [17,18], the CC parallel vector potential (i.e., the zonal current) in the short-wavelength regime is modified from that given by the extended fluid description [38–40].

The present work, based on the self-consistent NLGKE, thus, represents a novel and proper theoretical treatment on the CC spontaneous excitation by KAW via modulation instabilities. Our analysis, furthermore, allows, self-consistently, the nonlinearly excited CC frequency to be comparable to KAW frequency. The resulting equations governing the coupled nonlinear evolution of KAW and CC are complicated but straightforward to derive from the theoretical framework described in ref. [43]. As we will illustrate in the following, their solution shows that KAW modulation instability is suppressed at $[k_{\perp}\rho_i] \ll 1$; and suggests that filamentary structures including inductive $\delta E_{\parallel}$, with $[k_{\parallel}\rho_i] = 0$ and $[k_{\perp}\rho_i] \sim \mathcal{O}(1)$, can be efficiently generated by CC spontaneous emission from KAW. Thus, in general, KAW modulation instability could operate simultaneously with KAW parametric decay into sound wave and back-scattered KAW, with decay cross-section that also maximizes at $[k_{\perp}\rho_i] \sim \mathcal{O}(1)$ [43,44]. Our theoretical predictions are then, furthermore, verified by direct numerical simulations; showing excellent agreement with analytical predictions in both the modulational growth as well as the self-consistent electromagnetic field structures.

Theory. – Let us consider a uniform Maxwellian plasma immersed in a uniform $\mathbf{B}_0 = B_0 \hat{z}$. We further take $1 \gg \beta_i \sim \beta_i \gg m_e/c_i$ such that $v_{te} \gg v_{Ae} \gg v_{ti}$.

Here, for $j = e, i$ is subscript denoting electrons and ions, $\beta_j = 8\pi T_j/B_0^2$, $v_{je} = B_0/(4\pi n m_j)^{1/2}$ is the Alfvén speed, $v_{ti} = (T_i/m_i)^{1/2}$ is the thermal speed, and other notations are standard. Since frequencies are much smaller than that of the compressional Alfvén waves and $\beta_{j,e} \ll 1$, magnetic compression can be neglected and the scalar potential $\delta \phi$ and the parallel (to $\mathbf{B}_0$; i.e., $\hat{z}$) component of the vector potential $\delta A_{\parallel}$ can be adopted as field variables [43]. We investigate CC spontaneous excitation by a finite amplitude pump KAW, $\mathbf{Ω}_i = (\omega_i, k_B)$, via modulational instability, where the $\mathbf{Ω}$ notation is introduced for brevity to denote the 4-vector $(\omega, \mathbf{k})$ in Fourier space. Let $\Omega_k = (\omega_k, k_z)$ be the CC mode, and $\Omega_+ = (\omega_+ k_+)$ and $\Omega_- = (\omega_- k_-)$ be the respective upper and lower sideband KAW. We note that $k_z \cdot \hat{z} = 0$ and, formally, $|\omega_\pm| \sim |\omega_0|$, while frequency and wave vector matching conditions are $\omega_\pm = \omega_\pm \pm \omega_0$, $k_z = k_z \pm k_0$, and fluctuating fields can be expressed as, e.g., $\delta \phi = \sum_k \exp(i k \cdot \mathbf{x} - i \omega_k t) \delta \phi_k$.

The governing field equations for $\delta \phi$ and $\delta A_{\parallel}$ are the quasineutrality condition and vorticity equation. Assuming one ion species with unit electric charge $e$ and particle density $n$, the quasineutrality condition becomes [43]

$$\left(1 + T_i/T_e\right) \delta \phi_k = T_i/(ne) \int (J_k \delta g_{ke} - \delta g_{ke}) \, \mathrm{d}v. \tag{1}$$

Here, $J_k = J_0(k_0, p)$ with $J_0$ being the Bessel function, and $\rho = v_e/\Omega_e$ and the perturbed particle distribution function, $\delta g_k$, is given by

$$\delta f_k = - \langle e/\mathbf{T} \rangle F_M \delta \phi_k + \exp(-i \mathbf{p} \cdot k) \cdot \delta g_k, \tag{2}$$

with $F_M$ the Maxwellian equilibrium distribution function and $\rho \sim \Omega_e^{-1} \hat{z} \times \mathbf{v}$. The non-adiabatic particle response, $\delta g_k$, satisfies the following Frieman-Chen NLGKE [32]

$$\begin{align*}
(i \langle k||v|| - \omega_k) \delta g_k &= - \langle e/B_0 \rangle \Lambda^{k'}_\rho \langle \delta L_g \rangle_{k'} \delta g_{k'z} \\
- \langle \delta L_g \rangle_{k'} \delta g_{k'} &= - \langle \omega_k \rangle (\mathbf{v}/\mathbf{T}) F_M \langle \delta L_g \rangle_{k'k}, \tag{3}
\end{align*}$$

where $\Lambda^{k'}_{k''} = (k'_\perp \times k''_\perp) \cdot \hat{z}$, $k = k' + k''$, $k'$ and $k''$ are two wave vectors involved in the three-wave interaction with $k$, (...) denotes gyro-phase averaging, $\delta L_g = \exp(\mathbf{p} \cdot \nabla) \delta L$, $\delta L = \delta \phi - v_e \delta A_{\parallel}/c$ and $\langle \delta L_g \rangle_{k} = J_k \delta \rho - v_e \delta A_{\parallel}/c \equiv J_k \delta L_k$. Meanwhile, the vorticity equation can be cast as [43]

$$\begin{align*}
&\delta j_k = -i c^2 \omega_k \frac{k^2}{4\pi} \frac{v_{te}^2 b_k^2}{v_{te}^2} (1 - \Gamma_k) \delta \phi_k = - \Lambda^{k'}_{k''} \delta A_{\parallel k''} \\
&\times \frac{j_{k''} \delta j_{k''} - j_{k'} \delta j_{k'}}{B_0} \delta A_{\parallel k''} + \frac{cc}{B_0} \Lambda^{k''}_{k'} \int [(J_k J_{k'} - J_{k''}) \\
&\times \delta L_{k''} \delta g_{k''} - (J_k J_{k'} - J_k) \delta L_k \delta g_{k''}] \, \mathrm{d}v. \tag{4}
\end{align*}$$

with $\Gamma_k = \int J_k^2 (F_M/n) \, \mathrm{d}v = I_0(b_k) \exp(-b_k)$, $I_0$ is the modified Bessel function, $b_k = k^2_p r_i^2$, and $\delta j_{\parallel k}$ is given by the parallel Ampère’s law

$$\delta j_{\parallel k} = (c/4\pi) \delta L_{\parallel k} \delta A_{\parallel k}. \tag{5}$$

The first nonlinear term on the right-hand side of eq. (4) is the Maxwell stress, while the second one reduces to the well-known Reynolds stress for $b_k \ll 1$ [17].

Equations from (1) to (4) fully determine the evolution of $k_{\parallel} \neq 0$ fluctuations. Noting that the nonlinear response in eq. (1), applied to KAW sidebands, is dominated by electrons, one can obtain [10,17]

$$\sigma_{\perp} \delta \phi_{\perp} - \delta \psi_{\perp} = i \frac{e \sigma_0}{\omega_0 B_0} A_{\perp k} \left( \frac{\delta \phi_0}{\delta \phi_0} \right) (\delta \phi_2 - \delta \psi_2). \tag{6}$$
Here, \( \sigma_k = 1 + \tau (1 - \Gamma_k) \), \( \tau = T_e/T_i \), \( \delta \psi_k = \omega_k \delta A_{\parallel k}/(k_{||c}) \), \( \delta \psi_z = \omega_0 A_{\parallel z}/(k_{||c}) \), the subscripts 0, ± and z refer to KAW pump, sidebands, and CC, respectively; and we have introduced a column-vector notation where upper/lower rows correspond to fields with upper/lower subscripts on the left-hand side. Equation (6) may also be regarded as the generalized Ohm’s law. Meanwhile, substituting the lowest-order ion response, 

\[
\delta g^{(1)}_{k^\perp} = (e/T_i) F_{M_i} J_k \delta L_k,
\]

into eq. (4), we find

\[
(\omega_0 \pm \omega_z) b_{\pm} \left[ \frac{1}{b_k} \left( \frac{k_z^2 v_A^2}{\omega_k^2} \left( \frac{\omega_0 \pm \omega_z}{\omega_k^2} \right)^2 \delta \psi_{\pm} \right) \right] = i \frac{c}{B_0} \frac{\lambda_{k_0}}{\delta \psi \delta \phi},
\]

Equations (6) and (8) rely on the low-\( \beta \) approximation and are consistent with the corresponding equations obtained in refs. [38–40] by extending the fluid description of KAW to short wavelength. They can be considered to hold for \( \beta_e \sim \beta_z \lesssim b_k \) [17,18]. That is, in a parameter range consistent with the CC spontaneous excitation via modulational instability of the pump KAW, which, as shown below, is suppressed at long wavelengths. The pump KAW, meanwhile, satisfies the linear polarization condition, \( \delta \psi_0 = \sigma_0 \delta \phi_0 \), and the mode dispersion relation

\[
\epsilon_{Ak} |_{k = k_0} = \left[ 1 - \frac{\Gamma_k}{b_k} - \sigma_k \frac{k_z^2 v_A^2}{\omega_k^2} \right] = 0.
\]

Thus, eqs. (6) and (8) can be cast as

\[
(\omega_0 \pm \omega_z) b_{\pm} \epsilon_{Ak} \delta \phi_{\pm} = i \frac{c}{B_0} \frac{\lambda_{k_0}}{\delta \psi \delta \phi} \left[ \left( \frac{\Gamma_0 - \Gamma_z}{b_k} - \frac{b_\pm}{b_0} \right) \delta \phi_{\pm} \right] \left. \right|_{k = k_0} \times \left[ \left( \frac{1 - \Gamma_0}{1 \pm \omega_z/\omega_0} \right) \delta \psi_{\pm} \right] + \left( 1 - \Gamma_z \right) \delta \psi_{\pm},
\]

\[
(\omega_0 \pm \omega_z) b_{\pm} \epsilon_{Ak} \delta \psi_{\pm} = i \frac{c}{B_0} \frac{\lambda_{k_0}}{\delta \psi \delta \phi} \left[ \left( \frac{\Gamma_0 - \Gamma_z}{b_k} - \frac{b_\pm}{b_0} \right) \delta \psi_{\pm} \right] \left. \right|_{k = k_0} \times \left[ \left( \frac{1 \pm \omega_z}{\omega_0} \right) \left( 1 - \Gamma_{\pm} \right) \sigma_\pm \delta \phi_{\pm} \right] \left. \right|_{k = k_0} + \frac{\sigma_\pm}{\left[ \left( 1 - \Gamma_z - \frac{b_\pm}{b_0} \right) \delta \psi \right]} \left[ \left( \frac{\delta \phi_0}{\delta \phi_0} \right) \right] \left. \right|_{k = k_0}.
\]

Equations (10) and (11) connect the KAW sideband to the CC and KAW pump fluctuation level. Unlike the KAW pump, KAW sidebands have \( \epsilon_{\pm k} \neq 0 \) because \( \omega_\pm = \omega_0 \pm \omega_z \) is not the KAW normal mode frequency at \( k_\perp = k_\parallel = k_0 \). In fact, by direct substitution of \( \Omega_{\pm} = (\omega_\pm, k_\pm) \) into eq. (9), one can obtain

\[
\frac{b_{\pm} \epsilon_{Ak}}{\omega_0} = \frac{2}{1 - \left( \frac{\Gamma_0}{k_{||c}} \right)} \left( \frac{\pm \omega_z - \Delta_{\pm}}{1 \pm \omega_z/\omega_0} \right),
\]

\[
\Delta_{\pm} = \frac{b_{\pm} \sigma_{\pm} \left( 1 - \Gamma_0 \right) - \sigma_0 b_0 \left( 1 - \Gamma_{\pm} \right)}{2 \sigma_0 b_0 \left( 1 - \Gamma_{\pm} \right)}.
\]

where \( \Delta_{\pm} \) is the leading-order cells of the frequency mismatch of \( \Omega_{\pm} \) and the KAW normal mode frequency. As to the CC mode with \( k_{\parallel} = 0 \), eq. (4) yields

\[
- i \omega_0 \left( 1 - \Gamma_0 \right) \delta \phi_0 = - \frac{b_{\pm} - b_0}{1 - \omega_z/\omega_0} \left( \frac{b_{\pm} - b_0}{b_{\pm} - b_0} \right) \left( \frac{\delta \phi_0}{\delta \phi_0} \right) + \left( \frac{\delta \phi_0}{\delta \phi_0} \right),
\]

(13)

For the CC mode, however, eqs. (1) and (4) are not independent [16,17,45]. \( \delta \psi_z \) is, in turn, determined from the electron parallel current equation. In fact, noting eqs. (2) and (3), and for negligible electron inertia such that \( \delta \phi_{\parallel} \ll \left( |v_\parallel/c| \delta A_{\|} \right) \), one can show that \( \delta g_{k_z} = - (ne/T_e) F_{M_z} \delta \psi_z \) for \( k_z \neq 0 \). Meanwhile, for \( k_z = 0 \),

\[
\delta g_{k_z} = \frac{\epsilon_0 c}{B_0} \frac{\lambda_{k}^{2}}{\delta \psi_{z}} \left[ \left( \frac{\delta \phi_0}{\delta \phi_0} \right) \right] \left. \right|_{k = k_0} + \left( \frac{c}{B_0} \lambda_{k}^{2} \right) \left( \frac{\delta \phi_0 \delta \phi_{\parallel}}{\delta \phi_0 \delta \phi_{\parallel}} \right) \left. \right|_{k = k_0},
\]

(14)

Note that, in the limit of \( k_z^2 v_A^2/\omega_0^2 \ll 1 \), with \( \omega_0 \) the electron plasma frequency, \( \delta \psi_z \) = \( e \) \( \delta f_{e} \) \( \omega_0 \approx 0 \) formally coincides with the parallel electron force balance \( \delta F_{e} = \delta B_{z} \cdot \nabla \delta \phi_{e} \). In that limit, the present gyrokinetic description reduces to the extended fluid description of CC to short wavelength discussed in refs. [38–40]. More generally, the present gyrokinetic analysis of CC is based on eq. (13) and \( \delta \psi_{z} = 0 \), which, from eq. (14), yields [16,17,45]

\[
\delta \psi_z = \frac{i \epsilon_0 c}{\omega_0 B_0} \frac{\lambda_{k_0}}{\delta \psi \delta \phi} \left[ \left( \frac{\delta \phi_0}{\delta \phi_0} \right) \right] \left. \right|_{k = k_0} + \left( \frac{c}{B_0} \lambda_{k_0}^{2} \right) \left( \frac{\delta \phi_0 \delta \phi_{\parallel}}{\delta \phi_0 \delta \phi_{\parallel}} \right) \left. \right|_{k = k_0}.
\]

(15)

Equations (13) and (15), together with eqs. (10) to (12), define field structure and dispersion relation of CC spontaneous excitation via modulational instability of the pump KAW. Note that no additional frequency or wavelength ordering has been assumed for their derivation, other than \( 1 \gg \beta_e \sim \beta_z \gg m_e/m_i, k_z^2 v_A^2/\omega_0^2 \ll 1 \) and \( \beta_e \sim \beta_z \ll b_k \) (consistent with \( k_z^2 v_A^2/\omega_0^2 \ll 1 \) and \( b_k \sim O(1) \)); and the ordering underlying the nonlinear gyrokinetic description [32]. In particular, the frequency mismatch in eq. (12) could be significant; \( |\Delta_{\pm}/\omega_0| \sim O(1) \), as it is controlled by the CC wave vector. Note also that eqs. (10)–(15) are valid for \( |\omega_z/\omega_0| \) up to \( O(1) \). Below, we specialize to \( |\omega_z/\omega_0| \ll 1 \), since we are interested in the threshold for CC spontaneous excitation. We also consider \( k_z \perp b_k \) in order to maximize nonlinear interaction; thus, \( b_z = b_0 + b_\perp, \Delta_{\pm} = \Delta, \Gamma_{\perp} = \Gamma, \Gamma_0 = \Gamma_z \), etc. In this way, letting \( \omega_z = i \gamma_z \), eqs. (13) and (15) can be cast into the following form [17,18]:

\[
\left( \gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} \right) \delta \phi_z = - \alpha_\phi (\delta \phi_z - \delta \psi_z) + \beta_\phi \delta \psi_z,
\]

\[
\left( \gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} \right) \delta \psi_z = - \alpha_\psi (\delta \phi_z - \delta \psi_z) + \beta_\psi \delta \psi_z,
\]

(16)
dictating field structure and dispersion relation of CC spontaneous excitation via modulational instability of the pump KAW. Here \[17,18\]

\[\alpha_\phi = \left[ \frac{c}{B_0} k_x k_{\perp 0} \delta \phi_0 \right]^2 \frac{1}{1 - \Gamma_+} \left[ (\Gamma_0 - \Gamma_+ \frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_+}) + \delta \phi_0 (1 - \Gamma_+) \sigma_0 \right]^{-1}, \]

(17)

\[\beta_\phi = \left[ \frac{c}{B_0} k_x k_{\perp 0} \delta \phi_0 \right]^2 \frac{1}{1 - \Gamma_+} \left[ (\Gamma_0 - \Gamma_+ \frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_+}) + \delta \phi_0 (1 - \Gamma_+) \sigma_0 \right]^{-1}, \]

(18)

\[\alpha_\psi = \left[ \frac{c}{B_0} k_x k_{\perp 0} \delta \phi_0 \right]^2 \frac{\sigma_0}{1 - \Gamma_+} \left[ (\Gamma_0 - \Gamma_+ \frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_+}) + \Delta / \omega_0 \right] \sigma_0 \]

(19)

\[\beta_\psi = \left[ \frac{c}{B_0} k_x k_{\perp 0} \delta \phi_0 \right]^2 \frac{\sigma_0}{1 - \Gamma_+} \left[ (\Gamma_0 - \Gamma_+ \frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_+}) + \Delta / \omega_0 \right] \sigma_0 \]

(20)

The form of eqs. (16) for CC field structure and dispersion relation demonstrate that ESCC (described by \(\delta \phi_z\) only) and MSCC (described by \(\delta \psi_z\) only) are excited simultaneously by KAW, when the threshold condition is exceeded. Furthermore, FILR effects are crucial, since they determine the frequency mismatch between KAW and sideband; and they are responsible for the breaking of the Alfvénic State [46–49], i.e., the non-cancellation of Reynolds and Maxwell stresses that underlies the formation of CC [17,18]. Both assuming decoupled ESCC/MSCC and/or vanishing FILR corrections in the Reynolds stress could lead to erroneous conclusions on the spontaneous excitation of CC by KAW, as demonstrated below (cf., e.g., the recent analysis and summary of previous literature on this topic given in [42]).

In the long-wavelength limit, \(b_0, b_z \ll 1\), eqs. (17)–(20) yield \(\beta_\psi \sim \beta_\phi \sim \mathcal{O}(b_0)\alpha_\phi \sim \mathcal{O}(b_0)\alpha_\phi\), with

\[\left( \frac{\alpha_\phi}{\alpha_\psi} \right) = \left[ \frac{c}{B_0} k_x k_{\perp 0} \delta \phi_0 \right]^2 \frac{(3/4 + \tau) b_0}{b_0 + b_z} \left( \frac{b_0 + b_z}{b_0} \right). \]

(21)

Furthermore, \(\delta \psi_z / \delta \phi_z \simeq \alpha_\psi / \alpha_\phi \simeq b_z / (2b_0 + b_z) = \mathcal{O}(1)\) and the CC dispersion relation is

\[\gamma_\psi^2 + \Delta^2 = (\alpha_\psi - \alpha_\phi) < 0, \]

(22)

with \(\Delta = (\omega / 2) / (3/4 + \tau) b_2\) from eq. (12). Thus, CC excitation is suppressed at long wavelength regardless of the \(\tau = T_e / T_i\) value [17,18], consistently with some of the recent results of ref. [42] and in contrast with the analysis of refs. [39–41]. Artificially suppressing \(\delta \phi_z\) yields the incorrect ESCC dispersion relation, but still the correct qualitative conclusion that ESCC are not spontaneously excited for \(b_k \ll 1\). The analogous assumption that \(\delta \phi_z\) is suppressed, delivers the erroneous MSCC dispersion relation, as well as the erroneous claim that MSCC can be spontaneously excited for \(b_k \ll 1\) [42].

In the short-wavelength limit, \(b_0 \gg 1\), we also have \(\delta \psi_z / \delta \phi_z \sim \mathcal{O}(1)\), i.e., ESCC and MSCC are, again, generally coupled and cannot be excited separately. Furthermore, \(\beta_\psi \simeq \alpha_\psi (1 - \Gamma_+ - 2\Delta / \omega_0)(1 + \Gamma_+ + \Delta / \omega_0)^{-1}\), \(\beta_\phi \simeq \alpha_\phi (1 - \Gamma_+ - 2\Delta / \omega_0)(1 + \Gamma_+ + \Delta / \omega_0)^{-1}\), \(\alpha_\psi \simeq \alpha_\phi (1/2)(1 + \tau)(1 - \Gamma_+ \Delta / \omega_0)^{-1}\), with \(\alpha_\phi \simeq [(c / B_0) k_x] k_{\perp 0} \delta \phi_0 / (b_0 / (b_0 + b_z) (1 + \tau)^{-1} (1 + \Gamma_+ + \Delta / \omega_0)(1 + \Delta / \omega_0)^{-1})\), \(\Delta / \omega_0 \simeq b_z / (2b_0)\). It is readily recognized that \(\beta_\psi = \beta_\phi = 0\) for \(b_z = 0\), which sets an upper bound, \((k_x / k_{\perp 0}) u = 1\) for the CC excitation. For \(b_z \ll 1 \ll b_0\), the lower bound threshold is given by

\[b_{xz}(1 - \Gamma_+ x z) = \frac{4k_{\perp 0}^2 n^2}{2 \Gamma_{xz} - \tau - \Gamma_{xz}}, \]

(23)

where, without loss of generality, the pump KAW has been assumed in the form \(\delta B_{0x} = \hat{y} \delta B_x \sin(\omega_0 t - k_{xz} x - k_{0x} z)\). Thus, from now on, \(k_{\perp 0} = \hat{x} k_x\), while \(k_z = \hat{y} k_y\). Marginal
Fig. 2: (Colour online) Time evolution of $\delta B_y$, $\delta B_z$, and $\delta E_y$ in the pump KAW (black), daughter CC (red), and the matching KAW (green).

Stability curves from eqs. (16) are shown in fig. 1 (left) in the $(k_x,\rho_i,k_y\rho_i)$-plane for fixed $k_{||}\rho_i = 0.02$, $\tau = 1$ and $\beta_\parallel = \beta_\perp = 0.2$ and different values of $\delta B_y/B_0$. There, we also show the upper and lower stability boundaries for CC excitation, i.e., respectively, $(k_u/k_x)_u = 1$ and $k_{0f}\rho_i(\delta B_y/B_0)$ from eq. (23).

Simulation. – We have carried out direct numerical simulations in order to verify the theoretical predictions. The simulations are based on a kinetic ion-fluid electron hybrid model; where ions obey the fully kinetic Vlasov equation and electrons are treated as a massless fluid [50,51]. The computation scheme is fully non-linear, but our analysis of the modulational instability focuses only on the early linear stage of its exponential growth. Fixed parameters are those of fig. 1 (left), lengths are normalized to $\rho_i$, and the time to $\Omega_i^{-1}$. The pump mode is imposed everywhere as a steady driver with $\delta B_0 = (0,\delta B_y,0)\sin(\omega_0 t - k_x x - k_{||0} z)$ and specified wave number and frequency, which are consistent with eq. (9). Note, here, without loss of generality, $\omega_0 > 0$ and $k_{||0} > 0$. From $t = 0$–10, the system is filtered to keep only the Fourier mode $k_0$ of the initial pump, which allows a self-consistent development of the pump field structure. For $t > 10$, more Fourier modes are released in order to examine the excitation of the CC modes. The theoretically derived modulational instability growth rate (continuous line) vs. $\delta B_y/B_0$, based on eqs. (10)–(15), is compared with hybrid simulation results (open circles) for $(k_x,\rho_i,k_y\rho_i) = (0.8,0.6)$ (blue) and $(k_x,\rho_i,k_y\rho_i) = (1.0,0.8)$ (red) in fig. 1 (right). Error bars on numerical growth rates are mostly due to discrete particle noise in the simulations. Figure 2, meanwhile, shows the time evolution of $\delta B_y$, $\delta B_z$, and $\delta E_y$ in the resulting three-wave interaction for a case with constant pump amplitude $\delta B_y/B_0 = 0.5$ at $k_0 = (0.8,0,0.02)$. The black solid curves show the pump mode, for which $\delta B_y$ is constant in time. The $\delta E_y$ component of the pump is two orders of magnitude smaller than $\delta E_z$ (not shown). The excitation of the CC mode, with wave number $k_z = (0.6,0,0)$, is shown with the red curve. Both $\delta B_y$ and $\delta E_y$ increase nearly exponentially with time from $t = 10$, as fitted by the straight dotted line, reaching the saturation levels at $t \simeq 53$. The growth rate is measured to be $\gamma_\parallel/\Omega_i = 0.16$. No power is excited in the $\delta B_y$ component, consistent with $k_z = 0$ in the CC mode. The green curve depicts the matching KAW mode with $k_+ = (0.8,0.6,0.02)$, in which $\delta B_z$, $\delta B_y$, and $\delta E_y$ are all seen to also grow exponentially with $\gamma_\parallel/\Omega_i = 0.16$.

Conclusion. – This work has dealt with spontaneous CC excitation by KAW, i.e., with the linear phase of the modulational instability, which tends to isotropize the perpendicular KAW spectrum as a consequence of the condition $k_z \perp k_{||0}$ for maximizing the CC generation rate. Thus, our results have significant implications on cross-field transport. In fact, when the initial KAW spectrum is strongly anisotropic in the radial direction due to mode conversion at shear Alfvén resonances at the magnetopause [50,51], perpendicular transport can occur only after isotropization of KAW $k_{||}$-spectrum toward the East-West longitudinal direction via spontaneous CC excitation. These results, which crucially rely on FILR effects, have corresponding significant implications for spontaneous generation of zonal structures in fusion devices [15–17], which are the analogue of CC and could regulate plasma turbulence and ensuing transport.

The present work further suggests that filamentary structures including inductive $\delta E_\parallel$, with $|k_z\rho_i| = 0$ and $|k_{\perp}\rho_i| \sim O(1)$, can be efficiently generated by CC spontaneous emission from KAW, in direct competition with KAW parametric decay into sound wave and backscattered KAW [43,44]. Both processes could be operative simultaneously. The corresponding self-consistent nonlinear evolution remains unexplored, with possible implications on vortex and/or current filaments formation. Such studies require fully nonlinear simulations and (gyro-)kinetic analyses. In tokamak plasmas, the additional twist of complex geometry can also importantly modify the properties of the shear Alfvén continuous spectrum and mode converted KAW. Thus, nonlinear generation of zonal structures must be carried out bearing in mind that nonlinear gyrokinetic analyses in realistic magnetic equilibria are needed to provide an accurate description of nonlinear Alfvén wave physics.
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