Hybrid MHD-Gyrokinetic Simulations of Global Alfvén Modes in Fusion Plasmas

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Outline of the presentation

• Introduction and Motivation
• Numerical Model
• Numerical Simulations
  – Single-$n$ Simulations
  – Multiple-$n$ Simulations
  – Role of MHD and Energetic Particle Non-linearities
• Conclusions
The study of the effects of energetic particles (EPs), characterized by supra-thermal velocity, on magnetically confined plasmas approaching ignited conditions is a very relevant field of investigation in the magnetic confinement plasma community since several tens of years.

The main concern is that the mutual interaction of globally extended Alfvén modes and EPs (as, e.g., the fusion generated alpha particles and/or the energetic ions accelerated by auxiliary heating systems) could drive shear Alfvén modes unstable and, eventually, enhance the radial transport of the EPs themselves.

This can results, in turn, in increased difficulties in approaching and/or maintaining the ignited conditions (the EPs being displaced from the inner, hot core of the plasma discharge toward the edge, colder region before slowing down and heating the bulk species), or even damaging the vacuum vessel, if the EPs escape the plasma and hit the reaction chamber.

Single-\( n \) dynamics quite extensively studied in the past; Multiple-\( n \) effects on radial transport only recently addressed (\( n \): toroidal mode number).

Introduction and Motivation
Key ingredients of our model are:

- **Shear Alfvén waves** (to be studied in toroidal geometry => magnetically confined fusion in tokamak devices);
- Effect of **Energetic Particles (EPs)** on Alfvénic modes;
- The mutual interaction of **globally extended Alfvén modes and EPs**;
- Their effect on the **energetic particle radial density profile** (radial transport).

Hybrid MHD-Gyrokinetic model:

- **Magnetohydrodynamics** is used to describe the bulk plasma and Alfvén waves (Alfvén continua, various Alfvén modes, e.g., TAEs, RSAEs, ...);
- **Gyrokinetics** is used to describe the dynamics of the EPs, in order to keep the detail of the resonant interaction between EPs and MHD waves;
- The two plasma components (thermal and EPs) being coupled (W. Park et al., Phys. Fluids 1992) via the divergence of the pressure tensor term of the EPs entering in the extended momentum equation of the bulk plasma.
• Thermal (core) plasma:
  – described by reduced $O(\varepsilon_0^3)$ visco-resistive MHD equations in the limit of $\beta=0$
  ($\varepsilon_0 \equiv a/R_0$) \(\Rightarrow\) equilibria with shifted circular magnetic surfaces can be investigated
  – MHD fields: $\psi, \phi$ (poloidal magnetic flux function and electrostatic potential)

• Energetic Particle population:
  – described by the non-linear gyrokinetic Vlasov equation, expanded up to order $O(\varepsilon)$
    and $O(\varepsilon_B)$, with $\varepsilon \sim \rho_H/L_n$ the gyrokinetic ordering parameter and $\varepsilon_B \sim \rho_H/L_B < \varepsilon$, in the
    $k_\perp \rho_H << 1$ limit (guiding-center approximation);
  – coupling term between MHD and GK is the energetic particle pressure: $\Pi_\perp, \Pi_\parallel$;
  – fully retaining magnetic drift orbit widths;
  – solved by particle-in-cell (PIC) techniques.

$k_\perp$: perpendicular component of the wave vector;
$\rho_H$: energetic ion Larmor radius;
$L_n, L_B$: the equilibrium density and magnetic field scale lengths.

• Toroidal coordinates system ($r, \theta, \varphi$)
**Equilibrium:**

- $\varepsilon_0 = a/R_0 = 0.1; \ T_H/T_{H0} = 1, \ \rho_{H0}/a = 0.01, \ \nu_{H0}/\nu_{A0} = 1, \ m_H/m_i = 2; \ n_{H0}/n_i = 1.75 \times 10^{-3}$
- $q(r) = q_0 + (q_a - q_0)(r/a)^2$ with $q_0 = 1.1$ and $q_a = 1.9$
- $n_i \propto 1/q^2 \Rightarrow$ toroidal Alfvén gaps for different $n$ aligned
- EP equilibrium distribution function $F_{H,\text{eq}}$ is isotropic Maxwellian
- $n_H = n_{H0} \exp(-19.53 \ (1 - \psi/\psi_0)^2)$

**Fourier space** for perturbed quantities: $(m,n)$ and $(-m,-n)$ modes included in the simulations; $1 \leq n \leq 15$, $n \ q_0 \leq m \leq n \ q_a$

**Grid and particle per cell for GK:**
- $N_{\text{ppc}} = 8, \ N_{r,\text{GK}} = 256,$
- $N_{\theta,\text{GK}} = 160, \ N_{\varphi,\text{GK}} = 80,$
- $N_p = N_{\text{ppc}} N_{r,\text{GK}} N_{\theta,\text{GK}} N_{\varphi,\text{GK}} \approx 26.2 \times 10^6$

With $s$ defined as: $s = \sqrt{|\psi_{\text{eq}} - \psi_0|/|\psi_{\text{edge}} - \psi_0|}$
$\psi_{\text{eq}}$ the equilibrium magnetic poloidal flux function, and $\psi_0$ and $\psi_{\text{edge}}$ its values, at the magnetic axis and at the edge, respectively.
Single-\(n\) simulations on A1-A3 partitions of Marconi-fusion:

\[ \gamma \propto \omega_{*H} = k \cdot v_{*H} \propto nq \]

\((\omega_{*H} \text{ is the diamagnetic frequency of the “hot” particles; modes tap energy from EP spatial gradients})\)

\[ \gamma / \omega_{A0} \]

\[ \omega / \omega_{A0} \]

\(\gamma\) decreased by FOW effects (eigenfunction spatial width smaller than EP drifts)

Saturation occurs because of axisymmetric modification of EP distribution (in configuration and/or velocity space)
Single-\(n\) simulations-2

Frequency vs. \(r\) spectra (\(t\omega_{A0}=120\)):
Single-$n$ simulations-3

Frequency vs. $r$ spectra ($t\omega_{A0}=360$):

- $n=1$
- $n=2$
- $n=3$
- $n=4$
- $n=5$
- $n=6$
- $n=7$
- $n=8$
- $n=9$
- $n=10$
- $n=11$
- $n=12$
- $n=13$
- $n=14$
- $n=15$
Spectrograms (ω vs. t):
frequency chirping up and down after saturation

ω/ω0

n=1

n=2

n=3

n=4

n=5

n=6

n=7

n=8

n=9

n=10

n=11

n=12

n=13

n=14

n=15
HPC is important...

Typical Non-linear simulation on Marconi-fusion A1 partition:

- \( N_{\text{toroidal Fourier components}} = 10 \);
- \( M_{\text{poloidal Fourier components}} = 76 \);
- \( t\omega_{A0} = 355.2, \Delta t\omega_{A0} = 0.02 \Rightarrow N_{\text{steps, MHD}} = 17760, N_{\text{steps, GK}} = 5920 \) (+sub-cycling when required)
- 120 nodes, 36 cores/node: 4320 cores; elapsed time \( \approx 24h \)

Typical Non-linear simulation on Marconi-fusion A3 partition:

- \( N_{\text{toroidal Fourier components}} = 15 \);
- \( M_{\text{poloidal Fourier components}} = 142 \);
- \( t\omega_{A0} = 393.6, \Delta t\omega_{A0} = 0.02 \Rightarrow N_{\text{steps, MHD}} = 19680, N_{\text{steps, GK}} = 6560 \) (+sub-cycling when required)
- 120 nodes, 48 cores/node: 5760 cores; elapsed time \( \approx 24h \)
Multiple-$n$ simulations

- $n=1,\ldots,15$
- Both fluid (mode-mode) and EP non-linearities included
- $n=0$ not evolved (eventual formation of zonal structure not considered)
- Non-linear coupling drives all the modes unstable and makes them saturate almost simultaneously
- No evidence of “domino effect”
- Saturation amplitude of MHD fields is smaller than single-$n$ simulations

Single-$n$ simulations
Multiple-\(n\) simulations-2

Frequency vs. \(r\) spectra (\(t\omega_{A0}=150\)):
Comparison between multiple-\(n\) and single-\(n\) simulations, radial transport of EPs at saturation:

**Poloidal component of \(E_\theta\)**
- Multiple-\(n\) simulation saturates at larger values
- Multiple-\(n\) eigenfunctions and frequency spectrum at saturation are broader than in single-\(n\) simulations (\(n=4\))

**Radial transport**
- The EP radial density profile after saturation for the multiple-\(n\) simulation is broaden when compared with the most unstable, single-\(n\) case (\(n=4\))
Both MHD non-linearities and mode coupling through EP non-linearities are important

- In the multiple-\(n\) simulation, EP drive only (blue curves) already gives NL coupling (see, e.g., the \(n=1\) case, which, for the single-\(n\) simulation, i.e. with only EP drive as obtained by fluctuating fields with only \(n=1\) components, is stable);
- in the multiple-\(n\) simulation with EP drive plus fluid non-linearities (red curves), fluid non-linearities anticipate a bit in time, without changing appreciably the growth-rate, the growing for the sub-dominant modes (not for the dominant one, \(n=4\); the other stronger one, \(n=10\), is almost unchanged during its linear phase, but it is non-linearly driven at higher overshooting after the first roll over), thus typically making the individual \(n\) components to overshoot more compared with the multiple-\(n\) simulation with only EP non-linearities.
Conclusions.

- Comparison between single-$n$ and multiple-$n$ simulations of Alfvénic modes has been performed, using the HMGC code; multiple-$n$ simulations with the toroidal mode numbers $1 \leq n \leq 15$ have been considered.
- In single-$n$ simulations, the equilibrium considered (circular cross section, low inverse aspect ratio, $\varepsilon_0=0.1$), in presence of a Maxwellian EP population, result as either stable ($n=1$), weakly unstable ($n=2, 3, 13, 14, 15$) or unstable ($4 \leq n \leq 12$); a variety of modes are observed (TAEs, upper and lower KTAEs, EPMs). Weak or negligible EPs radial transport is observed at saturation, for all the toroidal mode numbers considered.
- In multiple-$n$ simulation, NL mode-mode coupling from MHD terms and mediated by EP term (three wave coupling), strongly drives sub-dominant modes already during the linear growth phase of the dominant modes; radial profiles of e.m. fields ($\psi, \phi$) and real frequencies are substantially different from linearly unstable, single-$n$ modes; all the toroidal modes saturate almost simultaneously, inducing enhanced EP transport (w.r.t. the single-$n$ simulations). No evidence of the so-called “domino effect” is observed.

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Publications, presentations

Marconi-fusion 1st cycle project HYMHDGK:
179000 node hours (153000 on A1 + 26000 on A2), starting from 17th October 2016 and ending on 31st December 2017.

Publications and presentations:

• G. Vlad et al. 2018: Nucl. Fusion, accepted manuscript (https://doi.org/10.1088/1741-4326/aaaed1)
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Multiple-\(n\) simulation

Toroidal mode numbers \(1 \leq n \leq 15\)

- **Standard** picture:
  1. strongest modes saturate first, because of non-linear (NL) energetic particle (EP) terms (e.g., flattening of EP radial density profile, at least for the resonant EP fraction);
  2. sub-dominant modes can, on turn, be driven unstable (or more unstable) because of the modifications to the EP distribution induced by the saturation of the dominant modes

- **Novel** observations from these set of simulations:
  1. NL mode-mode coupling from MHD terms, or mediated by EP term, strongly drives sub-dominant modes already during the linear growth phase of the dominant modes;
  2. sub-dominant modes driven non-linearly have field \((\psi, \phi)\) radial profiles and real frequencies substantially different from linearly unstable, single-\(n\) modes;
  3. all the toroidal modes saturate almost simultaneously, inducing an enhanced EP transport (enhanced w.r.t. the single-\(n\) simulations);
  4. On a longer time scale, after saturation of the faster modes, other subdominant modes can, in turn, be driven unstable (or more unstable) because of the modifications to the EP distribution (as \#2 above, not investigated here…)

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Mode coupling through the EP term (1).

Hybrid reduced $O(\varepsilon_0^3)$ MHD equations (HMG) (Briguglio et al., Phys. Plasmas 2, 3711 (1995); Wang et al., Phys. Plasmas 18, 052504 (2011)).

\[
\frac{\partial \psi}{\partial t} = \frac{R^2}{R_0} \nabla \psi \times \nabla \phi \cdot \nabla U + \frac{B_0}{R_0} \frac{\partial U}{\partial \phi} + \eta \frac{c^2}{4\pi} \Delta^* \psi + O(\varepsilon^4 \nu A_\phi),
\]

\[
\rho \left( \frac{D}{Dt} + \frac{2}{R_0} \frac{\partial U}{\partial Z} \right) \nabla^2 U + \nabla \cdot \left( \frac{D}{Dt} + \frac{1}{R_0} \frac{\partial U}{\partial Z} \right) \nabla U
= \frac{1}{4\pi} \mathbf{B} \cdot \nabla \Delta^* \psi + \frac{1}{R_0} \nabla \cdot \left[ R^2 (\nabla P + \nabla \cdot \Pi_H) \times \nabla \phi \right]
+ O\left( \varepsilon^4 \rho \frac{\nu^2 A}{a^2} \right),
\]

\[
\Pi_s(t, \mathbf{x}) = \frac{1}{m_s^2} \int d\mathbf{Z}_s \delta(\mathbf{x} - \mathbf{R}_s) \frac{\Omega_s \bar{M}_s}{m_s} \left[ \mathbf{I} + \mathbf{b} \mathbf{b} \left( \frac{\bar{V}^2 - \frac{\Omega_s \bar{M}_s}{m_s}}{m_s} \right) \right] \delta(\mathbf{x} - \mathbf{R})
\]

\[
a_\parallel = (e_s/c)(R_0/R)\psi; \quad U = -c\phi/B_0;
\]

\[
\psi \text{ is the magnetic stream function; } \phi \text{ is the e.s. potential; “s” stay for EP species, thermal ions, …}
\]

\[
Z^i = (\mathbf{R}, \mathbf{M}, \mathbf{V}) \text{ are the gyrocenter coordinates, } dZ^i/dt \text{ the phase-space velocities, } (dZ^i/dt)_{\text{pert}} \text{ the perturbed ones; } \bar{F}_s;_{\text{eq}} \text{ the equilibrium distribution function of the “s” EP species.}
\]

\[
\frac{d\mathbf{R}}{dt} = \bar{V} \mathbf{b} + \frac{e_s}{m_s \Omega_s} \mathbf{b} \times \nabla \phi - \frac{\bar{V}}{m_s \Omega_s} \mathbf{b} \times \nabla a_\parallel + \left[ \frac{\bar{M}}{m_s} + \frac{\bar{V}}{\Omega_s} \left( \bar{V} + a_\parallel \right) \right] \frac{\mathbf{b} \times \nabla \ln \mathbf{B}}{m_s}
\]

\[
\frac{d\mathbf{V}}{dt} = \frac{1}{m_s} \mathbf{b} \cdot \left\{ \left[ \frac{e_s}{\Omega_s} \left( \bar{V} + \frac{a_\parallel}{m_s} \right) \nabla \phi + \frac{\bar{M}}{m_s} \nabla a_\parallel \right] \times \nabla \ln \mathbf{B} \right\} + \frac{e_s}{m_s \Omega_s} \mathbf{b} \cdot \nabla a_\parallel \times \nabla \phi
- \frac{\Omega_s \bar{M}_s}{m_s} \mathbf{b} \cdot \nabla \ln \mathbf{B}
\]

Vlasov eq. for gyrocenter distribution function $F_s$:

\[
\left( \frac{\partial}{\partial t} + \frac{dZ^i}{dt} \frac{\partial}{\partial Z^i} \right) \bar{F}_s = 0
\]

Or, in term of $\delta F_s$:

\[
\left( \frac{\partial}{\partial t} + \frac{dZ^i}{dt} \frac{\partial}{\partial Z^i} \right) \delta F_s = - \left( \frac{dZ^i}{dt} \right)_{\text{pert}} \frac{\partial}{\partial Z^i} \bar{F}_s;_{\text{eq}}
\]
Mode coupling through the EP term (2).

Mode coupling through the EP term \( \nabla \cdot \Pi_H \) means that a toroidal mode number “\( n \)” gets a contribution from quantities related to the EPs characterized by modes “\( n_1 \)” and “\( n_2 \)” such that:

\[ n = n_1 + n_2 \] (three waves scheme)

These kind of terms are indeed present, as can be recognized schematically by the following:

\[ \Pi_H \propto \delta F_H: \]

\[
\left( \frac{\partial}{\partial t} + \left( \frac{d\tilde{Z}^i}{dt} \right)_{\text{eq}} \right) \frac{\partial}{\partial \tilde{Z}^i} \delta \tilde{F}_H = - \left( \frac{d\tilde{Z}^i}{dt} \right)_{\text{pert}} \frac{\partial}{\partial \tilde{Z}^i} \tilde{F}_{H,\text{eq}}
\]

After formally splitting the generalized velocities in the l.h.s. in unperturbed “unpert” and perturbed “pert” ones:

\[
\left[ \frac{\partial}{\partial t} + \left( \frac{d\tilde{Z}^i}{dt} \right)_{\text{eq}} \frac{\partial}{\partial \tilde{Z}^i} \right] \delta \tilde{F}_H = - \left( \frac{d\tilde{Z}^i}{dt} \right)_{\text{pert}} \frac{\partial}{\partial \tilde{Z}^i} \tilde{F}_{H,\text{eq}} - \left( \frac{d\tilde{Z}^i}{dt} \right)_{\text{pert}} \frac{\partial}{\partial \tilde{Z}^i} \delta \tilde{F}_H.
\]

And passing to toroidal Fourier components (equilibrium: “\( n=0 \)”; perturbed: “\( n \)”):

\[
\left[ \frac{\partial}{\partial t} + \left( \frac{d\tilde{Z}^i}{dt} \right)_{0} \frac{\partial}{\partial \tilde{Z}^i} \right] \delta \tilde{F}_{H,n} = - \left( \frac{d\tilde{Z}^i}{dt} \right)_{n} \frac{\partial}{\partial \tilde{Z}^i} \left( \tilde{F}_{H:0} + \delta \tilde{F}_{H:0} \right) - \Sigma_{\tilde{n} \neq 0} \left( \frac{d\tilde{Z}^i}{dt} \right)_{n-\tilde{n}} \frac{\partial}{\partial \tilde{Z}^i} \delta \tilde{F}_{H:\tilde{n}}
\]

From the last, convolution term, it can arise NL coupling between different \( n \)’s through the EP term.