

NAG Library Routine Document

S30QCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

S30QCF computes the Bjerk Sund and Stensland (2002) approximation to the price of an American option.

2 Specification

```
SUBROUTINE S30QCF(CALPUT, M, N, X, S, T, SIGMA, R, Q, P, LDP, IFAIL)
INTEGER          M, N, LDP, IFAIL
double precision X(M), S, T(N), SIGMA, R, Q, P(LDP,N)
CHARACTER*1      CALPUT
```

3 Description

S30QCF computes the price of an American option using the closed form approximation of Bjerk Sund and Stensland (2002). The time to maturity, T , is divided into two periods, each with a flat early exercise boundary, by choosing a time $t \in [0, T]$, such that $t = \frac{1}{2}(\sqrt{5} - 1)T$. The two boundary values are defined as $\tilde{x} = \tilde{X}(t)$, $\tilde{X} = \tilde{X}(T)$ with

$$\tilde{X}(\tau) = B_0 + (B_\infty - B_0)(1 - \exp\{h(\tau)\}),$$

where

$$h(\tau) = -(b\tau + 2\sigma\sqrt{\tau}) \left(\frac{X^2}{(B_\infty - B_0)B_0} \right),$$

$$B_\infty \equiv \frac{\beta}{\beta - 1}X, \quad B_0 \equiv \max\left\{X, \left(\frac{r}{r - b}\right)X\right\},$$

$$\beta = \left(\frac{1}{2} - \frac{b}{\sigma^2}\right) + \sqrt{\left(\frac{b}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}}.$$

with $b = r - q$, the cost of carry, where r is the risk-free interest rate and q is the annual dividend rate. Here X is the strike price and σ is the annual volatility.

The price of an American call option is approximated as

$$\begin{aligned} P_{\text{call}} = & \alpha(\tilde{X})S^\beta - \alpha(\tilde{X})\phi(S, t|\beta, \tilde{X}, \tilde{X}) + \\ & \phi(S, t|1, \tilde{X}, \tilde{X}) - \phi(S, t|1, \tilde{x}, \tilde{X}) - \\ & X\phi(S, t|0, \tilde{X}, \tilde{X}) + X\phi(S, t|0, \tilde{x}, \tilde{X}) + \\ & \alpha(\tilde{x})\phi(S, t|\beta, \tilde{x}, \tilde{X}) - \alpha(\tilde{x})\Psi(S, T|\beta, \tilde{x}, \tilde{X}, \tilde{x}, t) + \\ & \Psi(S, T|1, \tilde{x}, \tilde{X}, \tilde{x}, t) - \Psi(S, T|1, X, \tilde{X}, \tilde{x}, t) - \\ & X\Psi(S, T|0, \tilde{x}, \tilde{X}, \tilde{x}, t) + X\Psi(S, T|0, X, \tilde{X}, \tilde{x}, t), \end{aligned}$$

where α , ϕ and Ψ are as defined in Bjerk Sund and Stensland (2002).

The price of a put option is obtained by the put-call transformation,

$$P_{\text{put}}(X, S, T, \sigma, r, q) = P_{\text{call}}(S, X, T, \sigma, q, r).$$

4 References

Bjerk Sund P and Stensland G (2002) Closed form valuation of American options **Discussion Paper 2002/09 NHH Bergen Norway** <http://www.nhh.no/>

Genz A (2004) Numerical Computation of Rectangular Bivariate and Trivariate Normal and t Probabilities *Statistics and Computing* **14** 151–160

5 Parameters

- 1: CALPUT – CHARACTER*1 *Input*
On entry: determines whether the option is a call or a put.
 CALPUT = 'C'
 A call. The holder has a right to buy.
 CALPUT = 'P'
 A put. The holder has a right to sell.
Constraint: CALPUT = 'C' or 'P'.

- 2: M – INTEGER *Input*
On entry: the number of strike prices to be used.
Constraint: $M \geq 1$.

- 3: N – INTEGER *Input*
On entry: the number of times to expiry to be used.
Constraint: $N \geq 1$.

- 4: X(M) – **double precision** array *Input*
On entry: X(i) must contain X_i , the i th strike price, for $i = 1, 2, \dots, M$.
Constraint: $X(i) \geq z$ and $X(i) \leq 1/z$, where $z = \text{X02AMF}()$, the safe range parameter, for $i = 1, 2, \dots, M$.

- 5: S – **double precision** *Input*
On entry: S , the price of the underlying asset.
Constraint: $S \geq z$ and $S \leq 1/z$, where $z = \text{X02AMF}()$, the safe range parameter and $S^\beta < \frac{1}{z}$ where β is as defined in Section 3.

- 6: T(N) – **double precision** array *Input*
On entry: T(i) must contain T_i , the i th time, in years, to expiry, for $i = 1, 2, \dots, N$.
Constraint: $T(i) \geq z$, where $z = \text{X02AMF}()$, the safe range parameter, for $i = 1, 2, \dots, N$.

- 7: SIGMA – **double precision** *Input*
On entry: σ , the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
Constraint: SIGMA > 0.0.

- 8: R – **double precision** *Input*
On entry: r , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
Constraint: $R \geq 0.0$.

- 9: Q – **double precision** *Input*
 On entry: q , the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
 Constraint: $Q \geq 0.0$.
- 10: P(LDP,N) – **double precision** array *Output*
 On exit: the $m \times n$ array P contains the computed option prices.
- 11: LDP – INTEGER *Input*
 On entry: the first dimension of the array P as declared in the (sub)program from which S30QCF is called.
 Constraint: $LDP \geq M$.
- 12: IFAIL – INTEGER *Input/Output*
 On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CALPUT $\neq$ 'C' or 'P'.

IFAIL = 2

On entry, $M \leq 0$.

IFAIL = 3

On entry, $N \leq 0$.

IFAIL = 4

On entry, $X(i) < z$ or $X(i) > 1/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 5

On entry, $S < z$ or $S > 1/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 6

On entry, $T(i) < z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 7

On entry, $SIGMA \leq 0.0$.

IFAIL = 8

On entry, $R < 0.0$.

IFAIL = 9

On entry, $Q < 0.0$.

IFAIL = 11

On entry, $LDP < M$.

IFAIL = 14

On entry, $\beta \geq \frac{1}{z}$, where $z = X02AMF()$, the safe range parameter (see Section 3).

IFAIL = 15

Internal memory allocation failed.

7 Accuracy

The accuracy of the output will be bounded by the accuracy of the cumulative bivariate Normal distribution function. The algorithm of Genz (2004) is used, as described in the document for G01HAF, giving a maximum absolute error of less than 5×10^{-16} . The univariate cumulative Normal distribution function also forms part of the evaluation (see S15ABF and S15ADF).

8 Further Comments

None.

9 Example

This example computes the price of an American call with a time to expiry of 3 months, a stock price of 110 and a strike price of 100. The risk-free interest rate is 8% per year, there is an annual dividend return of 12% and the volatility is 20% per year.

9.1 Program Text

```
*      S30QCF Example Program Text
*      Mark 22 Release. NAG Copyright 2008.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          LDP, MMAX, NMAX
      PARAMETER        (LDP=50,MMAX=50,NMAX=50)
*      .. Local Scalars ..
      DOUBLE PRECISION Q, R, S, SIGMA
      INTEGER          I, IFAIL, J, M, N
      CHARACTER        PUT
*      .. Local Arrays ..
      DOUBLE PRECISION P(LDP,NMAX), T(NMAX), X(MMAX)
*      .. External Subroutines ..
      EXTERNAL         S30QCF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S30QCF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
*      Read problem parameters.
      READ (NIN,*) PUT
      READ (NIN,*) S, SIGMA, R, Q
      READ (NIN,*) M, N
*
      IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
```

```

*      Read array of strike/exercise prices, X
      READ (NIN,*) (X(I),I=1,M)
      READ (NIN,*) (T(I),I=1,N)
*
      IFAIL = 1
*
      CALL S30QCF(PUT,M,N,X,S,T,SIGMA,R,Q,P,LDP,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
        IF (PUT.EQ.'C' .OR. PUT.EQ.'c') THEN
          WRITE (NOUT,*) 'American Call :'
        ELSE IF (PUT.EQ.'P' .OR. PUT.EQ.'p') THEN
          WRITE (NOUT,*) 'American Put :'
        END IF
        WRITE (NOUT, '(A,1X,F8.4)') ' Spot      = ', S
        WRITE (NOUT, '(A,1X,F8.4)') ' Volatility = ', SIGMA
        WRITE (NOUT, '(A,1X,F8.4)') ' Rate      = ', R
        WRITE (NOUT, '(A,1X,F8.4)') ' Dividend  = ', Q
*
        WRITE (NOUT,*)
        WRITE (NOUT,*) ' Strike      Expiry      Option Price'
        DO 40 I = 1, M
          DO 20 J = 1, N
            WRITE (NOUT,99999) X(I), T(J), P(I,J)
          20 CONTINUE
        40 CONTINUE
      ELSE
        WRITE (NOUT,*)
        WRITE (NOUT,99998) IFAIL
      END IF
    END IF
*
99999 FORMAT (1X,2(F9.4,1X),6X,F9.4)
99998 FORMAT (1X,' ** S30QCF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

S30QCF Example Program Data

```

'C'      : Call = 'C', Put = 'P'
110.0 0.2 0.08 0.12 : S, SIGMA, R, Q
1 1      : M, N
100.0    : X(I), I = 1,2,...M
0.25     : T(I), I = 1,2,...N

```

9.3 Program Results

S30QCF Example Program Results

```

American Call :
Spot      = 110.0000
Volatility = 0.2000
Rate      = 0.0800
Dividend  = 0.1200

Strike      Expiry      Option Price
100.0000    0.2500      10.3340

```
