

NAG Library Routine Document

S21BJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21BJF returns a value of the classical (Legendre) form of the complete elliptic integral of the second kind, via the routine name.

2 Specification

double precision FUNCTION S21BJF(DM, IFAIL)

INTEGER

IFAIL

double precision

DM

3 Description

S21BJF calculates an approximation to the integral

$$E(m) = \int_0^{\frac{\pi}{2}} (1 - m \sin^2 \theta)^{\frac{1}{2}} d\theta,$$

where $m \leq 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$E(m) = R_F(0, 1 - m, 1) - \frac{1}{3}mR_D(0, 1 - m, 1),$$

where R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see S21BBF) and R_D is the Carlson symmetrised incomplete elliptic integral of the second kind (see S21BCF).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

1: DM – ***double precision*** *Input*

On entry: the argument m of the function.

Constraint: DM $\leq$ 1.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, –1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value –1 or 1 is recommended. If the output of error messages is undesirable, then

the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $DM > 1.0$; the function is undefined. On soft failure, the routine returns zero.

7 Accuracy

In principle S21BJF is capable of producing full *machine precision*. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the *machine precision*.

8 Further Comments

You should consult the S Chapter Introduction, which shows the relationship between this routine and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute R_F and R_D , see the routine documents for S21BBF and S21BCF, respectively.

9 Example

This example simply generates a small set of non-extreme arguments that are used with the routine to produce the table of results.

9.1 Program Text

```
*      S21BJF Example Program Text
*      Mark 22 Release. NAG Copyright 2006.
*      .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DM, F
      INTEGER          IFAIL, IX
*      .. External Functions ..
      DOUBLE PRECISION S21BJF
      EXTERNAL          S21BJF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S21BJF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      DM      S21BJF      IFAIL'
      WRITE (NOUT,*)

*
      DO 20 IX = 1, 3
        DM = IX*0.25D0
        IFAIL = 1

*
        F = S21BJF(DM,IFAIL)

*
        IF (IFAIL.GE.0) THEN
          WRITE (NOUT,99999) DM, F, IFAIL
        ELSE
```

```
                WRITE (NOUT,99998) IFAIL
                GO TO 40
            END IF
20 CONTINUE
*
40 CONTINUE
*
99999 FORMAT (1X,F7.2,F12.4,I5)
99998 FORMAT (1X,' ** S21BJF returned with IFAIL = ',I5)
END
```

9.2 Program Data

None.

9.3 Program Results

S21BJF Example Program Results

DM	S21BJF	IFAIL
0.25	1.4675	0
0.50	1.3506	0
0.75	1.2111	0
