A nonlinear Schrödinger equation for the description of geodesic-acoustic-modes in tokamaks

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Abstract

The geodesic-acoustic-mode (GAM) is a plasma oscillation observed in fusion reactors with toroidal geometry and is recognized to be the nonstationary branch of the zonal flows [1]. Prior studies [2] have established that as a direct consequence of nonlinear gyrokinetic theory, the GAM dynamics is well described by an equation of Schrödinger type - i. e. an equation whose linear contribution is exactly of the same form as the linear Schrödinger equation, while the nonlinear dynamics necessitates an integro-differential expression.

The presented work takes a closer look into the nonlinear contributions by deriving approximate, but well-defined analytic expressions from the (exact) integro-differential operators, using a state-of-the-art gyrokinetic framework [3]. In accordance with prior numerical studies [4, 5] a cubic nonlinearity is retrieved. The nonlinearity is found to act on a similar time scale as that characterizing the linear dispersion.

The cubic term stems from an interaction of quadratic structures generated by the GAMs (with oscillation frequencies that are either zero or twice the GAM frequency) with the GAM itself. Comparisons with gyrokinetic simulations using the global particle-in-cell code ORB5 [6] show very good quantitative agreement with the analytical theory.

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