Energetic particle nonlinear equilibria and transport processes in burning plasmas

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Lausanne-Varenna workshop
12–16 October 2020, Lausanne

AGENZIA NAZIONALE
PER LE NUOVE TECNOLOGIE, L'ENERGIA
E LO SVILUPPO ECONOMICO SOSTENIBILE

EUROfusion

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Introduction: motivations

- Predicting the dynamics of a burning plasma on long time scales, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g. ITER;
- the crucial role of energetic particles as mediators of cross scale couplings, see Zonca et al. 2015b; Chen and Zonca 2016, must be properly described;
- a first-principle-based, self-consistent approach is crucial;

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- a first-principle-based, self-consistent approach is crucial;

- Extending gyrokinetic simulations to these time scales is a challenging task from the computational resource point of view, i.e. $\sim 10^{24}$ grid points;
- simplifying assumptions based on physics understanding and first principles must be introduced leading to the definition of a reduced model for the dynamics.

Introduction: purpose statement

We aim at providing general expressions describing plasma transport in the phase space, particularly energetic particles, on long time scales and at introducing a framework to solve these equations within different levels of approximation.

By means of this approach it will be possible to:

- describe the physics of next generation fusion experiments where alpha particles will play a key role;
- characterize cross-scale couplings in burning plasmas.

Introduction: Outline

- Introduction;
- Transport equations;
- Zonal state;
- Zonal field structures;
- Summary & Conclusions.

Introduction: multiple scales analysis

- A multiple scales approach has been applied to extend gyrokinetic simulations to long time scales;
- macro-scales characterize radial plasma profiles, i.e. L, τ_T, while micro-scales describe “fast” variation of physical quantities, i.e. ρ_L, ω^{-1};
- transport equations for radial profiles are derived by means of a systematic separation of scales between macroscopic equilibrium and fluctuations.
- intermediate spatio-temporal scales are suppressed. Is it possible to use the same approach retaining meso-scales produced by the dynamics?

Introduction: what are meso-scales?

- Radial transport requires the study of flux surface averaged equations:

$$\langle \partial_t n \rangle_\psi = - \langle \boldsymbol{\nabla} \cdot (n\boldsymbol{V}) \rangle_\psi ;$$

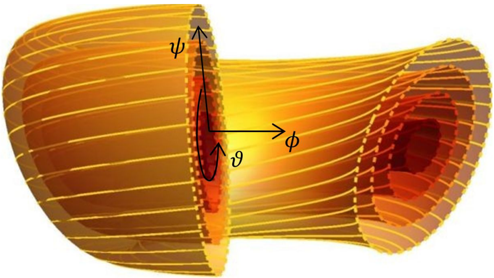

Figure: Courtesy of J. Ball

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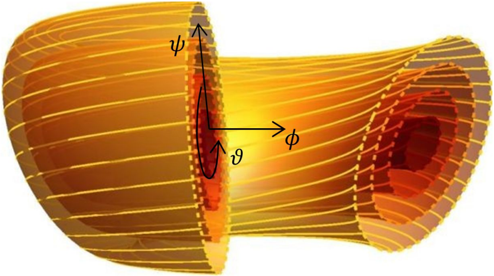
$$\langle \partial_t n \rangle_\psi = - \langle \boldsymbol{\nabla} \cdot (n\boldsymbol{V}) \rangle_\psi \, ;$$

- finite contributions only from non-linear terms with vanishing toroidal and poloidal mode-numbers;
- in principle they have arbitrary radial length-scale.



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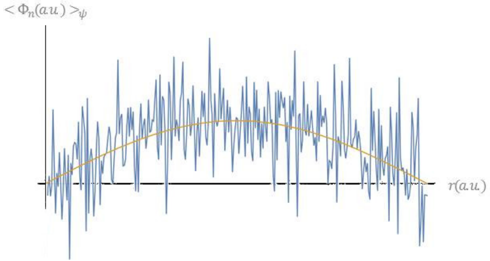

Introduction: why meso-scales are important?

Meso-scales on ITER . . .

- peculiar feature of ITER: interplay of meso-scale structures with micro-scales generated by energetic particles, e.g. $\rho_{LE} \sim (\rho_L L)^{1/2}$;
- unique role of energetic particles, which act as mediators between the micro- and the macro- scales;

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. . . and DTT (Divertor Tokamak Test facility)

- $T_E/T_i \sim (4\rho*)^{-1} \Rightarrow \rho_{LE} \sim (\rho_L L)^{1/2}$ with dimensionless parameters close to ITER;
- using minority heating by ICRH and/or NBI, cross-scale couplings should be similar;

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- using minority heating by ICRH and/or NBI, cross-scale couplings should be similar;

- need for transport equations for the mesoscales extending previous analysis;
- non Maxwellian distribution functions must be taken into account, i.e. phase space transport;
- meso-scales do not emerge only due to EP physics, e.g. ITBs, L-H transitions;
- derivation based on the theory of Phase space zonal structures, see Zonca et al. 2015a; Chen and Zonca 2016; Falessi and Zonca 2019;

Introduction: phase space zonal structures

Zonal structures & Zonal field structures

- Mode mode coupling between fluctuating fields can generate toroidal symmetric structures in the density and temperature profiles unaffected by rapid collision-less dissipation, i.e. Landau damping, called zonal structures;
- their counterpart for vector and scalar potentials are called zonal field structures, e.g. $\delta \boldsymbol{E}_r = -\boldsymbol{\nabla}\psi \partial_\psi \delta\phi(\psi)$.

Phase space zonal structures (PSZS)

- fluctuations collision-less undamped in the phase space are called phase space zonal structures and they importantly regulate turbulence saturation level by scattering instability turbulence to shorter radial wavelength stable domain;
- they are coherent micro/meso-scale radial corrugations of the distribution function Chen and Zonca 2016. They are associated to deviations from the local thermodynamic equilibrium and thus they may enhance collisional transport.

Transport equations: ordering assumptions

- following Hinton and Hazeltine 1976, Frieman and Chen 1982, we describe slowly evolving equilibria: $\omega^{-1}\partial_t \ln p_0 \sim \mathcal{O}(\delta^2)$;
- The reference magnetic equilibrium is: $\boldsymbol{B}_0 = F\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\psi$;
- We introduce the following gyrokinetic ordering for fluctuating quantities:

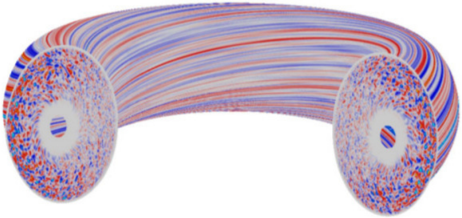

Figure: Courtesy of Y. Xiao et al., PoP 2015.

$$\frac{cE}{Bv_{th}} \sim \frac{|\partial/\partial t|}{|\Omega|} \sim \left|\frac{\delta B}{B}\right| \sim \frac{\boldsymbol{\nabla}_\parallel}{\boldsymbol{\nabla}_\perp} \sim \frac{\boldsymbol{k}_\parallel}{\boldsymbol{k}_\perp} \sim \mathcal{O}(\delta).$$

Transport equations: PSZS equation

- We want to explicit identify the part of the toroidally symmetric distribution function unaffected by rapid collisionless dissipation that is the PSZS;
- we start by writing the nonlinear Gyrokinetic equation:

$$\partial_t(DF) + \boldsymbol{\nabla} \cdot (D\dot{\boldsymbol{X}}F) + \partial_{\mathcal{E}}(D\delta\dot{\mathcal{E}}F) = 0$$

- D is the Jacobian of the velocity space and $\dot{\boldsymbol{X}} = \dot{\boldsymbol{X}}_0 + \delta\dot{\boldsymbol{X}}$ is the gyrocenter velocity due to magnetic equilibrium and to fluctuating fields;

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- we decompose F as $F = F_0 + \delta F$ where F_0 is the steady state solution of the lowest order gyrokinetic equation, i.e.:

$$F_0(\psi,\theta,\mathcal{E},\mu) = e^{-iQ_z}F_{B0}(\psi,\theta,\mathcal{E},\mu) = F_{B0}(\psi - \frac{Fv_\parallel}{\Omega},\mathcal{E},\mu) = F_{B0}(\bar{\psi}(\psi,\theta,\mathcal{E},\mu),\mathcal{E},\mu)$$

  where we have introduced the drift/banana center shift operator e^{iQ_z} with $Q_z \equiv F(v_\parallel/\Omega)k_z/(d\psi/dr)$;
- we can calculate $D(\delta\dot{\boldsymbol{X}}\cdot\boldsymbol{\nabla} + \delta\dot{\mathcal{E}}\partial_{\mathcal{E}})F_{B0}(\bar{\psi}(\psi,\theta,\mathcal{E},\mu),\mathcal{E},\mu)\ldots$

Transport equations: PSZS equation

…and re-write the toroidally symmetric component of the Gyrokinetic equation:

$$D(\partial_t + \dot{\boldsymbol{X}}_0 \cdot \boldsymbol{\nabla}) \left(F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 + \frac{F}{B_0} \frac{\partial F_0}{\partial \bar{\psi}} \langle \delta A_{\parallel g} \rangle_z \right) +$$
$$+ D \frac{e}{m} \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 \partial_t \langle \delta L_g \rangle_z + \frac{1}{J} \frac{\partial}{\partial \theta} (J D \delta \dot{\theta} \delta F) + \frac{1}{J} \frac{\partial}{\partial \psi} (J D \delta \dot{\psi} \delta F) + \frac{\partial}{\partial \mathcal{E}} (J D \delta \dot{\mathcal{E}} \delta F) = 0$$

where $\langle \delta L_g \rangle_z = \hat{I}_0 \left(\delta \phi_z - \frac{v_\parallel}{c} \delta A_{\parallel z} \right) + \frac{m}{e} \mu \hat{I}_1 \delta B_{\parallel z}$ and J is the Jacobian of the curvilinear coordinate system;

- this naturally leads to the definition of G_z:

$$G_z = F_z - \frac{e}{m} \langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 + \frac{F}{B_0} \langle \delta A_{\parallel g} \rangle_z \frac{\partial}{\partial \bar{\psi}} F_0$$

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- we now apply e^{iQ_z} and write the governing equation for $G_B = e^{iQ_z} G_z$ …

Transport equations: PSZS equation

…we obtain the following equation:

$$\partial_t(JDG_B) + \partial_\theta G_B = e^{iQ_z}\Big[-\frac{e}{m}\frac{\partial}{\partial \mathcal{E}}|_{\bar{\psi}} F_0 \partial_t (JD\langle \delta L_g\rangle_z) - \frac{1}{J}\frac{\partial}{\partial\theta}(JD\delta\dot{\theta}\delta F) - \frac{1}{J}\frac{\partial}{\partial\psi}(JD\delta\dot{\psi}\delta F) - \frac{\partial}{\partial\mathcal{E}}(JD\delta\dot{\mathcal{E}}\delta F)\Big]$$

- it can be shown that e^{iQ_z}, up to the required order, commute with partial derivatives;

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$$- \frac{1}{J}\frac{\partial}{\partial\theta}(JD\delta\dot\theta\delta F) - \frac{1}{J}\frac{\partial}{\partial\psi}(JD\delta\dot\psi\delta F) - \frac{\partial}{\partial\mathcal{E}}(JD\delta\dot{\mathcal{E}}\delta F)\Big]$$

- it can be shown that e^{iQ_z}, up to the required order, commute with partial derivatives;
- finally, integrating over θ and introducing the bounce/transit average $\overline{[\ldots]} = \tau_b^{-1}\oint \frac{d\ell}{v_\parallel}[\ldots]$ we obtain:

$$\partial_t \overline{G}_B = -\overline{e^{iQ_z}\frac{e}{m}\frac{\partial}{\partial\mathcal{E}}|_{\bar\psi}F_0\partial_t\langle\delta L_g\rangle_z} - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}(\tau_b\overline{e^{iQ_z}\delta\dot\psi\delta F}) - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}(\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F})$$

Transport equations: results

- We can re-write the low frequency (transport) component of δf_z in term of only $\delta\overline{G}_B$;
- taking the time derivative of the surface averaged velocity integral (see Falessi 2017; Falessi and Zonca 2019) and neglecting the role of parallel nonlinearity:

$$\partial_t \langle\langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{\left(e^{-iQ_z} \hat{I}_0 \right)}\, \overline{\left(e^{iQ_z} \hat{I}_0 \right)} \right] \frac{\partial F_0}{\partial \mathcal{E}} \partial_t \delta\phi_z \right\rangle_v + \\ - \frac{1}{V'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{\left(e^{-iQ_z} \hat{I}_0 \right)}\, \overline{\left[c e^{iQ_z} R^2 \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla} \langle \delta L_g \rangle \delta G \right]} \right\rangle_v \right\rangle_\psi$$

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- this equation describes the radial oscillations on any length-scale of the density profile in the absence of collisions and assuming GK ordering;
- mesoscales are spontaneously created by turbulence;
- F_0 is never assumed to be Maxwellian.

Transport equations: recap

- the previous derivation can be repeated to describe energy transport and, in general, any transport equation;
- results known in literature are recovered considering the Maxwellian special case and macroscopic PSZS;
- in the next slides we will show that this is consistent with effectively redefining F_0 including mesoscales spontaneously produced by the dynamics;
- a phase space transport equation will be derived generalizing the conceptual framework already introduced;
- this is of crucial importance to study long time scale gyrokinetic or hybrid simulation of EP transport, where the non-Maxwellian features and the role of wave-particle resonances is most important;

Transport equations: PSZS equation

$$\partial_t \bar{G}_B = -\overline{e^{iQ_z} \frac{e}{m} \frac{\partial}{\partial \mathcal{E}} |_{\bar{\psi}} F_0 \partial_t \langle \delta L_g \rangle_z} - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} (\tau_b \overline{e^{iQ_z} \delta \dot{\psi} \delta F}) - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} (\tau_b \overline{e^{iQ_z} \delta \dot{\mathcal{E}} \delta F})$$

- this expression can be used to derive any transport equation and it is a generalization of the expression derived in Falessi and Zonca 2019 retaining the effect of parallel non linearity;
- in particular, using the quasi-linear approximation we can derive general transport expressions in arbitrary geometry, see Chen 1999;

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- in particular, using the quasi-linear approximation we can derive general transport expressions in arbitrary geometry, see Chen 1999;
- this naturally leads to define a reference distribution function F_0 at each instant of time. Going back to the the previous derivation. . . ;

Zonal state: renormalization of F_0

…and rewriting terms such as:

$$\delta \dot{\boldsymbol{X}}|_z \cdot \boldsymbol{\nabla} \delta F_z = \delta \dot{\boldsymbol{X}}|_z \cdot \boldsymbol{\nabla} e^{-iQ_z} (\delta \bar{F}_{Bz} + \delta \tilde{F}_{Bz})$$

- $e^{-iQ_z} \delta \bar{F}_{Bz}$, analogously to F_0, is a function of the constants of motion;

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- $e^{-iQ_z}\delta\bar{F}_{Bz}$, analogously to F_0, is a function of the constants of motion;
- re-doing the previous derivation, we obtain a newly defined G_z:

$$G_z = F_z - \frac{e}{m}\langle\delta L_g\rangle_z \frac{\partial}{\partial\mathcal{E}}|_{\bar\psi} F_{0*} + \frac{F}{B_0}\langle\delta A_{\parallel g}\rangle_z \frac{\partial}{\partial\bar\psi} F_{0*}$$

where $F_{0*} = F_0 + e^{-iQ_z}\delta\bar{F}_{Bz}$ is the renormalized F_0;

- PSZS are the macro-/meso- scopic component of F_{0*}. They define a self-consistent nonlinear equilibrium in the presence of a finite level of fluctuations;

Zonal state: orbit averaging

- particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- it follows from previous slides that PSZS equation is connected with the macro-meso- scopic component, i.e. $[\ldots]_S$, unperturbed orbit-averaged distribution function (Falessi et al. 2019):

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$

where $\overline{(...)} = \tau_b^{-1} \oint d\theta/\dot{\theta}\,(\ldots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\ldots)(\bar{\psi}, \theta)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\tilde{\psi}(\theta)$;

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- this is equivalent to bounce/transit averaging a quantity shifted with the e^{iQ_z} operator;
- this expression describe transport processes in the phase space due to fluctuations or collisions and sources;

Zonal state: neighboring nonlinear equilibria

- Having defined PSZS, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;

$$F_z = \overline{F_{z0}} + \overline{\delta F_z} + \delta\tilde{F}_z$$

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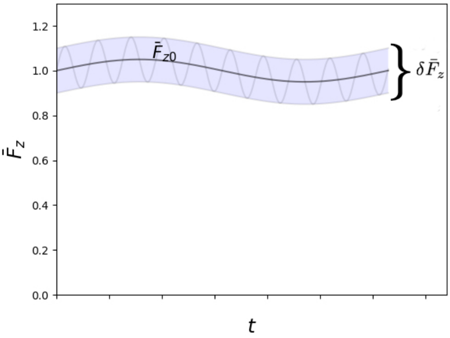
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal field structures, form a zonal state, see Falessi and Zonca 2019.

$$F_z = \overline{F_{z0}} + \overline{\delta F_z} + \delta\tilde{F}_z$$



Zonal state: governing equations

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{C^g_{z0}} + [\bar{S}]_S$$

- expanding the collision operator we obtain:

$$\bar{C}^g_{z0} = \overline{C^g_z\left[\overline{F}_{z0}, \overline{F}_{z0}\right]} + \left[\overline{C^g_z\left[\overline{F}_{z0}, \delta F_z\right]} + \overline{C^g_z\left[\delta F, \delta F\right]}\right]_S$$

- first term balance $[\bar{S}]_S$ while the second, in the presence of a Maxwellian reference state, describes neoclassical transport;
- corrections to neoclassical transport are given by the first and the third terms;

$$\frac{\partial}{\partial t}\overline{\delta F}_z + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \overline{F_{z0}}\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \overline{F_{z0}}\right)_z}\right] +$$
$$+ \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_F = \overline{C^g}_z - \overline{C^g}_{z_0} + [\bar{S}]_F$$

Zonal state: CGL equilibrium

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations;
- this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium . . .

Zonal state: CGL equilibrium

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations;
- this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium ...
- following Cary and Brizard 2009; Brizard and Hahm 2007, we write every moment of the zonal distribution function using their push-forward representation, e.g.:

$$\mathbf{J}_z(r) = e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left(T^{-1}\mathbf{v}\right)\delta(\mathbf{X}+\rho-\mathbf{r})\Big[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0+ \\ -\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\Big]+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\Big[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\Big]$$

- we can substitute F_z for the PSZS $\bar{F}_{z0}=[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}]_S$;
- we obtain, from a multipole expansion, an equilibrium that is consistent with an anisotropic CGL pressure tensor;

Zonal state: CGL equilibrium

- the current $\mathbf{J}_z$ and the pressure tensor $\mathbf{P}$ satisfy the following force balance equation:

$$\sigma\frac{\mathbf{J}_z \times \mathbf{B}}{c} = \boldsymbol{\nabla} P_{\parallel} + (\sigma - 1)\boldsymbol{\nabla}(\frac{B^2}{8\pi}) + \frac{B^2}{4\pi}\boldsymbol{\nabla}_{\perp}\sigma$$

where $\sigma = 1 + \frac{4\pi}{B^2}(P_{\perp} - P_{\parallel})$;

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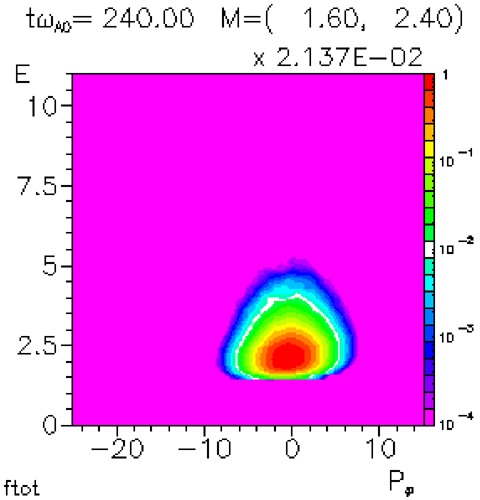
  where $\sigma = 1 + \frac{4\pi}{B^2}(P_{\perp} - P_{\parallel})$;
- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

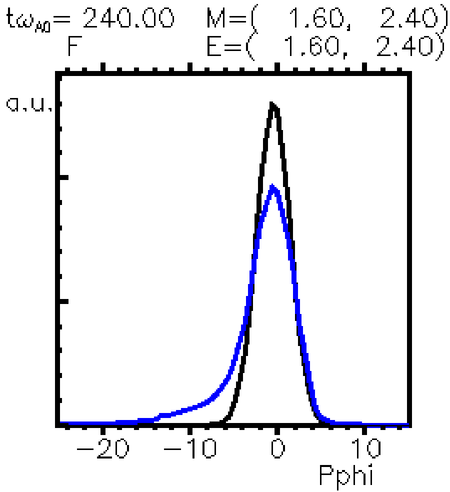
$$\Delta^* \psi + \boldsymbol{\nabla} \ln \sigma \cdot \boldsymbol{\nabla} \psi = -\frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2}\frac{\partial G}{\partial \psi}$$

  where $G \equiv \sigma F^2/2$ is a flux function;

Zonal state: Example

- PSZS have been extracted from an EPM simulation by the HMGC hybrid code, i.e. see Briguglio et al. 1995, and more recently by HYMAGIC:





$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = 0$$

Zonal state: MET project

- Phase space transport has been studied by means of a hierarchy of verified and validated reduced models within the MET EUROfusion Enabling Research Project with F. Zonca as P.I. (see poster 15 on Thursday);
- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;
- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$;
- the second and final level of simplification is the Quasilinear model.

Zonal state: recap

- PSZS are a possible definition of nonlinear equilibrium distribution function;
- together with zonal field structures and residual orbit averaged particle response they define the zonal state;
- this definition is particularly important in collisionless burning plasmas, where one cannot always describe transport via evolution of macroscopic radial profiles of a reference Maxwellian;
- present approach generally applies to near-integrable Hamiltonian systems.

Zonal field structures: governing equations

- Computing the zonal state require equations for zonal field structures, i.e. the long lived component of toroidal symmetric fields;
- assume, for simplicity, that the zonal state is characterized predominantly by the scalar potential $\delta\phi_z$;
- we can study its self-consistent evolution in the absence of symmetry breaking fluctuations;

Zonal field structures: governing equations

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- assume, for simplicity, that the zonal state is characterized predominantly by the scalar potential $\delta\phi_z$;
- we can study its self-consistent evolution in the absence of symmetry breaking fluctuations;
- the scalar potential zonal field structure is obtained substituting the orbit averaged distribution function into the quasi neutrality condition:

$$\sum_s \left\langle \frac{e^2}{m} \frac{\partial F_0}{\partial \mathcal{E}} \right\rangle_v \delta\phi_z + \sum_s \left\langle e \hat{I}_0 \delta G_z \right\rangle_v = 0$$

Zonal field structures: scalar potential

- We can write the kinetic equation for δG_B:

$$(\partial_t + \omega_b \partial_{\vartheta c})\delta G_B = -e^{-iQ_z}\left[\frac{e}{m}\frac{\partial F_0}{\partial \mathcal{E}}\hat{I}_0 \partial_t \delta\phi_z + NL\right]$$

  nonlinear terms, i.e. $(\delta\dot{\boldsymbol{X}}\cdot\boldsymbol{\nabla}\delta F + \delta\dot{\mathcal{E}}\partial_{\mathcal{E}}F)_z$, have been indicated as NL;
- we have introduced the action angle variables ϑ_c such that $\omega_b = \dot{\vartheta}_c$ and ζ_c such that $\dot{\zeta}_c = \bar{\omega}_d$
- ω_b and ω_d are, respectively, the bounce/transit frequency and precession drift frequency;

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- ω_b and ω_d are, respectively, the bounce/transit frequency and precession drift frequency;
- introducing the expansion parameter $\omega_{NL}/\omega \ll 1$ we can explicitly write the solution $\delta G_B = \sum_l e^{il\vartheta_c}\delta G_{Bl}$;
- substituting the $l = 0$ component into the flux surface averaged quasi neutrality condition . . .

Zonal field structures: scalar potential

. . . We obtain the following expression:

$$
\sum_s \frac{V'_\psi}{4\pi^2}\frac{e^2}{m_s^2}\left\langle\left\langle\frac{\partial F_0}{\partial \mathcal{E}}\delta\phi_z - e^{-iQ_z}\overline{\hat{I}_0 e^{iQ_z}\hat{I}_0\frac{\partial F_0}{\partial \mathcal{E}}\delta\phi_z}\right\rangle_v\right\rangle_\psi =
$$

$$
\sum_s\sum_\sigma\int d\mathcal{E}d\mu\tau_B e\frac{i}{\omega}\overline{e^{iQ_z}\overline{\hat{I}_0 e^{iQ_z}NL}}
$$

Zonal field structures: scalar potential

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- F_0 is an arbitrary (renormalized) anisotropic distribution function;
- neglecting the non linear term we can study the polarizability in arbitrary geometry and realistic F_0;

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- F_0 is an arbitrary (renormalized) anisotropic distribution function;
- neglecting the non linear term we can study the polarizability in arbitrary geometry and realistic F_0;
- results known in literature, e.g. Wang and Hahm 2009; Lu et al. 2019, are recovered in the proper limits;
- GAM/EGAM dispersion relations retaining finite Larmor radius and finite orbit width effects, see Qiu, Zonca, and Chen 2010; Qiu, Chen, and Zonca 2008 can be derived;

Zonal field structures: scalar potential

$$
\partial_t \left\langle \langle \delta f_z \rangle_v \right\rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{\left(e^{-iQ_z} \hat{I}_0 \right)} \, \overline{\left(e^{iQ_z} \hat{I}_0 \right)} \right] \frac{\partial F_0}{\partial \mathcal{E}} \partial_t \delta \phi_z \right\rangle_v +
$$

$$
- \frac{1}{V'_\psi} \frac{\partial}{\partial \psi} \left\langle \left\langle V'_\psi \overline{\left(e^{-iQ_z} \hat{I}_0 \right)} \, \overline{\left[c e^{iQ_z} R^2 \nabla \phi \cdot \nabla \langle \delta L_g \rangle \delta G \right]} \right\rangle_v \right\rangle_\psi
$$

- we can verify that the scalar potential zonal field structure governing equation is consistent with the density transport equation;

Zonal field structures: scalar potential

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$$-\frac{1}{V'_\psi}\frac{\partial}{\partial\psi}\left\langle\left\langle V'_\psi\overline{\left(e^{-iQ_z}\hat{I}_0\right)}\overline{\left[ce^{iQ_z}R^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\left\langle\delta L_g\right\rangle\delta G\right]}\right\rangle_v\right\rangle_\psi$$

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- we can verify that the scalar potential zonal field structure governing equation is consistent with the density transport equation;
- in both cases, on macro-scales we obtain the usual transport equations while meso-/micro- scopic physics describes polarization effects;
- collisions and souces can be readily added;

Summary & Conclusions

- Need of predictive models on timescales comparable with magnetic fusion experiments, i.e. ITER, properly describing EP;
- extend kinetic simulations on this time scale is challenging. A possible solution is represented by a multiple scale approach and reduced models;

$t_{sim} \sim \tau_E$

10^{24} grid points!!!

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- we have introduced the concept of zonal state to describe the evolution of the plasma between neighboring nonlinear equilibria;

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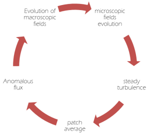
- we have introduced a conceptual framework which allow to study EP phase space transport over long time scales.



Thank you for your attention.