When the chorus sings …
… and laboratory plasmas join in (*)

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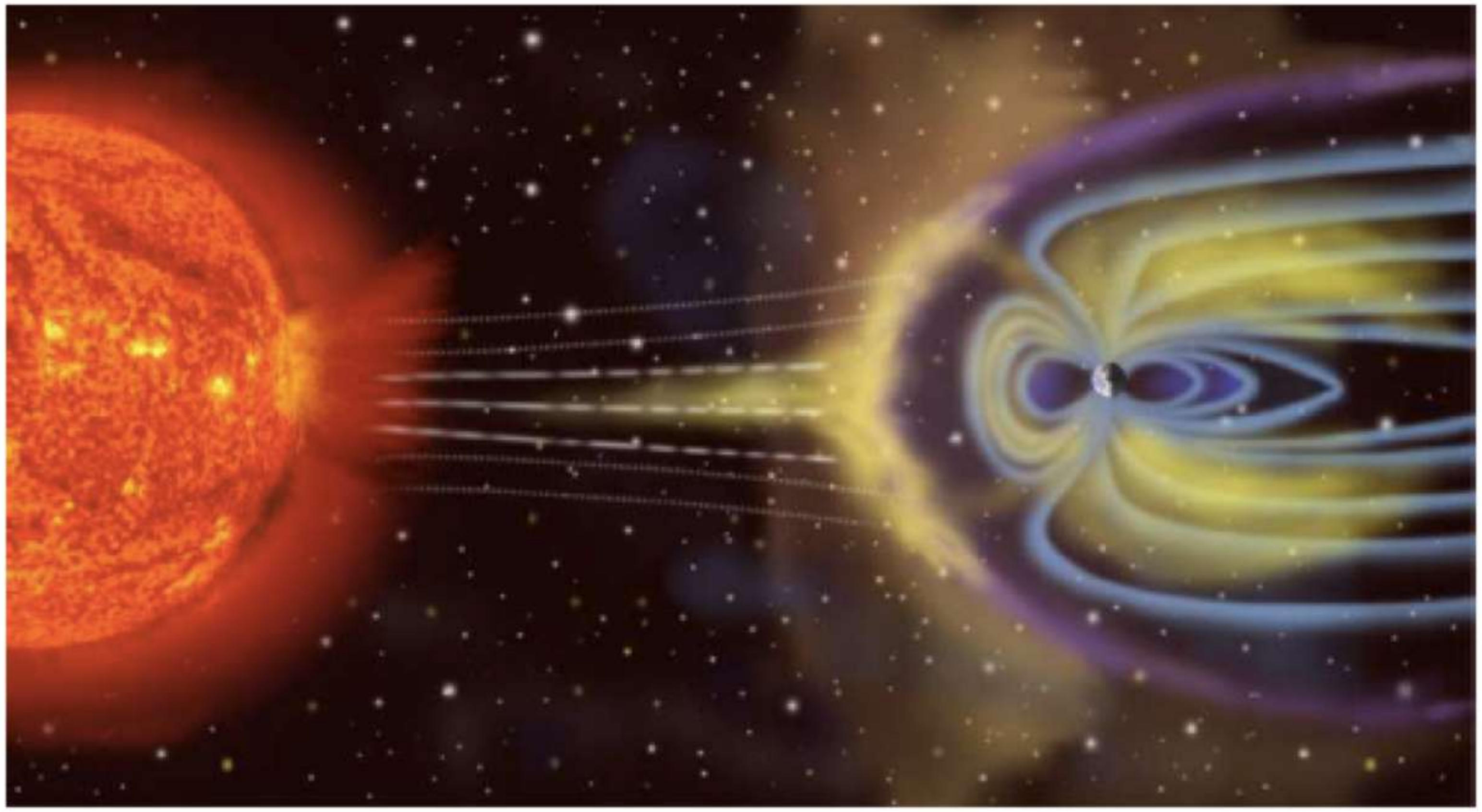
What is chorus



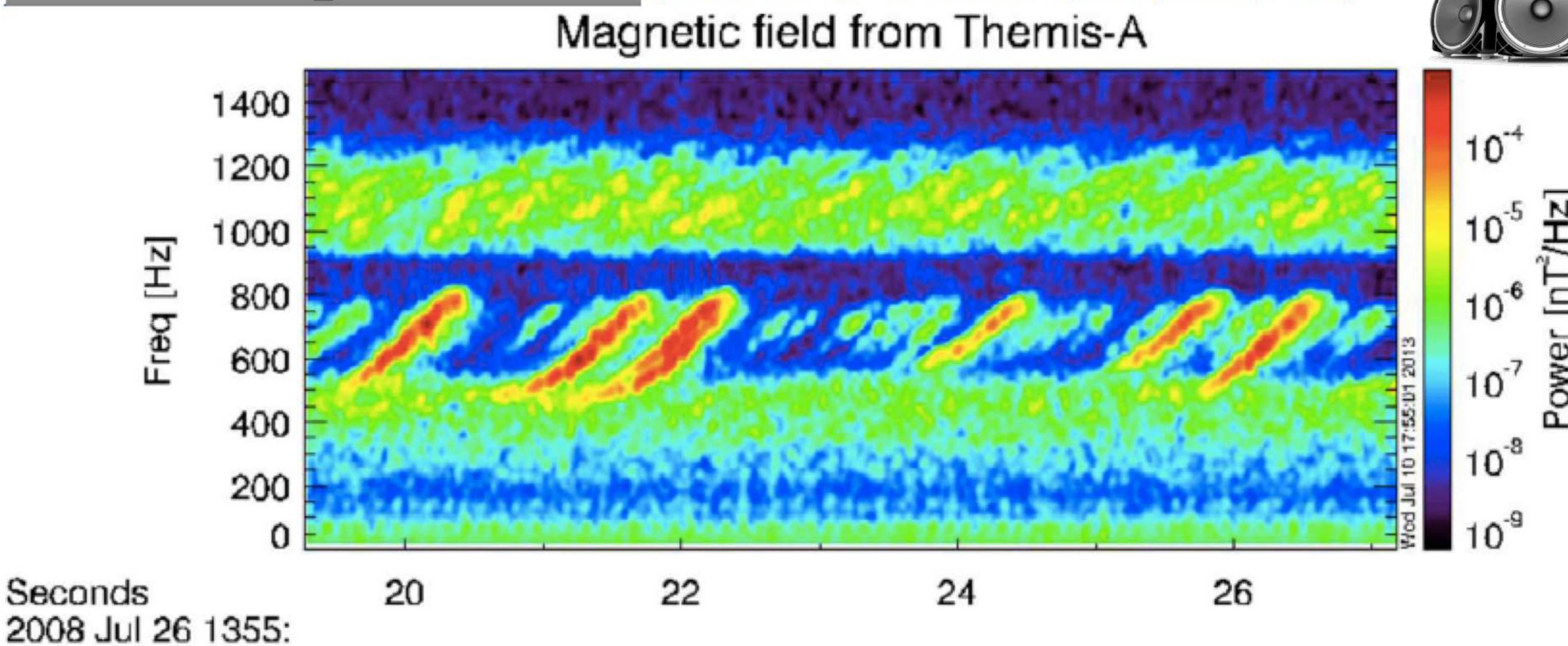
Source: http://sec.gsfc.nasa.gov/popscise.jpg

- The Earths magnetospheric plasma, under certain conditions, can amplify specific types of electromagnetic waves.
- In particular, the whistler waves, which are electromagnetic fluctuations propagating at less than the non-relativistic electron cyclotron frequency, can occur in $\sim 0.1s$ bursts with either rising or falling tones.

Chorus chirping

- These bursts are known as chorus because of their characteristic chirping.

[Source: https://www.ucalgary.ca/above/science/chorus]



- Chorus excitation and nonlinear dynamics is one of the long-studied physics problems of Earths magnetosphere due to its implications for particle acceleration and distribution in the radiation belts.

Chorus and fusion plasmas - I

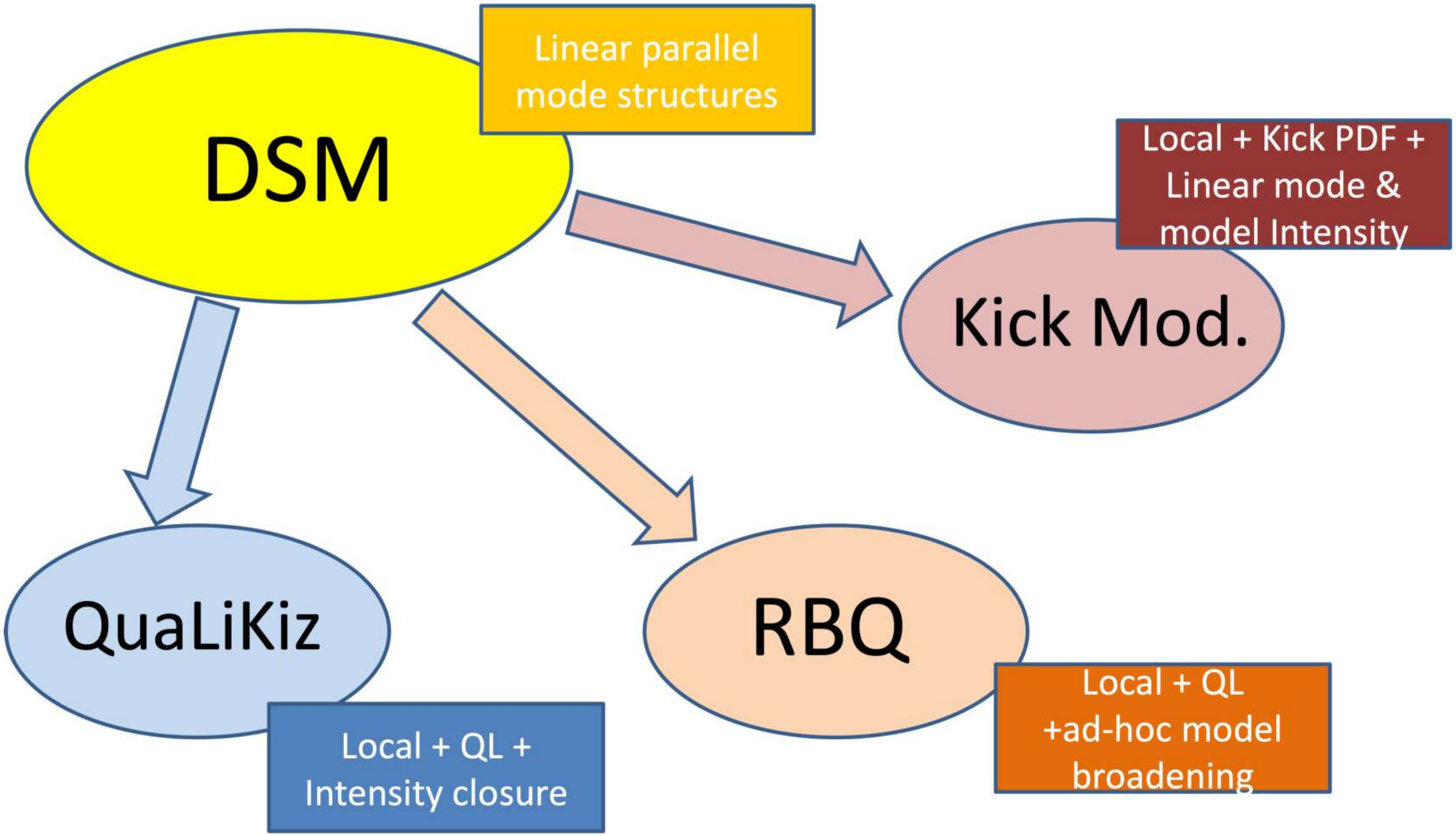
- This talk: focus on (nearly) parallel propagating whistler giving rise to rising tone chorus generation.
  - Self-consistent nonlinear dynamics of non-uniform, non-autonomous system with one degree of freedom
  - Fundamental plasma physics problem of general interest and practical applications
  - Connection with reduced models with application to magnetized fusion plasmas [Chen&Zonca RMP 2016]

- Nonlinear wave particle interaction time during chorus wave generation is typically of the same order as the characteristic nonlinear time scale.
  - Nonlinear mode evolution (τ_{NL}) occurs self-consistently with particle transport (τ_{tr})
  - Novelty of the present work [Tao, Chen & Zonca 2017, 2019] with respect to previous analyses assuming $\tau_{\mathrm{NL}} \gg \tau_{\mathrm{tr}}$

Chorus and fusion plasmas - II

Direct connection with Dyson Schrödinger Model (DSM) :

(ZCFQ seminar Nov 20/2020)



Numerical/Theoretical framework

- **Numerical simulation results summarized:**
    - A "Trap-Release-Amplify" Model of Chorus Waves, published in Journal of Geophysical Research: Space Physics (PDF)

- **Theoretical framework fully analyzed:**
    - A theoretical framework of chorus wave excitation (arXiv preprint arXiv:2107.03151; submitted to JGR; PDF)
    - Nonlinear dynamics and phase space transport by chorus emission (arXiv preprint arXiv:2106.15026; tutorial submitted to RMPP; PDF)

Wave packet evolution - I

- Evolution equation for the fluctuation spectrum using wave-packets
  - $\delta \boldsymbol{E}_\perp(z,t) = e^{iS_k(z,t)}\delta\bar{\boldsymbol{E}}_{\perp k}(z,t)$; $\omega_k = -\partial_t S_k,\ k = \partial_z S_k$
  - $\delta\bar{\boldsymbol{E}}_{\perp k}(z,t) = |\delta\bar{\boldsymbol{E}}_{\perp k}(z,t)|e^{i\varphi_k(z,t)}$; $I_k(z,t) \equiv (\partial D_w/\partial k)|\delta \boldsymbol{E}_{\perp 0k}(z,t)|^2$

- Wave packets satisfy the Whistler wave dispersion relation

$$D_w = 1 + \frac{\omega_p^2}{\omega(\Omega-\omega)} - \frac{k^2c^2}{\omega^2} = 0\,. \qquad \Rightarrow \qquad k = K(z,\omega)$$

$$\left(1 + \frac{\omega_p^2}{\omega(\Omega-\omega)}\right)\delta\bar{\boldsymbol{E}}_{\perp k} - \frac{k^2c^2}{\omega^2}\delta\bar{\boldsymbol{E}}_{\perp k} = -\frac{4\pi i}{\omega}\delta\bar{\boldsymbol{J}}_{hk}$$

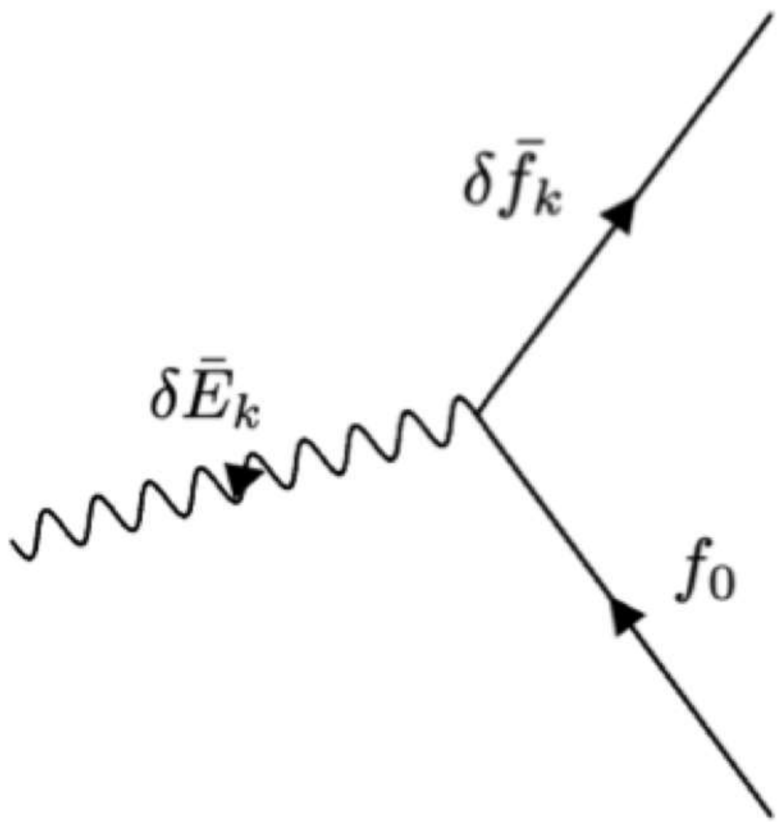
- Wave packet propagation is entirely constructed from

$$W(z,t,\omega) + i\Gamma(z,t,\omega) \equiv -\frac{4\pi i}{\omega_k \partial D_w/\partial\omega_k}\frac{\delta\boldsymbol{J}_h\cdot\delta\boldsymbol{E}_\perp^*}{|\delta\boldsymbol{E}_\perp(z,t)|^2}\bigg|_{k=K(z,\omega)}$$

$\delta\boldsymbol{J}_h \equiv$ supra-thermal electron current

Linear electron response

- **Compute electron current from particle response:**



$$\delta\bar{f}_k = \frac{e}{m}\mathcal{P}_k v_\perp \delta\bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \ ,$$

Electron response

$$\begin{aligned}\bar{\partial}_{\mathcal{E}k} &\equiv \left[\frac{k}{\omega_k}\frac{\partial}{\partial v_\parallel} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\right] \\ &= \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial \mu}\right] \ ;\end{aligned}$$

$$\delta\bar{\boldsymbol{J}}_{hk\perp} = -e\left\langle \boldsymbol{v}_\perp e^{i\alpha}\delta\bar{f}_k\right\rangle$$

$$\mathcal{P}_k = \left[\Omega + k v_\parallel - \omega_k\right]^{-1}$$

Linear propagator

$$-\frac{4\pi i}{\omega}\frac{\delta\bar{\boldsymbol{J}}_{hk}\cdot\delta\bar{\boldsymbol{E}}^*_{\perp k}}{|\delta\bar{\boldsymbol{E}}_{\perp k}(z,t)|^2}\bigg|_{k=K(z,\omega)} = \left\langle \frac{4\pi e}{\omega}\frac{v_\perp}{2}\frac{\delta\bar{E}^*_k\delta\bar{f}_k}{|\delta\bar{E}_k|^2}\right\rangle$$

$$= \frac{\omega_p^2}{n\omega}\left\langle \frac{v_\perp^2/2}{\Omega + k v_\parallel - \omega}\left[\frac{k}{\omega}\frac{\partial f_0}{\partial v_\parallel} + \left(1 - \frac{k v_\parallel}{\omega}\right)\frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right]\right\rangle$$

Wave packet evolution - II

- Intensity evolution equation

$$\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right) I_k(z,t) = 2\Gamma(z,t,\omega) I_k(z,t)$$

- Phase evolution equation

$$\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right) \varphi_k(z,t) = -W(z,t,\omega)$$

- As noted already, all relevant physics is included in the wave-particle interaction term:

$$W(z,t,\omega) + i\Gamma(z,t,\omega) \equiv - \frac{4\pi i}{\omega_k \partial D_w/\partial\omega_k} \frac{\delta \boldsymbol{J}_h \cdot \delta \boldsymbol{E}_\perp^*}{|\delta \boldsymbol{E}_\perp(z,t)|^2}\bigg|_{k=K(z,\omega)}$$

- It must be computed self-consistently, in the presence of evolving finite-amplitude fluctuation spectrum:
  - $\Rightarrow$ $\tau_{\rm NL} \sim \tau_{\rm tr}$ (unlike existing theories that assume $\tau_{\rm NL} \gg \tau_{\rm tr}$)
  - $\Rightarrow$ Renormalization problem for the long wavelength (spatial averaged) particle response

Self consistent NL description

- **Basic underlying idea:**
  - We focus on the nonlinear particle response without fast temporal or spatial dependences, which correspond to the self-interaction of the wavenumber of interest with itself. (*)
  - This nonlinear response obeys an evolution equation, which may look like a "quasilinear" description at first glance but is, indeed, much more general as will be shown and discussed in the following.
  - The present approach has a close connection of with the Dyson equation and the so-called "principal series" of secular terms.

(*) Acknowledgment: D. F. Escande

Holes and clumps

- **Formation of structures in phase space:**
  - For rising tone chorus, phase locked particles gain energy and momentum but loose parallel energy

$$\dot{\mu} B\omega = \Omega\dot{\mathcal{E}} \, , \qquad \Rightarrow \qquad \omega d\mu = (\Omega/B) d\mathcal{E} \, .$$

  - Structures are formed near resonance:

Holes: lower density w.r.t. unperturbed surrounding

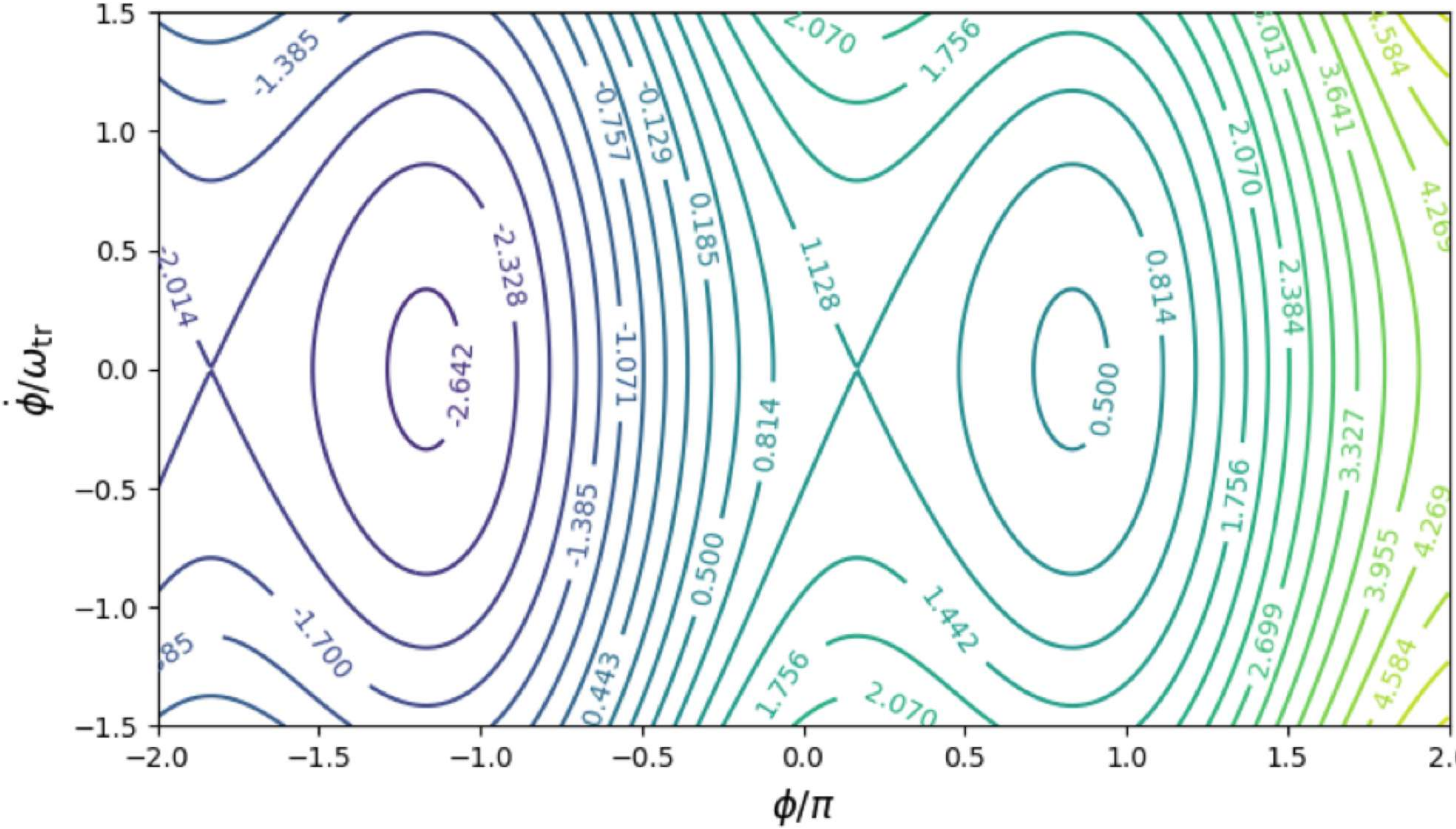
Clumps: higher density w.r.t. unperturbed surrounding

$$f_0 = \frac{n_e}{(2\pi)^{3/2} w_{\parallel e} w_{\perp e}^2} \exp\left(-\mathcal{E}/w_{\parallel e}^2 + A\mu B_e / w_{\perp e}^2 \right) \, ,$$

$$A = w_{\perp e}^2 / w_{\parallel e}^2 - 1$$

$$\begin{aligned} \Delta f_0 &= f_0(\mathcal{E}, \mu) - f_0(\mathcal{E} + \Delta\mathcal{E}, \mu + \Delta\mu) \, , \\ &\propto \frac{\Delta\mathcal{E}}{w_{\parallel e}^2} - AB_e \frac{\Delta\mu}{w_{\perp e}^2} = -AB_e \frac{\Delta\mu}{w_{\parallel e}^2} \left(\frac{A}{A+1} - \frac{\omega}{\Omega_e} \right) \, . \end{aligned}$$

Holes and clumps



$$\ddot{\phi} = \omega_{\mathrm{tr}}^2 \left(\sin\phi - R\right)$$

$$(1/2)\dot{\phi}^2 + \omega_{\mathrm{tr}}^2 \left(\cos\phi + R\phi\right) = \mathrm{const}$$

$$R = \frac{1}{\omega_{\mathrm{tr}}^2}\left[\left(1 - \frac{v_r}{v_g}\right)^2 \frac{\partial\omega}{\partial t} + \left(\frac{kv_\perp^2}{2\Omega} - \frac{3}{2}v_r\right)\frac{\partial\Omega}{\partial z}\right]$$

Self-consistent NL electron response

- **Asymptotic expansion:**
  - Formally solving NL Vlasov using small amplitude expansion

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \dots$$

$$\left(\frac{\partial}{\partial t} + v_\parallel \frac{\partial}{\partial z} + \Omega \frac{\partial}{\partial \alpha}\right) f - \frac{e}{m}\left(\delta \boldsymbol{E}_\perp + \frac{\boldsymbol{v} \times \delta \boldsymbol{B}_\perp}{c}\right) \cdot \frac{\partial}{\partial \boldsymbol{v}} f = 0$$

$$f(\boldsymbol{v}, z, t) = f_0(\boldsymbol{v}, z) + \frac{1}{2}\sum_k \left(e^{iS_k(z,t)+i\alpha}\delta \bar{f}_k(\boldsymbol{v}, z, t) + c.c.\right)$$

$$\begin{aligned} f^{(2)} \quad = \quad & \frac{1}{2}\sum_{k,k'}\left(e^{iS_k(z,t)+iS_{k'}(z,t)+2i\alpha}\delta \bar{f}_{k+k'}(\boldsymbol{v}, z, t) + c.c.\right) \\ & + \frac{1}{2}\sum_{k,k'}\left(e^{iS_k(z,t)-iS_{k'}(z,t)}\delta \bar{f}_{k-k'}(\boldsymbol{v}, z, t) + c.c.\right) \ . \end{aligned}$$

Nonlinear electron response

- **Asymptotic expansion:**
  - Formally solving NL Vlasov using small amplitude expansion

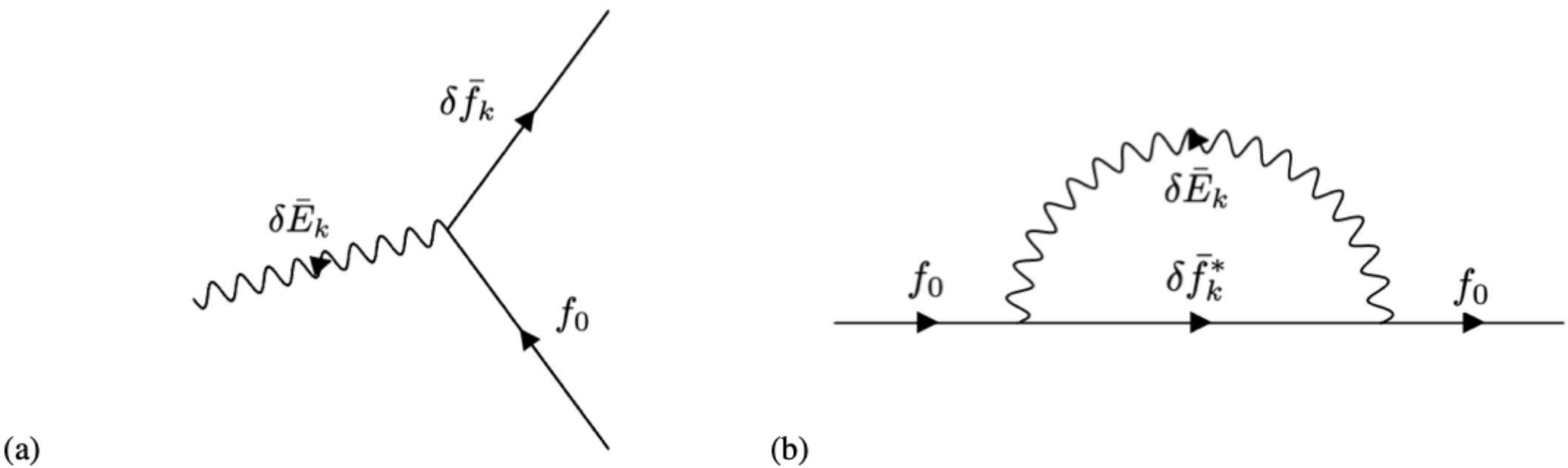
$$
\begin{aligned}
&\left[2\Omega+(k+k')v_\parallel-(\omega_k+\omega_{k'})-i(\partial_t+v_\parallel\partial_z)\right]\delta\bar{f}_{k+k'}=\frac{e}{m}\frac{v_\perp}{4}\left\{\delta\bar{E}_k\left[\bar{\partial}_{\mathcal{E}k}\right.\right.\\
&\left.\left.-\frac{1}{v_\perp^2}\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}_{k'}+\delta\bar{E}_{k'}\left[\bar{\partial}_{\mathcal{E}k'}-\frac{1}{v_\perp^2}\left(1-\frac{k'v_\parallel}{\omega_{k'}}\right)\right]\delta\bar{f}_k\right\},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\partial}_{\mathcal{E}k} &\equiv \left[\frac{k}{\omega_k}\frac{\partial}{\partial v_\parallel}+\left(1-\frac{kv_\parallel}{\omega_k}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\right]\\
&= \left[\frac{\partial}{\partial\mathcal{E}}+\left(1-\frac{kv_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial\mu}\right];
\end{aligned}
$$

$$
\begin{aligned}
&\left[(k-k')v_\parallel-(\omega_k-\omega_{k'})-i(\partial_t+v_\parallel\partial_z)\right]\delta\bar{f}_{k-k'}=\frac{e}{m}\frac{v_\perp}{4}\left\{\delta\bar{E}_k\left[\bar{\partial}_{\mathcal{E}k}+\frac{1}{v_\perp^2}\right.\right.\\
&\left.\left.\times\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}^*_{k'}-\delta\bar{E}^*_{k'}\left[\bar{\partial}_{\mathcal{E}k'}+\frac{1}{v_\perp^2}\left(1-\frac{k'v_\parallel}{\omega_{k'}}\right)\right]\delta\bar{f}_k\right\}. \qquad (52)
\end{aligned}
$$

Nonlinear electron response

- **Renormalization via same-k interactions:**
  - Consider the NL contribution without fast spatiotemporal scales
  - Effect of self-interactions of the wave number of interest with itself



Nonlinear electron response

- **Renormalization via same-k interactions:**
  - Secular growth on short time scale

$$f_0 \to f_0 + \sum_k \mathbb{R}\mathrm{e}\delta \bar{f}_0 \ ,$$

$$(\partial_t + v_\parallel \partial_z)\delta \bar{f}_0 = \frac{e}{m}\frac{v_\perp}{4}\sum_k i \left\{ \delta \bar{E}_k \left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2} \right. \right.$$
$$\left. \left. \times \left(1 - \frac{k v_\parallel}{\omega_k} \right) \right] \delta \bar{f}_k^* - \delta \bar{E}_k^* \left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2}\left(1 - \frac{k v_\parallel}{\omega_k} \right) \right] \delta \bar{f}_k \right\}$$

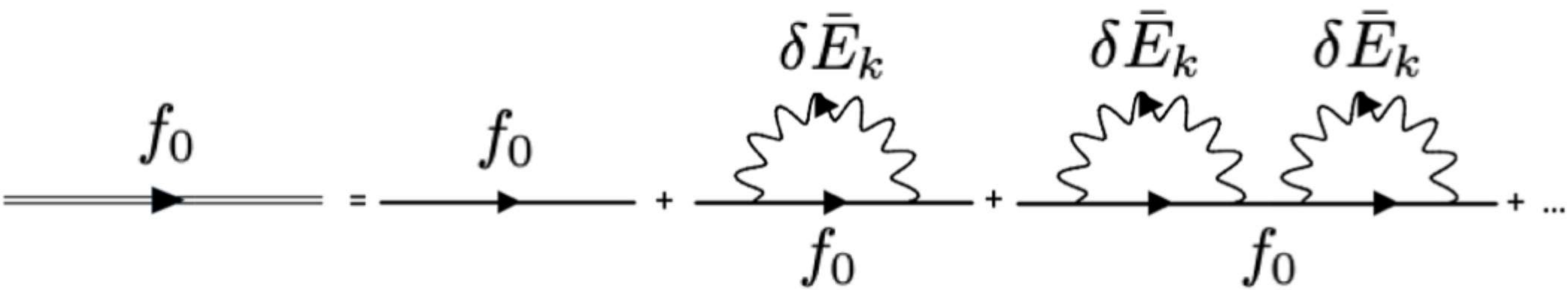
$$\delta \bar{f}_k = \frac{e}{m}\mathcal{P}_k v_\perp \delta \bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \ ,$$

$$\mathcal{P}_k = \left[\Omega + k v_\parallel - \omega_k \right]^{-1}$$

Nonlinear electron response

- **Renormalization via same-k interactions:**
  - By iterative solution one obtains the dominant renormalized response ➔ **Dyson – like equation (DSE)**

$$(\partial_t + v_\parallel \partial_z) f_0 = \frac{e}{m}\frac{v_\perp}{4}\mathbb{R}\mathrm{e}\sum_k i\left\{\delta\bar{E}_k\left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2}\right.\right.$$
$$\left.\left.\times\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}_k^* - \delta\bar{E}_k^*\left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2}\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}_k\right\}$$



$$\delta\bar{f}_k = \frac{e}{m}\mathcal{P}_k v_\perp \delta\bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \,,$$
$$\mathcal{P}_k \equiv \left[\Omega + kv_\parallel - \omega_k - i(\partial_t + v_\parallel\partial_z) - \Delta_k\right]^{-1}$$

resonance broadening

Resonance Broadening

- **Coupling of $\delta \bar{f}_{k+k'}$ with (k')* and $\delta \bar{f}_{k-k'}$ with k' (Dupree 66):**

$$
\begin{aligned}
\delta \bar{f}_k &= \frac{e}{m} \mathcal{P}_k v_\perp \delta \bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \, , \\
\mathcal{P}_k &\equiv \left[\Omega + k v_\parallel - \omega_k - i(\partial_t + v_\parallel \partial_z) - \Delta_k \right]^{-1}
\end{aligned}
$$

resonance broadening

$$\Delta_k \equiv \Delta_{1k} + \Delta_{2k}$$

$$
\begin{aligned}
\bar{\partial}_{\mathcal{E}k} &\equiv \left[\frac{k}{\omega_k} \frac{\partial}{\partial v_\parallel} + \left(1 - \frac{k v_\parallel}{\omega_k} \right) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \right] \\
&= \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k} \right) \frac{1}{B} \frac{\partial}{\partial \mu} \right] \, ;
\end{aligned}
$$

$$
\begin{aligned}
-i\Delta_{1k} &\equiv i \frac{e^2}{m^2} \frac{v_\perp^2}{8} \sum_{k'} \delta \bar{E}^*_{k'} \left[\bar{\partial}_{\mathcal{E}k'} + \frac{2}{v_\perp^2} \left(1 - \frac{k' v_\parallel}{\omega_{k'}} \right) \right] \\
&\quad \times \mathcal{P}_{k+k'} \delta \bar{E}_{k'} \left[\bar{\partial}_{\mathcal{E}k'} - \frac{1}{v_\perp^2} \left(1 - \frac{k' v_\parallel}{\omega_{k'}} \right) \right] \, , \\
-i\Delta_{2k} &\equiv i \frac{e^2}{m^2} \frac{v_\perp^2}{8} \sum_{k' \neq k} \delta \bar{E}^*_{k'} \bar{\partial}_{\mathcal{E}k'} \mathcal{P}_{k-k'} \delta \bar{E}_{k'} \left[\bar{\partial}_{\mathcal{E}k'} + \frac{1}{v_\perp^2} \left(1 - \frac{k' v_\parallel}{\omega_{k'}} \right) \right]
\end{aligned}
$$

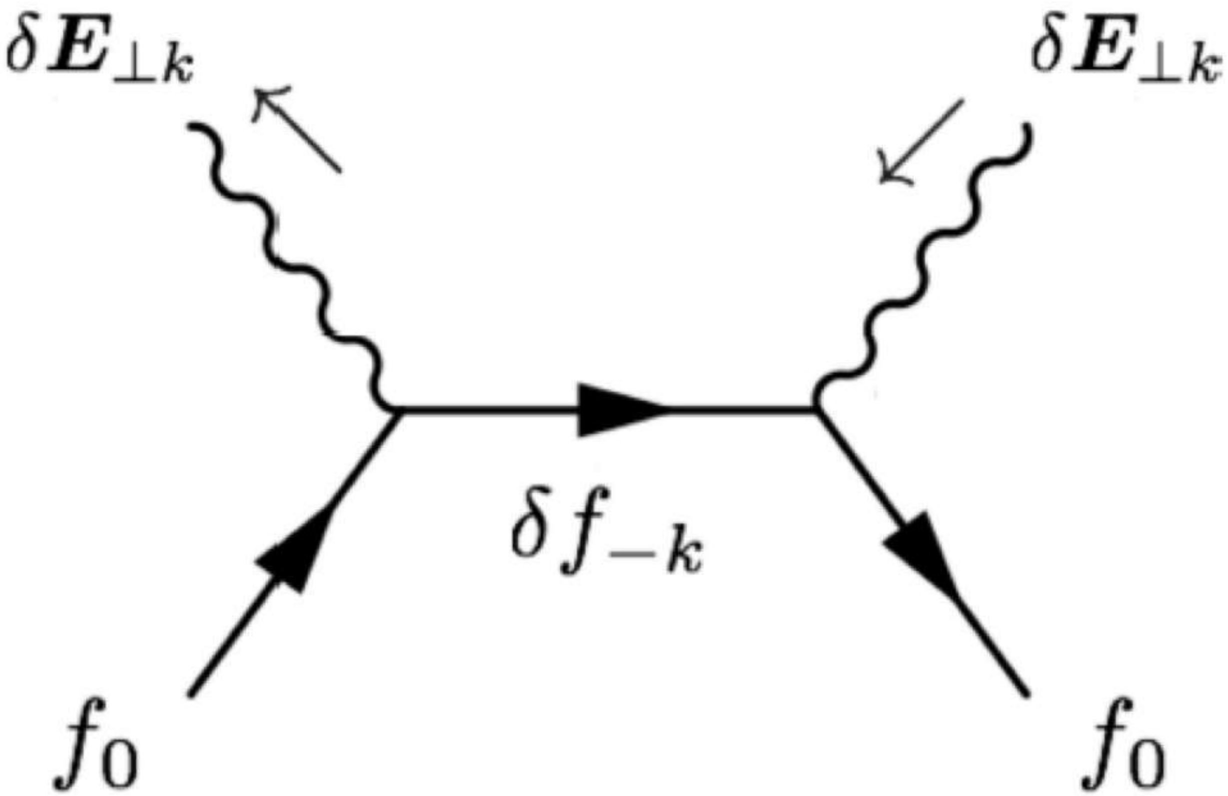
Self consistent NL description

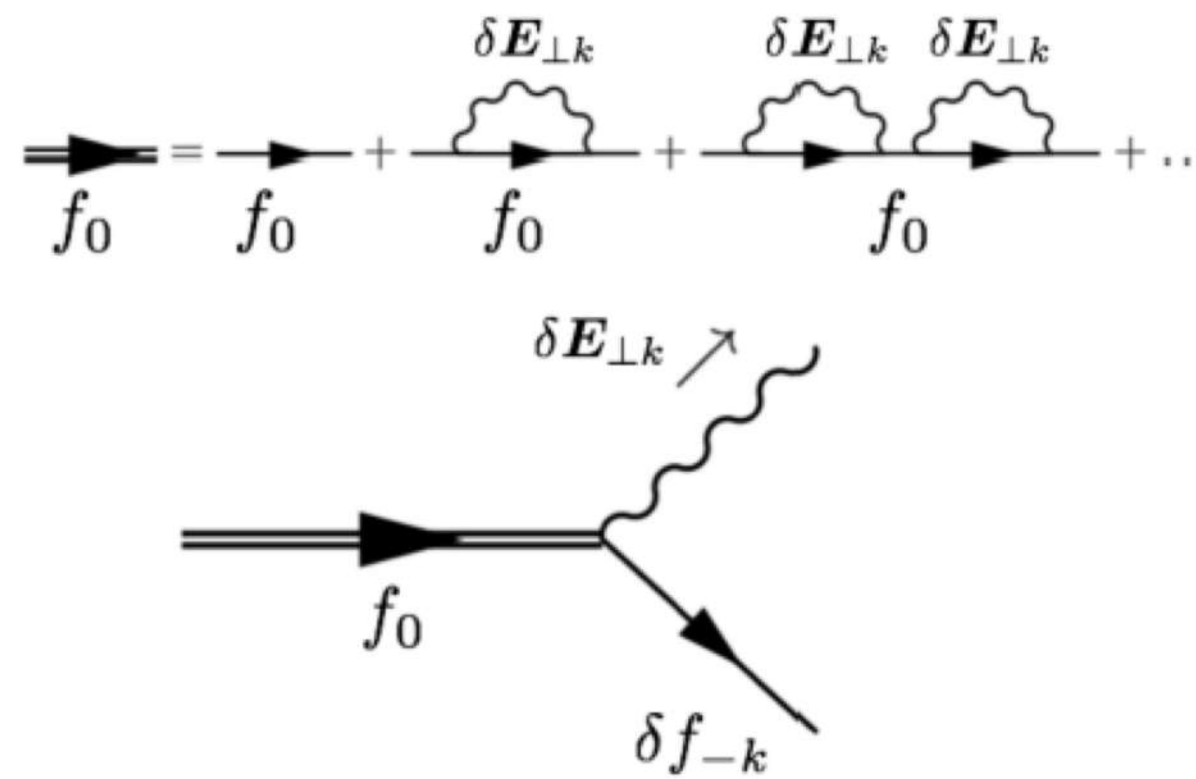
Brief recap :

- Perturbation expansion of particle response in the weak field intensity limit, yields secular terms dominated by emission and absorption of the same k-fluctuation (called diagonal interactions).
- At ℓ-th order, distortion of the reference state, f_0 contains, $\propto t^\ell$ secularity, while k_0 fluctuation contains $\propto t^\ell e^{-i\omega_{k_0}t}$ behavior.
- For growing modes, $\propto t^\ell$ is replaced by $\propto (\omega_k/\gamma_k)^\ell$ behavior [Montgomery 63], [Al'tshul' & Karpman 65], [Mima 73]
  - $\Rightarrow$ rising tone chorus has a quasi-coherent fluctuation spectrum with $|\gamma_k/\omega_k| \ll 1$, with a typical nonlinear evolution time, $\tau_{\rm NL} \sim \gamma_k^{-1}$
  - $\Rightarrow$ for quasi-coherent fluctuation spectrum, particle response is obtained as asymptotic expansion in $|\omega_k|^{-1}\tau_{\rm NL}^{-1} \ll 1$ [Chen & Zonca RMP16]

Renormalized particle response

- Nonlinear dynamics takes place through the self-consistent non-linear evolution of the supra-thermal particle distribution function $f_0(t)$, which fully accounts for the supra-thermal electron current $\delta \boldsymbol{J}_h$ (closes self-consistent nonlinear equations). [Zonca, Tao & Chen 2017]

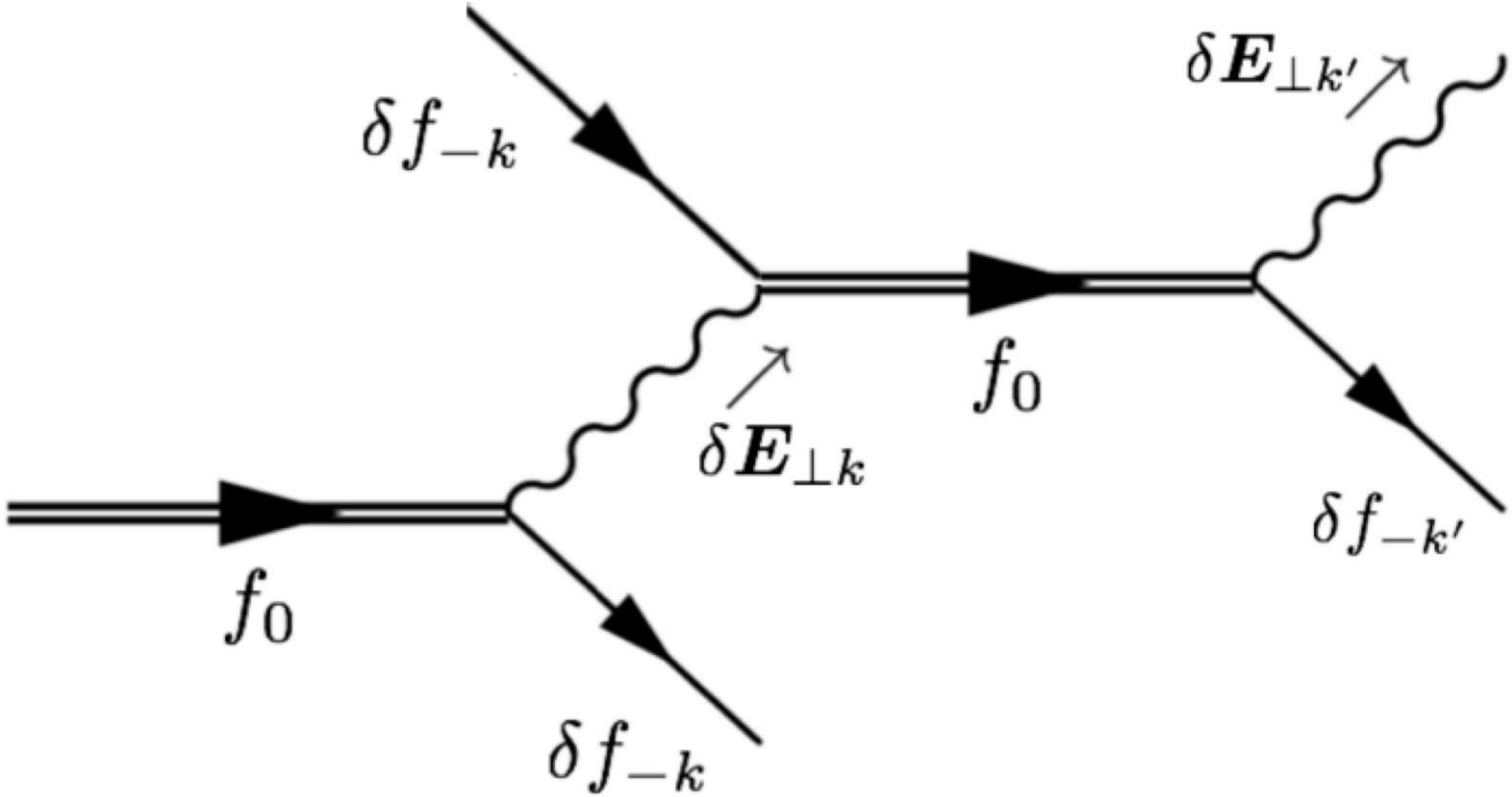




- The renormalized particle response is obtained via a Dyson-like equation approach.

Dyson description of chorus chirping

- Within this theoretical framework, the rising tone chorus chirping is due to the subsequent emission and reabsorption of wave-packets that maximize wave-particle power extraction [Zonca, Tao & Chen 2017]
  $\Rightarrow$ Consistent with, e.g., [Vomvoridis et al. 1982] and [Omura et al. 2008].

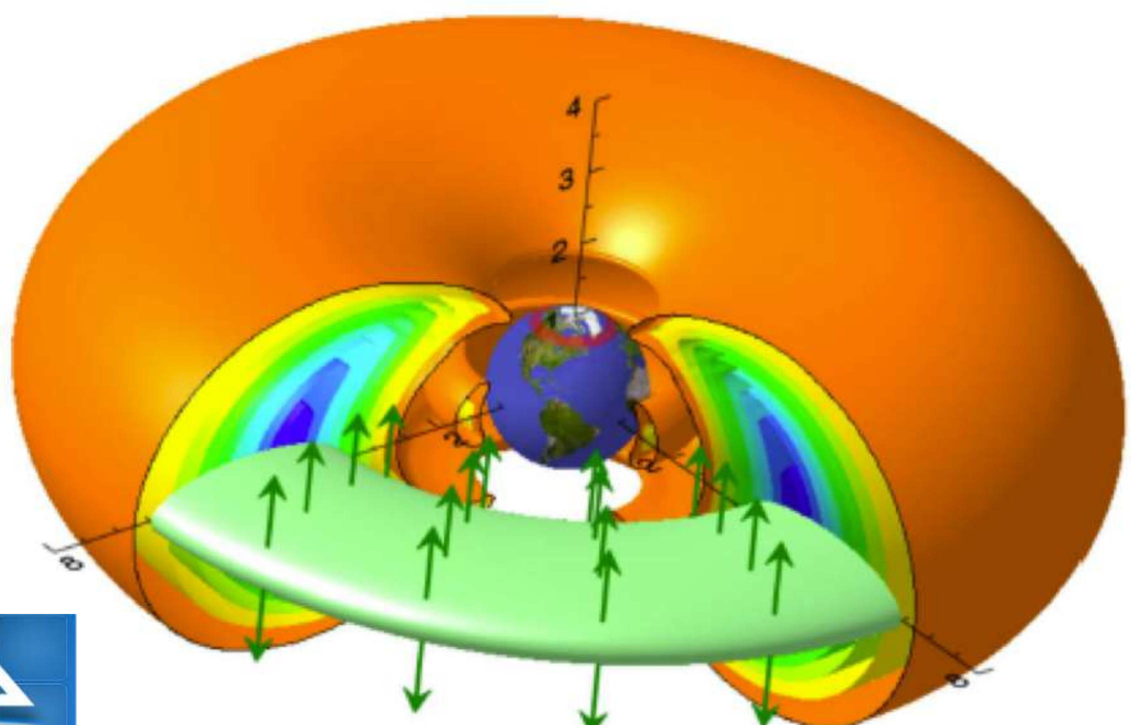


Reduced Dyson description

Reduced Dyson description:

- Derive evolution equation for nonlinear growth and frequency shift
- Based on the existence of a narrow (quasi-coherent) spectrum and its resonant excitation
- Use localized source to ignore magnetic field non-uniformity

$$W(z,t,\omega) + i\Gamma(z,t,\omega) =$$
$$= \frac{\omega(\Omega_e - \omega)^2}{\Omega_e} \frac{n_e}{n} \zeta^4 \left\langle \frac{v_\perp^2}{2} \mathcal{P}_k \left[\frac{k v_\perp^2}{2\omega} \frac{\partial \hat{f}_0}{\partial v_\parallel} - \frac{\Omega_e}{\omega} \hat{f}_0 \right]_{k=K(z=0,\omega)} \right\rangle$$



[Source: https://www.ucalgary.ca/above/science/chorus]

$$\begin{aligned} \delta \bar{f}_k &= \frac{e}{m} \mathcal{P}_k v_\perp \delta \bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \,, \\ \underset{\text{propagator}}{\mathcal{P}_k} &\equiv \left[\Omega + k v_\parallel - \omega_k - i(\partial_t + v_\parallel \partial_z) - \underset{\text{resonance broadening}}{\Delta_k} \right]^{-1} \end{aligned}$$

Reduced Dyson description

- **Reduced Dyson description:**

$$\bar{\Gamma}_{NL}(\omega) = \left[\Omega_e \frac{\partial}{\partial\omega} - \frac{2\Omega_e^2}{k^2\langle v_\perp^2\rangle}\left(1 - \frac{v_{r\omega}}{v_{g\omega}}\right)\right]\Omega_e\frac{\partial}{\partial\omega} \times \sum_{k'}\left[\frac{(\omega'-\omega)^2}{4\Omega_e^2} + \Omega_e^{-2}\partial_t^2\right]^{-1} \times \frac{\langle\langle\omega_{\mathrm{tr}k'}^4\rangle\rangle}{4\Omega_e^4(1 - v_{r\omega'}/v_{g\omega'})^4}\left(\frac{\bar{\Gamma}(\omega)+\bar{\Gamma}(\omega')}{2}\right).$$

$$\Omega_e^{-1}\partial_t\bar{W}(\omega) = \left[\Omega_e \frac{\partial}{\partial\omega} - \frac{2\Omega_e^2}{k^2\langle v_\perp^2\rangle}\left(1 - \frac{v_{r\omega}}{v_{g\omega}}\right)\right]\Omega_e\frac{\partial}{\partial\omega} \times \sum_{k'}\frac{(\omega'-\omega)}{2\Omega_e}\left[\frac{(\omega'-\omega)^2}{4\Omega_e^2} + \Omega_e^{-2}\partial_t^2\right]^{-1} \times \frac{\langle\langle\omega_{\mathrm{tr}k'}^4\rangle\rangle}{4\Omega_e^4(1 - v_{r\omega'}/v_{g\omega'})^4}\left(\frac{\bar{\Gamma}(\omega)+\bar{\Gamma}(\omega')}{2}\right),$$

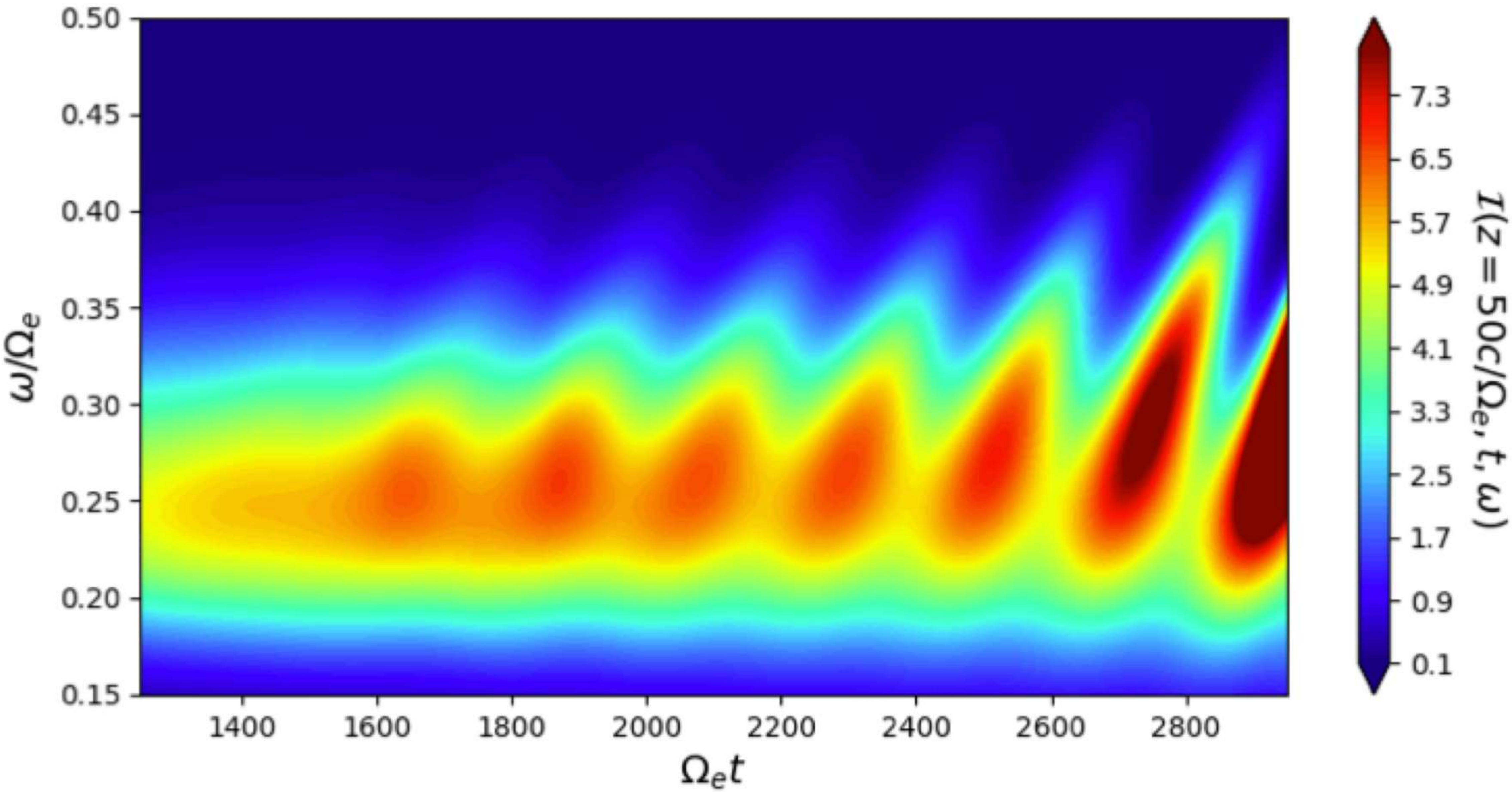
- **Narrow spectrum: recovers Vomvoridis 1982**
  - Analytic result derived for the first time
  - Self similar solution that ballistically propagates

$$\frac{\partial^2}{\partial t^2}\bar{\Gamma}_{NL}(\omega) \simeq \left(\sum_{k'}\frac{\langle\langle\omega_{\mathrm{tr}k'}^4\rangle\rangle}{4\left(1 - v_{r\omega'}/v_{g\omega'}\right)^4}\right)\frac{\partial^2}{\partial\omega^2}\bar{\Gamma}_{NL}(\omega)$$

$$\frac{\partial\omega}{\partial t} = \pm\frac{1}{2}\frac{\langle\langle\omega_{\mathrm{tr}k}^4\rangle\rangle^{1/2}}{(1 - v_{r\omega}/v_{g\omega})^2}$$

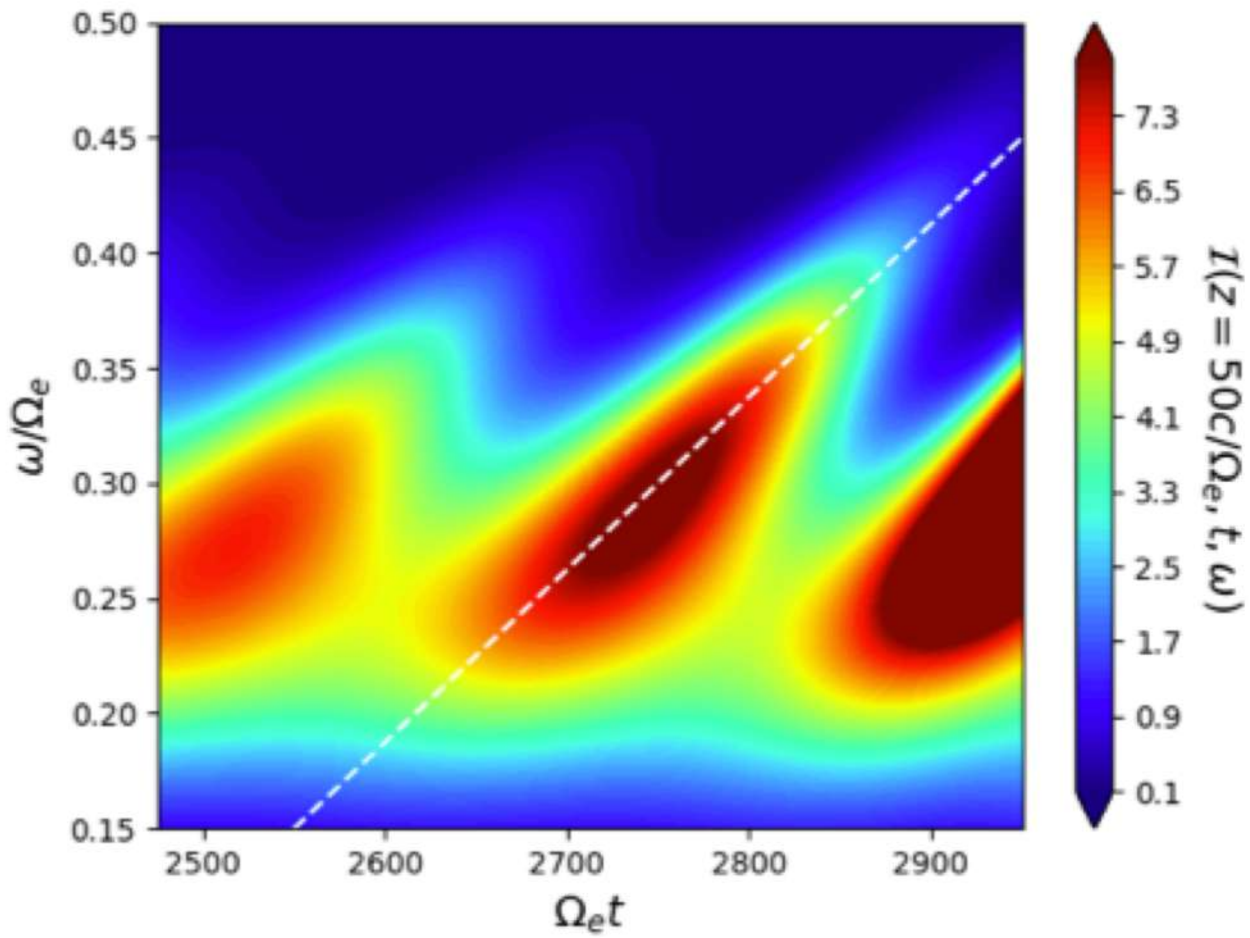
Numerical results

- **Reduced Dyson description:**

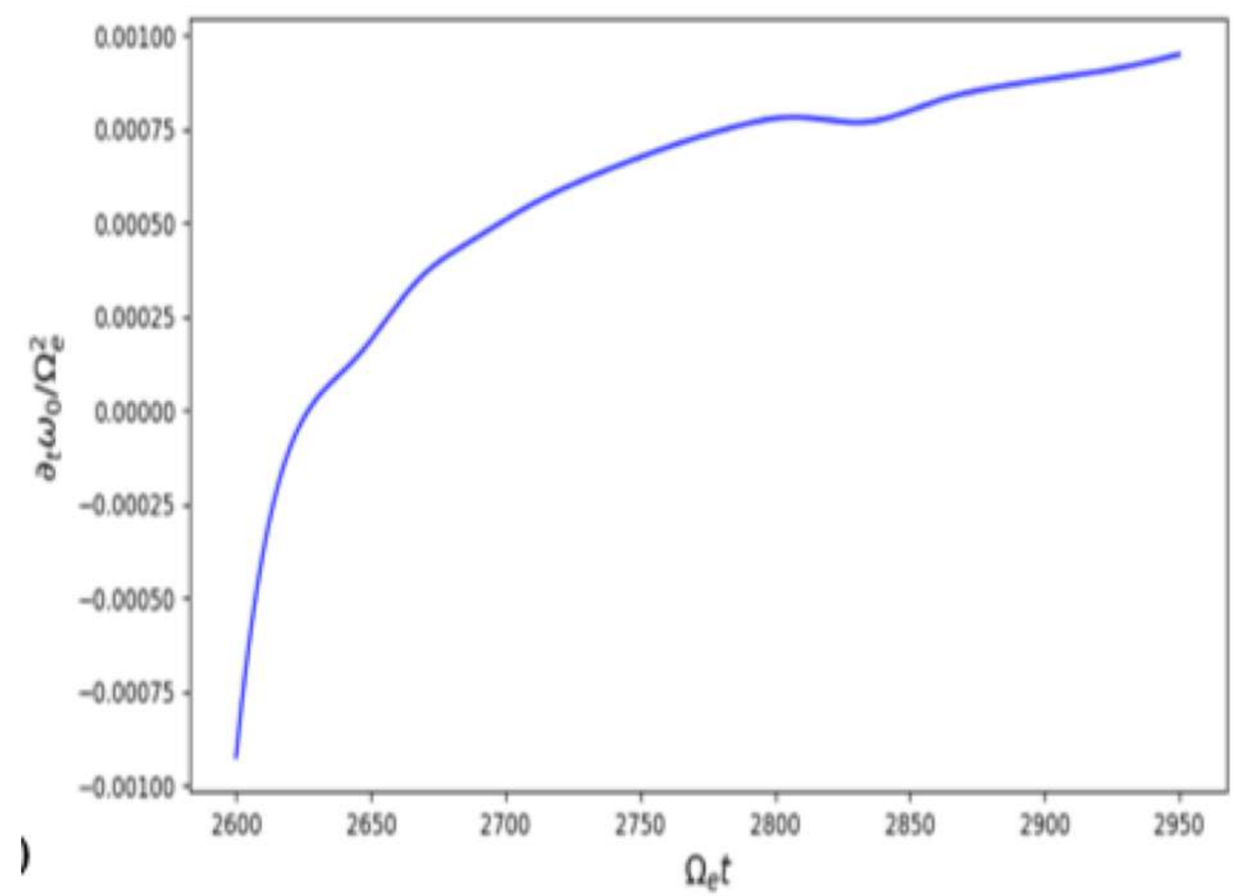


Numerical results

- **Reduced Dyson description**
  - Chirping rate 7.5 x 10-4 consistent with PIC simulation

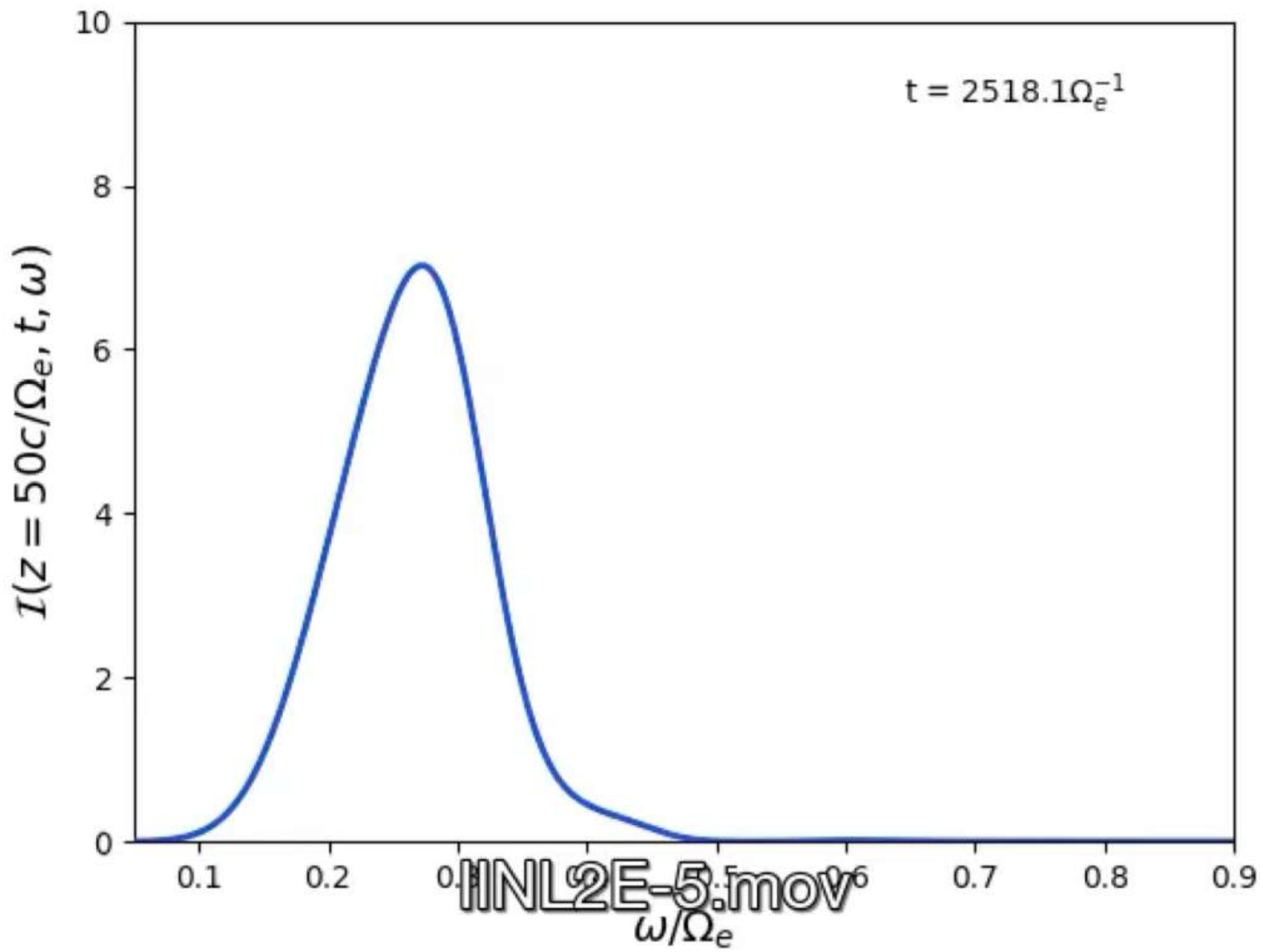


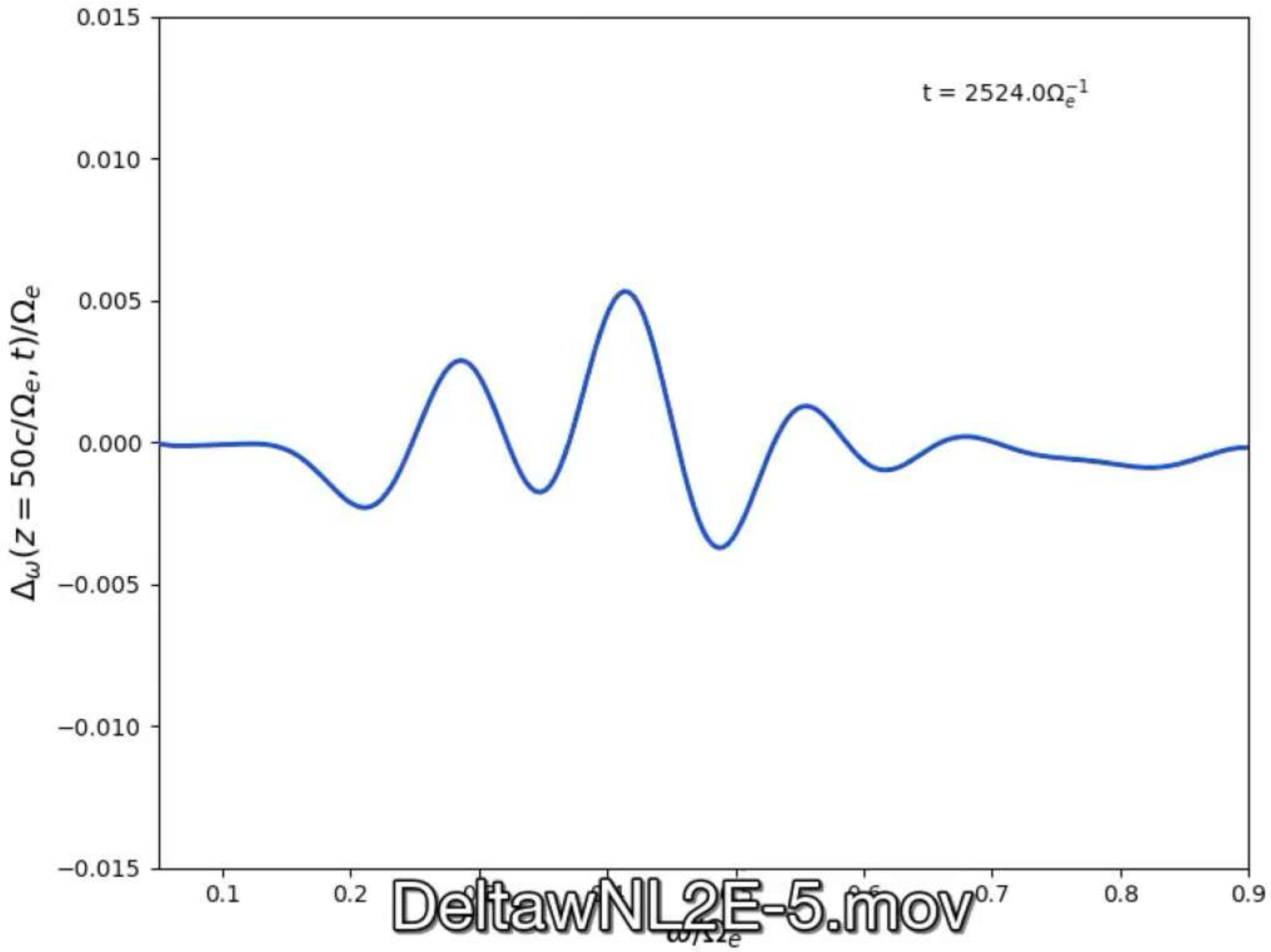
(b)



A one - chirp show

- **A one – chirp show: single chorus element**





Universal chirping phenomena

L. Chen and F. Zonca, RMP **88**, 015008 (2016)

- Wide variety of frequency sweeping phenomena
- Universal process when:
  - Spontaneous emission (instability)
  - Non-perturbative (dispersiveness, continuum)
  - Resulting in secular propagation of phase space resonance structures at rate ~amplitude
  - E.g., chorus chirping (Vomvoridis, 1982; …)

$$\frac{\partial \omega}{\partial t} = \pm \frac{1}{2} \frac{\left\langle \left\langle \omega_{trk}^4 \right\rangle \right\rangle^{1/2}}{(1 - v_{r\omega}/v_{g\omega})^2}$$

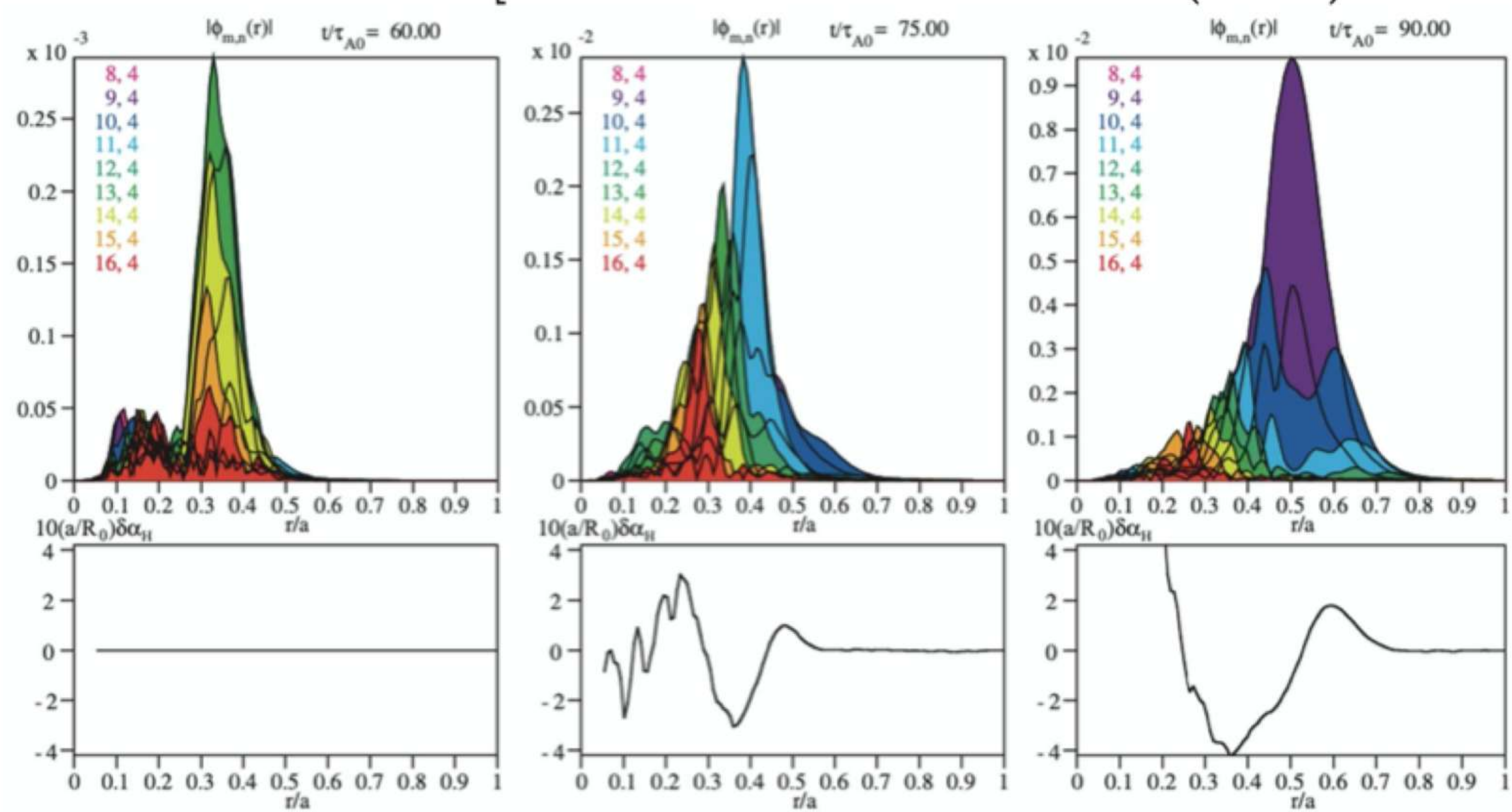
- arXiv:2107.03151; PDF
- arXiv:2106.15026 PDF

  - Predicted and observed in fusion plasmas: (EPM, fishbones)

Back to fusion plasmas - EPM

Theoretical framework systematically verified by NL Hybrid MHD GK simulations (HMGC): (Xin Wang seminar Apr. 09/2021; Sergio Briguglio seminar Apr. 23/2021; Chirping explained)

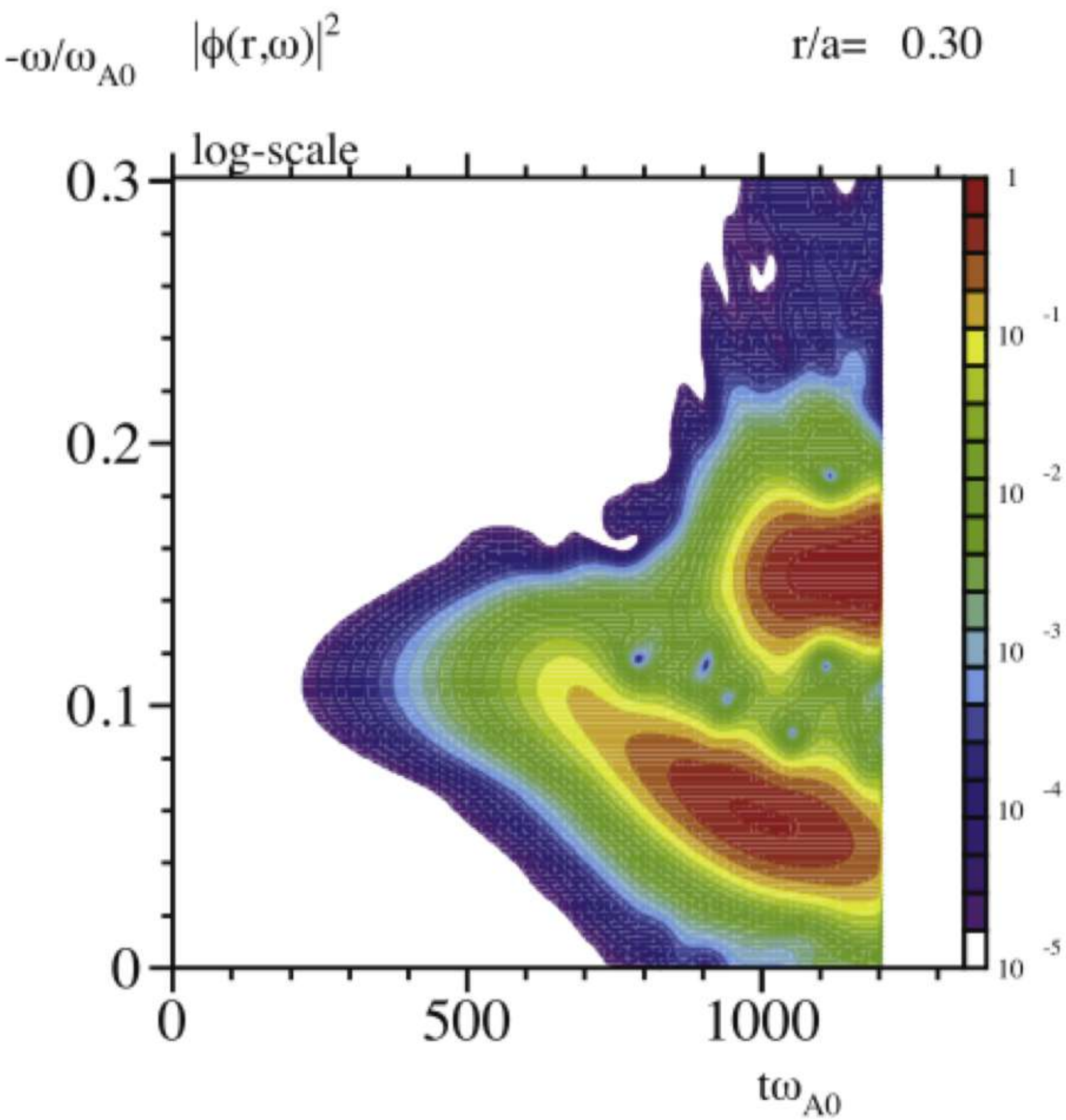
Evidence of EP avalanche [Z. et al Nucl. Fusion 45 (2005) 477 – 484]

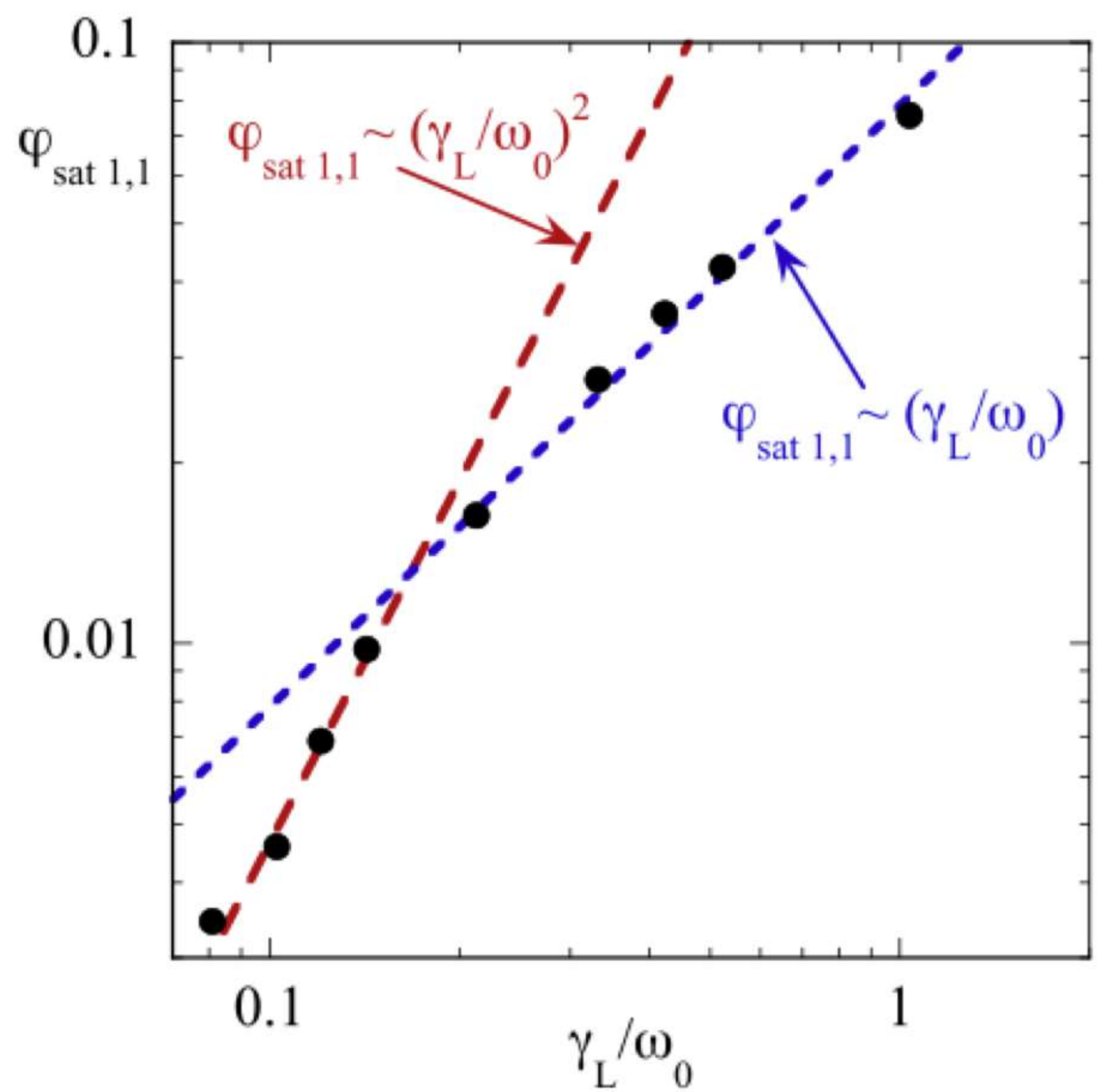


Back to fusion plasmas - fishbones

Theoretical framework systematically verified by NL Hybrid MHD GK simulations (HMGC&HYMAGYC):

(G. Vlad et al. New J. Phys. 18 (2016) 105004)





XHMGC e-fishbones; weakly hollow q profile

A hierarchy of models

Connecting Chorus with DSM:

- The DSM is made of two components: the Dyson Schwinger Equation (DSE) and NLSE
- The NLSE evolves the spectrum consistent with the nonlinear equilibrium solution; the DSE evolves the nonlinear equilibrium given the spectrum
- The chorus problem is the analogue of DSM, where the NLSE is replaced by the WKE

Recovering Quasilinear from DSM:

- This correspondence of DSM to chorus problem allows to illustrate how Quasilinear description is recovered in the proper limit

Quasilinear limit

Dyson – like equation (DSE)

$$(\partial_t + v_\parallel \partial_z) f_0 = \frac{e}{m}\frac{v_\perp}{4}\mathbb{R}\mathrm{e}\sum_k i\left\{\delta\bar{E}_k\left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2}\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}_k^* - \delta\bar{E}_k^*\left[\bar{\partial}_{\mathcal{E}k} + \frac{1}{v_\perp^2}\left(1-\frac{kv_\parallel}{\omega_k}\right)\right]\delta\bar{f}_k\right\}$$

Closed by …

$$\delta\bar{f}_k = \frac{e}{m}\mathcal{P}_k v_\perp \delta\bar{E}_k \bar{\partial}_{\mathcal{E}k} f_0 \, ,$$

$$\mathcal{P}_k \equiv \left[\Omega + kv_\parallel - \omega_k - i(\partial_t \cancel{+} v_\parallel \partial_z) \cancel{-\Delta}_k\right]^{-1}$$

resonance broadening

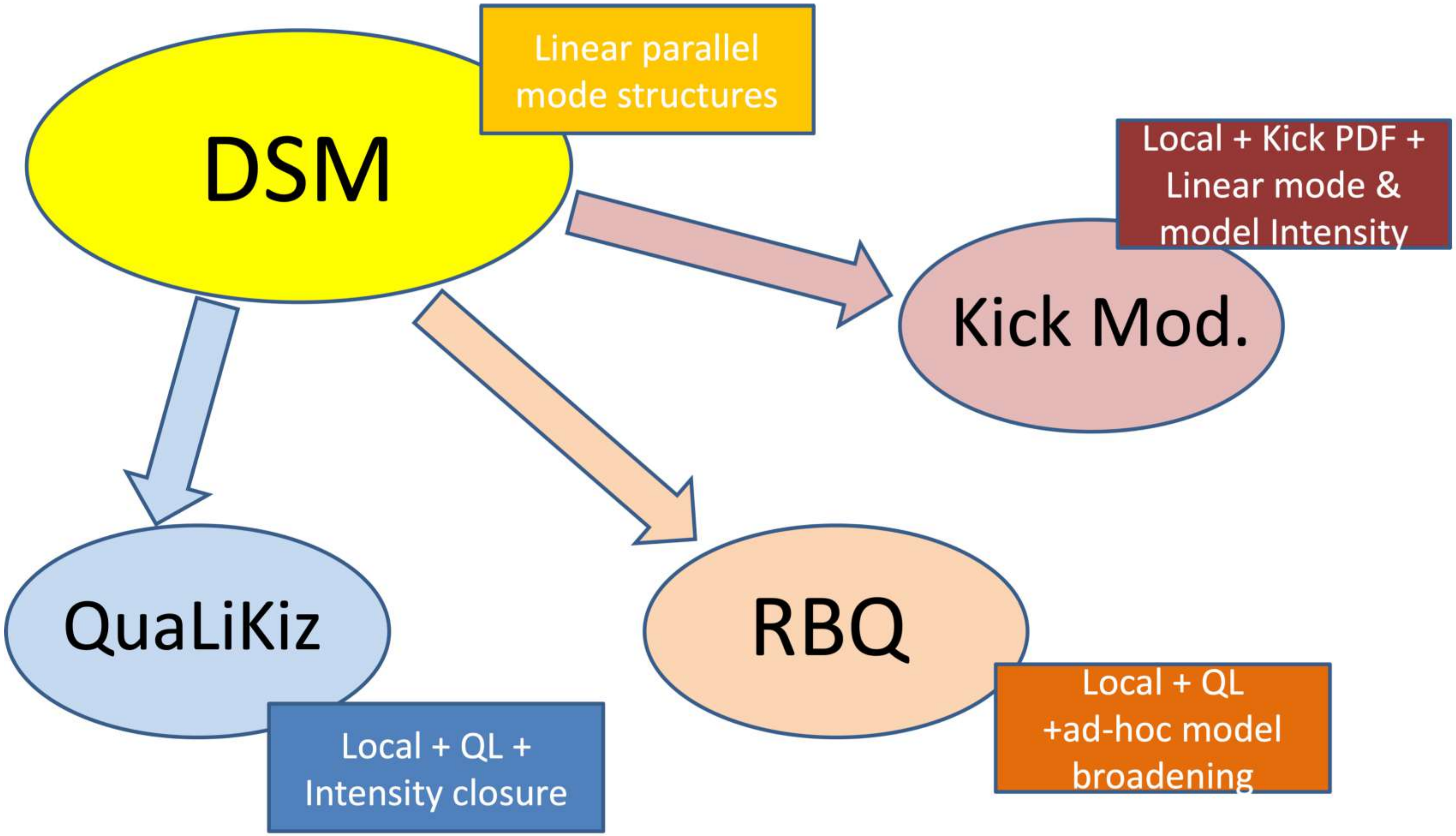
slow spatiotemporal dep.

1. Propagator becomes the linear one
2. The spectrum is considered broad (w.r.t. ω_{trap}) small Kubo #
3. Resonances overlap and $\omega_{trap} >> 1/\tau_{diff} \sim 1/\tau_{NL}$
4. Near marginal stability: γ even smaller than $1/\tau_{diff} \sim 1/\tau_{NL}$

Chorus and fusion plasmas - II

Direct connection with Dyson Schrödinger Model (DSM) :

(ZCFQ seminar Nov 20/2020)



Conclusions

- The theoretical framework for chorus emission is fully developed based on the DSE+WKE solution. Supported by extensive numerical verification.
- It describes universal chirping phenomena in space/lab plasmas
- The chorus problem is the analogue of DSM, where the NLSE is replaced by the WKE. This provides a direct route to demonstration that the DSE+WKE/NLSE solution reduces to QL in the proper limit
- This illustrates the DSM as a rubust framework for a hierarchy of reduced models, yielding QL and local CG
- These results suggest the systematic application of DSM to a broader class of problems dealing with transport in phase space (beyond AE/EPM/fishbones) ➔ ATEP/TSVV#10 EUROfusion Proj.

THANK YOU!

Your questions are welcome