

NAG Library Routine Document

S21BHF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21BHF returns a value of the classical (Legendre) form of the complete elliptic integral of the first kind, via the routine name.

2 Specification

double precision FUNCTION S21BHF(DM, IFAIL)

INTEGER IFAIL

double precision DM

3 Description

S21BHF calculates an approximation to the integral

$$K(m) = \int_0^{\frac{\pi}{2}} (1 - m \sin^2 \theta)^{-\frac{1}{2}} d\theta,$$

where $m < 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$K(m) = R_F(0, 1 - m, 1),$$

where R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see S21BBF).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

1: DM – ***double precision*** *Input*

On entry: the argument m of the function.

Constraint: DM < 1.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, –1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value –1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the

recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $DM > 1.0$; the function is undefined. On soft failure, the routine returns zero.

$IFAIL = 2$

On entry, $DM = 1.0$; the function is infinite. On soft failure, the routine returns the largest machine number (see X02ALF).

7 Accuracy

In principle S21BHF is capable of producing full *machine precision*. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the *machine precision*.

8 Further Comments

You should consult the S Chapter Introduction, which shows the relationship between this routine and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithm used to compute R_F , see the routine document for S21BBF.

9 Example

This example simply generates a small set of non-extreme arguments that are used with the routine to produce the table of results.

9.1 Program Text

```
*      S21BHF Example Program Text
*      Mark 22 Release. NAG Copyright 2006.
*      .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DM, F
      INTEGER          IFAIL, IX
*      .. External Functions ..
      DOUBLE PRECISION S21BHF
      EXTERNAL          S21BHF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S21BHF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      DM          S21BHF      IFAIL'
      WRITE (NOUT,*)
*
      DO 20 IX = 1, 3
         DM = IX*0.25D0
         IFAIL = 1
*
         F = S21BHF(DM,IFAIL)
*
```

```
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) DM, F, IFAIL
      ELSE
        WRITE (NOUT,99998) IFAIL
        GO TO 40
      END IF
20  CONTINUE
*
40  CONTINUE
*
99999 FORMAT (1X,F7.2,F12.4,I5)
99998 FORMAT (1X,' ** S21BHF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

None.

9.3 Program Results

S21BHF Example Program Results

DM	S21BHF	IFAIL
0.25	1.6858	0
0.50	1.8541	0
0.75	2.1565	0
