Proceedings of the 12th Asia Pacific Physics Conference
JPS Conf. Proc. **1**, 011001 (2014)

Physics of Alfvén Waves*

Liu CHEN[1,2] and Fulvio ZONCA[3,1]

[1]*Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou 310027, China*
[2]*Department of Physics and Astronomy, University of California, Irvine CA 92697-4575, U.S.A.*
[3]*Associazione Euratom-ENEA sulla Fusione, C. R. Frascati, C. P. 65-00044 Frascati, Italy*

E-mail: liuchen@zju.edu.cn

(Received August 9, 2013)

Alfvén waves discovered by Hannes Alfvén (1942, *Nature* **150** 405) are fundamental electromagnetic oscillations prevalent in magnetically confined plasmas existing in the nature and laboratories. Alfvén waves play important roles in the heating, stability, and transport of magnetized plasmas. The anisotropic nearly incompressible shear Alfvén wave is particularly interesting since, due to inhomogeneities and geometries in realistic plasmas, its wave spectra consist of both the regular discrete and the singular continuous components. In this paper, we will discuss interesting features of the spectral properties of Alfvén waves, spontaneous wave excitations via resonance with supra-thermal particles, and nonlinear physics issues dealing with both wave-particle as well as wave-wave interactions.

KEYWORDS: Alfvén Waves, Gyrokinetics, Nonlinear Phenomena, Parametric Effects, Mode Coupling, Wave-wave interactions, Wave-particle interactions.

1. Introduction

Hydromagnetic Alfvén waves, discovered by Hannes Alfvén [1], are fundamental low-frequency electromagnetic oscillations in magnetized plasmas. Alfvén waves are found to be prevalent in nature and laboratory plasmas. For example, Alfvén waves are often found to be excited by supra-thermal (energetic) charged particles, produced by either geomagnetic storms or plasma heating, in space and fusion plasmas. Alfvén waves, manifested as geomagnetic oscillations, are also often observed within the Earth's magnetosphere, due to solar wind perturbations. Since Alfvén waves carry electromagnetic perturbations, charged particles can exchange energy and momentum with the waves; resulting in acceleration, heating, and transports across the confining magnetic field. Alfvén waves, thus, have been proposed as the mechanism for the solar corona heating [2]. Meanwhile, rapid loss of energetic/alpha particles due to Alfvén wave instabilities is a major issue of concern in burning fusion plasmas, such as ITER [3, 4].

2. Shear Alfvén Waves in nonuniform plasmas

In uniform plasmas, it is well known that Alfvén waves consist of the compressional and shear Alfvén waves (CAW and SAW). While SAW is anisotropic, with $\omega \simeq k_\parallel v_A$, CAW is isotropic, with $\omega \simeq k v_A$. Here, ω being the wave frequency, $\boldsymbol{k}$ being the wave vector, $k_\parallel \equiv \boldsymbol{k} \cdot \boldsymbol{b}_0$, v_A is the Alfvén speed, $\boldsymbol{b}_0 \equiv \boldsymbol{B}_0/B_0$, and $\boldsymbol{B}_0$ the equilibrium confining magnetic field. Due to its lower frequency and near incompressibility, SAW is generally easier to be excited than CAW. Furthermore, since, for SAW, the wave group velocity is along $\boldsymbol{B}_0$, $\boldsymbol{v}_g \simeq v_A \boldsymbol{b}_0$; and energetic/alpha charged particles also move mainly along $\boldsymbol{B}_0$ with velocities comparable to v_A, it is, in general, easier for SAW than CAW

*Supported by ITER-CN, NSFC, USDOE, and USNSF Grants, and EURATOM.

to satisfy the wave-particle resonance condition. SAW has, thus, been the focus of extensive research in both space and fusion plasmas; and will also be the focus of the present paper.

In nonuniform plasmas; *e.g.*, tokamaks; radial nonuniformities render SAW frequency radially varying; *i.e.*, $\omega^2 = k_\parallel^2(r)v_A^2(r) \equiv \omega_A^2(r)$ and SAW spectrum becomes a continuum. In terms of initial perturbations, fluctuations such as fluid displacement, $\delta\boldsymbol{\xi}$, then behave time asymptotically as

$$|\delta\boldsymbol{\xi}| \propto \exp\left[-i\omega_A(r)t\right] \quad . \tag{1}$$

That SAW oscillations exhibit a continuous spectrum has been clearly demonstrated by satellite observations [5].

Equation (1) indicates that the corresponding radial wave number

$$|k_r| \equiv |d\delta\boldsymbol{\xi}/dr| \,/\, |\delta\boldsymbol{\xi}| \propto \left|\omega_A'(r)\right| t \quad ; \tag{2}$$

i.e., perturbations become "singular" as $t \to \infty$. This "singular" behavior is, of course, consistent with the existence of SAW resonant absorption at steady state [6, 7] and the physics of linear mode conversion to the kinetic Alfvén wave (KAW) [8] with the following dispersion relation, for $b_i = k_\perp^2\rho_i^2 < 1$,

$$\omega^2 = \omega_A^2\left[1 + b_i\left(T_e/T_i + 3/4\right)\right] \quad . \tag{3}$$

In realistic fusion devices, *e.g.*, axisymmetric tokamak plasmas, there are, in addition to the radial nonuniformities, the poloidal asymmetries. A SAW wave-packet propagating along a given magnetic field line will, thus, experience periodic variations; *i.e.*, a lattice symmetry, which then leads to the appearance of "gaps" in the SAW continuum [9]. Additional equilibrium variations, meanwhile, constitute as "defects" to this lattice [10] and, thereby, introduce discrete localized bound states, known as Alfvén Eigenmodes within the SAW gaps [11]. Thus, in realistic fusion plasmas, SAW consists of both continuous and discrete spectra, which are of fundamental importance to linear as well as nonlinear SAW dynamics.

3. Kinetic excitations by energetic particles

In fusion devices, the plasma typically consists of a thermal and a supra-thermal or energetic component, produced by external heating or fusion reactions. These energetic charged particles (EP) have dynamics frequencies, typically, in the range of SAW frequency and, thus, render SAE-EP resonance feasible. In addition, EPs have finite density and pressure gradients, which provide the necessary free energy to excite SAW instabilities if there is finite force acting on the resonant EPs. Since SAW frequency is much lower than the cyclotron frequency, the EPs mainly experience gyrophase-averaged force due to finite $\delta E_\parallel$, $\delta B_\parallel$, and $(\boldsymbol{v}_d \times \delta\boldsymbol{B}_\perp)\cdot\boldsymbol{b}_0$, where $\delta E_\parallel = \delta\boldsymbol{E}\cdot\boldsymbol{b}_0$, $\delta B_\parallel = \delta\boldsymbol{B}\cdot\boldsymbol{b}_0$, $\boldsymbol{v}_d$ denotes the ∇B and $\boldsymbol{\kappa} \equiv \boldsymbol{b}_0\cdot\nabla\boldsymbol{b}_0$ (magnetic curvature) drift, and $\delta\boldsymbol{E}$ and $\delta\boldsymbol{B}$ are perturbed wave fields.

One of the best known example of collective SAW instabilities excited by EPs is the "fishbone" instability observed during the perpendicular beam injection experiments in the PDX toroidal device [12]. Observations provided the following insights: (1) the mode frequency is in resonance with EP precessional frequency in the toroidal direction; leading to secular motion in the major radius direction; and (2) "fishbone" instability is driven by the finite EP pressure gradients. More specifically, since in a toroidal device magnetic field B decreases with the major radius and the magnetic moment $\mu = v_\perp^2/2B$ is an adiabatic invariant, magnetically trapped EPs whose (bounce) average radial position is shifted outward (inward) loose (gain) energy. With EP pressure peaking at inside, there is, thus, a net loss of EP energy and, thereby, a net gain of electromagnetic field energy or instability. Qualitatively speaking, "fishbone" instability is analogous to the Rayleigh-Taylor instability with resonant EP being the heavier-on-the-top fluid and electromagnetic force being the gravitational force.

Analytically, we note first that, given a SAW instability, its eigenmode structure generally consists of two regions; one regular and one singular associated with the SAW continuum. Asymptotically matching solutions in these two regions then leads to the following "fishbone" dispersion relation [13–19]

$$i\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k(\omega) \ ; \tag{4}$$

which takes on a form of generalized energy principle. In Eq. (4), $\Lambda(\omega)$ corresponds to the kinetic energy or inertia; while $\delta\hat{W}_f$ and $\delta\hat{W}_k$ correspond to the potential energy due to thermal plasma and EPs, respectively. In terms of effects, $\Lambda(\omega)$ determines the structure of continuum along with gaps, $\delta\hat{W}_f$ determines the existence of discrete AE; *i.e.*, the strength of "defect", and $\delta\hat{W}_k(\omega)$ provides the instabilities mechanisms as well as additional unstable branches via the frequency dependence of $\delta\hat{W}_k(\omega)$. In a simple but relevant limit, we have $i\Lambda \propto i\omega$ corresponding to damping due to SAW continuum resonant absorption, $\delta\hat{W}_f \simeq 0$, and

$$\delta\hat{W}_k \propto \int \frac{\mathcal{E}\bar{\omega}_d}{\bar{\omega}_d - \omega}\frac{\partial F_{EP}}{\partial r} d\boldsymbol{v} \ , \tag{5}$$

where $\mathcal{E} = v^2/2$, $\bar{\omega}_d \propto \mathcal{E}$ is the precessional frequency of magnetically trapped particles, and $F_{EP} = F_{EP}(\mathcal{E}, r)$ is the EP distribution function. $\mathcal{E}\partial F_{EP}/\partial r$, thus, corresponds to the instability drive associated with the finite EP pressure gradient, and the $(\bar{\omega}_d - \omega)$ term corresponds to the wave-particle resonance as well as gives rise to a new unstable branch due to EP at $\omega \simeq \bar{\omega}_d(\mathcal{E}_{inj})$ with $\mathcal{E}_{inj}$ being the beam-injection energy. This new unstable mode has been dubbed as the Energetic Particle Modes (EPM) [17]. We also note that SAW instabilities (geomagnetic pulsations) can be collectively excited by energetic ring-current protons injected into the Earth's inner magnetosphere during periods of geomagnetic storms [20].

4. Nonlinear physics

Ultimately, we need to develop the nonlinear physics in order to properly construct the SAW turbulent spectrum and assess the associated heating/acceleration and transports. In terms of EP-driven SAW instabilities, there are two possible nonlinear routes. One corresponds to nonlinear SAW-EP interactions. The other route is via nonlinear wave-wave interactions and the resultant spectral wave energy transfer.

4.1 Nonlinear SAW-EP interactions: "bump-on-tail" and "fishbone" paradigms

The nonlinear dynamics of phase-space holes and clumps have been extensively investigated after the pioneering work by Bernstein, Greene and Kruskal (BGK) [21]; and used for analyzing nonlinear behaviors of 1D uniform Vlasov plasmas [22–24]. These studies have been extended to take into account sources and collisions and widely adopted by Berk, Breizman and coworkers (cf. the recent review [25]), proposing the 1D uniform beam-plasma system as paradigm for nonlinear behaviors of AEs near marginal stability [26]. This "bump-on-tail" paradigm offers obvious advantages of adopting a simple 1D system for complex dynamics studies and has been used for interpretation of various experimental evidences of SAW collective instabilities excited by EPs in tokamak plasmas [25–29]. However, the self-consistent nonlinear interactions between the SAW collective instabilities and EPs, in general, consist of complicated dynamics significantly influenced by nonuniformities and geometries in realistic plasmas; which could naturally be expected when fluctuation induced radial excursions of EP orbits become comparable with the mode radial wavelength, $\lambda_\perp$ [30]. The important differences between EP-SAW interactions in nonuniform toroidal plasmas and the beam-plasma system can be described by the nonlinear physics of "fishbone" instability discussed in Sec. 3; *i.e.*, the "fishbone" paradigm.

In this paper, we will focus on the "fishbone" paradigm. Experimental observations and dedicated simulations [31, 32] have suggested the following essential features: (1) as the mode evolves nonlinearly, the mode frequency chirps downward (*i.e.*, $\dot{\omega} < 0$); and (2) correspondingly, resonant

EPs execute sizable radial excursions resulting in a radially much less peaked distributions. More detailed simulation studies [32], further, indicate that, as $\dot{\bar{\omega}}_d \propto \dot{\mathcal{E}} < 0$, wave-EP resonant interaction is maintained even in the nonlinear stage. That is, $\dot{\omega} \simeq \dot{\bar{\omega}}_d$ and one has phase-locking. Wave-EP decoupling in this case occurs only via the finite radial extent of the wave mode structures.

Taking these essential features into consideration, we can then construct a corresponding analytical theory [33, 34]. Within this theoretical framework, the instabilities remain governed by the formally linear "fishbone" dispersion relation, Eq. (4); except F_{0EP} now evolves nonlinearly in time due to radial redistributions of resonant EPs. More specifically, the time evolution of F_{0EP} occurs via emission and absorption of toroidally symmetry-breaking perturbations (toroidal mode number $n = 1$ in the case of "fishbone" mode), and the governing equation is analogous to the well-known Dyson equation [35]. In the simplifying limit of long wave-particle correlation due to phase locking, the governing equation for F_{0EP} becomes

$$\frac{\partial}{\partial t} F_{0EP} \simeq S_{EP}(t) + \left(\frac{\partial}{\partial t}\right)^{-1} \frac{1}{H(r)} \frac{\partial}{\partial r} \left[|\delta u_n|^2 \frac{\partial}{\partial r} H(r) F_{0EP} \right] \quad . \tag{6}$$

Here, $S_{EP}(t)$ stands for collisions and EP source, $H(r)$ is an $O(1)$ geometrical constant, and δu_n is the radial velocity due to plasma displacement. Equation (6) takes on the form of a wave equation; suggesting the resonant EPs convect outward with the radial speed $\simeq |\delta u_n| \propto |\delta B_{\perp n}|$. As the "fishbone" mode has a finite radial mode width, r_s; it then can be expected that the instability growth will diminish when the resonant EPs leave the mode structure in a time shorter than the characteristic time scale for the wave-EP energy exchange; γ_L^{-1}, with γ_L being the linear instability growth rate. The saturation amplitude, therefore, is estimated to be $|\delta u_n|_{sat} \sim r_s \gamma_L$. Simulation results [36] have indeed observed the predicted scaling with γ_L.

4.2 Nonlinear wave-wave interactions

One useful perspective on the nonlinear wave-wave interactions among SAWs is analyzing how to break the pure Alfvénic state. This is an unique feature of nonlinear SAW where, under some minimal constraints, dominant nonlinearities vanish and finite-amplitude SAW can self-consistently sustain itself on a long-time scale. In the followings, we will first briefly review the pure Alfvénic state, and discuss how to break it. More specifically, we discuss the effects of finite ion compressibility and the SAW parametric decay process. We then demonstrate the importance of geometries by examining the nonlinear excitation of zonal structures by Toroidal Alfvén Eigenmodes (TAE).

Let us consider an infinite uniform plasma, using the one-fluid magnetohydrodynamic (MHD) description. The governing equation is then

$$\varrho_m(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} \cdot \underline{\underline{\boldsymbol{P}}} + \boldsymbol{J} \times \boldsymbol{B}/c \quad , \tag{7}$$

where ϱ_m is the mass density, $\boldsymbol{u}$ the fluid velocity, $\underline{\underline{\boldsymbol{P}}}$ the pressure tensor, and $\boldsymbol{J}$ is the current density. Expressing quantities in terms of the equilibrium and perturbed components; *e.g.*, $\varrho_m = \varrho_{m0} + \delta\varrho_m$, and noting $\boldsymbol{u}_0 = 0$, $\boldsymbol{J}_0 = 0$, and $\boldsymbol{\nabla} \cdot \underline{\underline{\boldsymbol{P}}}_0 = 0$, Eq. (7) becomes

$$(\varrho_{m0} + \delta\varrho_m)(\partial_t + \delta\boldsymbol{u} \cdot \boldsymbol{\nabla})\delta\boldsymbol{u} = -\boldsymbol{\nabla} \cdot \delta\underline{\underline{\boldsymbol{P}}} + (\delta\boldsymbol{J} \times \boldsymbol{B}_0 + \delta\boldsymbol{J} \times \delta\boldsymbol{B})/c \quad . \tag{8}$$

We now note that, for SAWs, magnetic compression is generally negligible; *i.e.*, $\delta\boldsymbol{B} \simeq \delta\boldsymbol{B}_\perp$. Furthermore, noting SAW is nearly incompressible and, hence, $\boldsymbol{\nabla} \cdot \delta\boldsymbol{u} \simeq 0$ or $\delta\varrho_m \simeq 0$ and $\boldsymbol{\nabla} \cdot \delta\underline{\underline{\boldsymbol{P}}} \simeq 0$, Eq. (8) can then be cast into the following form

$$\varrho_{m0}\partial_t\delta\boldsymbol{u} = \boldsymbol{F}_p^{(2)} + \delta\boldsymbol{J} \times \boldsymbol{B}_0/c \quad . \tag{9}$$

Here, $\boldsymbol{F}_p^{(2)}$ is the nonlinear ponderomotive force, given by

$$\boldsymbol{F}_p^{(2)} = -\boldsymbol{\nabla}|\delta\boldsymbol{B}|^2/8\pi - \mathbf{Mx} - \mathbf{Re} \ \ ; \tag{10}$$

with

$$\mathbf{Mx} \simeq -(\delta\boldsymbol{B}_\perp \cdot \boldsymbol{\nabla})\delta\boldsymbol{B}_\perp/4\pi \ \ , \tag{11}$$

and

$$\mathbf{Re} \simeq \varrho_{m0}(\delta\boldsymbol{u} \cdot \boldsymbol{\nabla})\delta\boldsymbol{u}_\perp \ \ . \tag{12}$$

Mx and **Re** are, respectively, the Maxwell and Reynolds stresses.

If we further adopt the ideal MHD assumption $\delta E_\parallel = 0$, we can then derive, from Eq. (9), the following nonlinear SAW equation

$$c^2\left[(\boldsymbol{b}_0 \cdot \boldsymbol{\nabla})^2 - v_A^{-2}\partial_t^2\right]\nabla_\perp^2\delta\psi + 4\pi\partial_t(\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)}) = 0 \ \ ; \tag{13}$$

where the nonlinear term $\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)}$ is given by

$$\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)} = -(c/B_0)\boldsymbol{b}_0 \cdot \boldsymbol{\nabla}\times[\mathbf{Re} + \mathbf{Mx}] \ \ , \tag{14}$$

and

$$\delta\boldsymbol{u}_\perp = (c/B_0)\boldsymbol{b}_0 \times \boldsymbol{\nabla}\delta\psi \ \ .$$

Equation (14) indicates that, for perturbations satisfying the following Walén relation [37],

$$\delta\boldsymbol{u}_{\perp W}/v_A = \pm\delta\boldsymbol{B}_{\perp W}/B_0 \ \ , \tag{15}$$

$\mathbf{Re} + \mathbf{Mx} = 0$ and $\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)} = 0$. Equation (15), meanwhile, corresponds to

$$[\partial_t \mp v_A\boldsymbol{b}_0 \cdot \boldsymbol{\nabla}]\delta\psi_W = 0 \ \ . \tag{16}$$

That is, a SAW propagating either along or against $\boldsymbol{B}_0$ satisfies the nonlinear SAW equation, Eq. (13) for any arbitrary amplitude, $\delta\psi_W$. This is the celebrated pure Alfvénic state. In other words, within the present constraints, a large-amplitude SAW, satisfying $\omega = \pm k_\parallel v_A$ can exist for a long time scale without being broken by nonlinear processes. To break this pure Alfvénic state, one would need to either introduce higher-order nonlinearities and, thereby, examine dynamics at longer time scales [38, 39] or, as adopted in the present work, remove the fundamental constraints employed here; *i.e.*, incompressibility, ideal MHD assumption, and homogeneity/geometry.

Since ion sound wave (ISW) compresses plasmas along $\boldsymbol{B}_0$, we may break the pure Alfvénic state by considering parametric decays of a pump SAW, $\Omega_0 = (\omega_0, \boldsymbol{k}_0)$, to a daughter SAW, $\Omega_- = (\omega_-, \boldsymbol{k}_-)$, and an ISW, $\Omega_s = (\omega_s, \boldsymbol{k}_s)$; where $\Omega_- = \Omega_s - \Omega_0$. This decay process was analyzed first by Sagdeev and Galeev [40] in the macroscopic MHD limit. The result indicates that the decay instability is a backscattering process; *i.e.*, Ω_- is propagating counter to Ω_0 along $\boldsymbol{B}_0$; and the dispersion relation is given by

$$\epsilon_s\epsilon_{A-} = C_I|e\delta\psi_0/T_e|^2 \ \ ; \tag{17}$$

where ϵ_s and ϵ_{A-} are, respectively, the linear dielectric constants of ISW and SAW,

$$C_I = O\left(k_\perp^2\rho_i^2\right)\cos^2\theta \ \ , \tag{18}$$

and θ is the angle between $\boldsymbol{k}_{0\perp}$ and $\boldsymbol{k}_{-\perp}$. Thus, the decay maximizes around $\theta = 0, \pi$; *i.e.*, when $\boldsymbol{k}_{-\perp}$ is nearly parallel to $\boldsymbol{k}_{0\perp}$. This, as we will see later, carries important implications to the transport processes.

In the micro/meso scales, electron-ion decoupling is enhanced and SAW transforms into kinetic Alfvén wave (KAW). The corresponding parametric decay process for KAWs was carried out first in the drift-kinetic limit [8] and, recently, in the gyrokinetic limit [41]. The corresponding parametric dispersion relation is then given by

$$\epsilon_{sK}\epsilon_{A-K} = C_K |e\delta\psi_0/T_e|^2 \quad ; \tag{19}$$

where ϵ_{sK} and ϵ_{A-K} are, respectively, the linear kinetic dielectric constants of ISW and KAW, and, for $|k_\perp \rho_i| \lesssim 1$,

$$C_K = O\left[\left(\frac{\Omega_i}{\omega_0}\right)^2 (k_\perp \rho_i)^6\right] \sin^2\theta \quad , \tag{20}$$

Note that Eq. (20) indicates that the KAW decay process maximizes around $\theta \simeq \pm\pi/2$; *i.e.*, $\boldsymbol{k}_{0\perp} \perp \boldsymbol{k}_{-\perp}$, and $|k_\perp \rho_i| \sim O(1)$. Both predictions have recently been observed in numerical simulations [42].

Equations (18) and (20) indicate that the decay instabilities are quantitatively and qualitatively different in the MHD and kinetic regimes. Specifically, we note that for $1 \gtrsim |k_\perp \rho_i|^2 > |\omega_0/|\Omega_i| \lesssim O(10^{-2})$, typically; we have $C_K > C_I$ and the nonlinear decays enter into the kinetic regime. More significantly, as noted above, $\boldsymbol{k}_{-\perp} \perp \boldsymbol{k}_{0\perp}$ in the kinetic regime, in contrast to $\boldsymbol{k}_{-\perp} \parallel \boldsymbol{k}_{0\perp}$ in the MHD regime. Taking, for example, Ω_0 being a radially localized Alfvén eigenmode, such that $\boldsymbol{k}_{0\perp} \simeq k_{0r}\hat{\boldsymbol{r}}$, the MHD theory would then predict the decay SAW also tends to peak around $\boldsymbol{k}_{-\perp} \simeq k_{-\perp}\hat{\boldsymbol{r}}$ and, hence, produces little transport. On the contrary, the kinetic theory would predict $\boldsymbol{k}_{-\perp} \simeq k_{-\perp}\hat{\boldsymbol{\theta}}$ and, hence, significant transports.

Finally, in order to examine how geometries break the pure Alfvénic state, we shall consider specifically the case of spontaneous excitation of zonal structures by TAE [43]. Zonal structures are coherent micro/meso-scale radially varying corrugations. Typically, they are spontaneously excited via the amplitude modulations of the driving waves or instabilities. As the zonal structures tend to scatter the driving waves or instabilities to the generally stable short radial wavelength regime, they provide a route to nonlinear damping; and, thus, an important self regulatory mechanism of plasma instabilities. Here, we shall adopt the ideal MHD and incompressibility constraints. In this model, as noted earlier, SAW is composed of a continuous and discrete spectra; with TAE being the discrete AE considered here. The zonal structure, meanwhile, is the zero-frequency one [44]. As the SAW continuum with $\omega^2 \simeq k_\parallel^2 v_A^2$ satisfies the pure Alfvénic state within the given constraints, there is negligible coupling to the zonal structures. On the other hand, for discrete AEs residing within the SAW continuum gap such as TAE, effects due to Reynolds and Maxwell stresses do not cancel and zonal structures can indeed be spontaneously excited. Furthermore, in this specific case, it is found that, due to kinetic and geometry effects, the zonal currents play more significant role than the often cited zonal flows [44, 45].

5. Summary and Discussions

In this paper, we present essential features of linear as well as nonlinear physics of the anisotropic shear Alfvén waves (SAW) in realistic magnetic confined plasmas. We show how nonuniformities and geometries have rendered understanding the underlying physics mechanisms into a rich and interesting subject of investigations.

In the linear physics, we note that SAW spectral features consist of continuous spectra, frequency gaps, and discrete Alfvén eigenmodes (AEs) within the gaps. Collective SAW instabilities; both the AEs and the energetic particle modes (EPM); *e.g.*, the "fishbone" mode, can be readily excited by supra-thermal (energetic) particles via wave-particle resonances by tapping the finite energetic particle pressure gradient.

In the nonlinear physics, we illustrate the nonlinear wave-particle interactions using the "fishbone" paradigm; where the wave-particle resonance is kept over a long-time scale (*i.e.*, phase locking) due to the mode frequency chirping. Wave-EP decoupling then occurs due to the limited spatial extent of the mode. As to nonlinear wave-wave interactions, we first present the pure Alfvénic state and then discuss various effects which can break the Alfvénic state and result in significant nonlinear interactions; such as parametric decays and spontaneous excitations of zonal structures.

Looking back, investigations on the physics of Alfvén waves have clearly been benefitted by the cross fertilization between space and fusion plasma physics research, and this positive and fruitful trend can be expected to continue. Looking ahead, furthermore, with the recent rapid advances in diagnostics in both laboratory devices and satellites as well as in the capabilities of careful large-scale first-principle simulations, one may expect even more significantly novel and deeper understandings in the physics of Alfvén waves; which is both intellectually exciting and practically important to laboratory fusion energy and space physics research.

Acknowledgments

This work was supported by ITER-CN, NSFC, USDOE, and USNSF Grants, and EURATOM.

References

[1] H. Alfvén: Nature **150** (1942) 405.
[2] J. A. Ionson: Astrophys. J. **254** (1982) 318.
[3] K. Tamabechi, J. R. Gilleland, Y. A. Sokolov, R. Toschi, and ITER Team: Nucl. Fusion **31** (1991) 1135.
[4] A. Fasoli, C. Gormenzano, H. L. Berk, B. N. Breizman, S. Briguglio, D. S. Darrow, N. N. Gorelenkov, W. W. Heidbrink, A. Jaun, S. V. Konovalov, R. Nazikian, J. Noterdaeme, S. E. Sharapov, K. Shinohara, D. Testa, K. Tobita, Y. Todo, G. Vlad, and F. Zonca: Nucl. Fusion **47** (2007) S264.
[5] M. J. Engebretson, L. J. Zanetti, T. A. Potemra, W. Baumjohann, H. Lühr, and M. H. Acuna: J. Geophys. Res. **92** (1987) 10053.
[6] L. Chen and A. Hasegawa: Phys. Fluids **17** (1974) 1399.
[7] L. Chen and A. Hasegawa: J. Geophys. Res. **79** (1974) 1024.
[8] H. Hasegawa and L. Chen: Phys. Fluids **19** (1976) 1924.
[9] C. E. Kieras and J. A. Tataronis: J. Plasma Phys. **28** (1982) 395.
[10] L. Chen and F. Zonca: Nucl. Fusion **47** (2007) S727.
[11] C. Z. Cheng, L. Chen, and M. S. Chance: Ann. Phys. (N.Y.) **161** (1985) 21.
[12] K. McGuire, R. Goldston, and M. Bell et al.: Phys. Rev. Lett. **50** (1983) 891.
[13] L. Chen, R. B. White, and M. N. Rosenbluth: Phys. Rev. Lett. **52** (1984) 1122.
[14] H. Biglari and L. Chen: Phys. Rev. Lett. **67** (1991) 3681.
[15] M. Liljeström and J. Weiland: Phys. Fluids B **4** (1992) 630.
[16] S. Tsai and L. Chen: Phys. Fluids B **5** (1993) 3284.
[17] L. Chen: Phys. Plasmas **1** (1994) 1519.
[18] L. Chen and F. Zonca: Phys. Scr. **T60** (1995) 81.
[19] F. Zonca and L. Chen: Plasma Phys. Control. Fusion **48** (2006) 537.
[20] K. Takahashi: Adv. Space. Res. **8** (1988) 427.
[21] I. B. Bernstein, J. M. Greene, and M. D. Kruskal: Phys. Rev. **108** (1957) 546.
[22] H. L. Berk, C. W. Nielson, and K. W. Roberts: Phys. Fluids **13** (1970) 980.
[23] T. H. Dupree: Phys. Rev. Lett. **25** (1970) 789.
[24] T. H. Dupree: Phys. Fluids **15** (1972) 334.
[25] B. N. Breizman and S. E. Sharapov: Plasma Phys. Control. Fusion **53** (2011) 054001.
[26] H. L. Berk and B. N. Breizman: Phys. Fluids B **2** (1990) 2246.
[27] H. L. Berk, B. N. Breizman, and M. Pekker: Phys. Rev. Lett. **76** (1996) 1256.
[28] H. L. Berk, B. N. Breizman, and N. V. Petiashvili: Phys. Lett. A **234** (1997) 213.
[29] B. N. Breizman, H. L. Berk, M. Pekker, F. Porcelli, G. V. Stupakov, and K. L. Wong: Phys. Plasmas **4** (1997) 1559.
[30] F. Zonca, S. Briguglio, L. Chen, G. Fogaccia, and G. Vlad: Nucl. Fusion **45** (2005) 477.

[31] G. Y. Fu, W. Park, H. R. Strauss, J. Breslau, J. Chen, S. Jardin, and L. E. Sugiyama: Phys. Plasmas **13** (2006) 052517.

[32] G. Vlad, S. Briguglio, G. Fogaccia, F. Zonca, V. Fusco, and X. Wang: Nucl. Fusion **53** (2013) 083008.

[33] F. Zonca, P. Buratti, A. Cardinali, L. Chen, J. Dong, Y. Long, A. V. Milovanov, F. Romanelli, P. Smeulders, L. Wang, Z. Wang, C. Castaldo, R. Cesario, E. Giovannozzi, M. Marinucci, and V. Pericoli Ridolfini: Nucl. Fusion **47** (2007) 1588.

[34] F. Zonca, P. Buratti, A. Cardinali, L. Chen, J. Dong, Y. Long, A. V. Milovanov, F. Romanelli, P. Smeulders, L. Wang, Z. Wang, C. Castaldo, R. Cesario, E. Giovannozzi, M. Marinucci, and V. Pericoli Ridolfini: arXiv:0707.2852v1 [physics.plasma-ph] (2007).

[35] L. M. Al'tshul' and V. I. Karpman: Sov. Phys. JETP **22** (1966) 361.

[36] G. Vlad, S. Briguglio, G. Fogaccia, F. Zonca, C. Di Troia, V. Fusco, and X. Wang: Proceedings of the 24th International Conference on Fusion Energy, 2012, http://www–naweb.iaea.org/napc/physics/FEC/FEC2012/papers/77_THP603.pdf.

[37] C. Walén: Ark. Mat. Astron. Fys. **30A** (1944) 1.

[38] H. Alfvén: *Cosmical Electrodynamics* (Clarendon, Oxford, UK, 1950).

[39] W. M. Elsasser: Rev. Mod. Phys. **28** (1956) 135.

[40] R. Z. Sagdeev and A. A. Galeev: *Nonlinear Plasma Theory* (W. A. Benjamin Inc., 1969).

[41] L. Chen and F. Zonca: Europhys. Lett. **96** (2011) 35001.

[42] Y. Lin, J. R. Johnson, and X. Wang: Phys. Rev. Lett. **109** (2012) 125003.

[43] L. Chen and F. Zonca: Phys. Rev. Lett. **109** (2012) 145002.

[44] P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm: Plasma Phys. Control. Fusion **47** (2005) R35.

[45] A. Hasegawa, C. G. Maclennan, and Y. Kodama: Phys. Fluids **22** (1979) 2122.