

NAG Library Routine Document

S21BFF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21BFF returns a value of the classical (Legendre) form of the incomplete elliptic integral of the second kind, via the routine name.

2 Specification

double precision FUNCTION S21BFF(PHI, DM, IFAIL)
 INTEGER IFAIL
double precision PHI, DM

3 Description

S21BFF calculates an approximation to the integral

$$E(\phi | m) = \int_0^\phi (1 - m \sin^2 \theta)^{\frac{1}{2}} d\theta,$$

where $0 \leq \phi \leq \frac{\pi}{2}$ and $m \sin^2 \phi \leq 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$E(\phi | m) = \sin \phi R_F(q, r, 1) - \frac{1}{3} m \sin^3 \phi R_D(q, r, 1),$$

where $q = \cos^2 \phi$, $r = 1 - m \sin^2 \phi$, R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see S21BBF) and R_D is the Carlson symmetrised incomplete elliptic integral of the second kind (see S21BCF).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

1:	PHI – <i>double precision</i>	<i>Input</i>
2:	DM – <i>double precision</i>	<i>Input</i>

On entry: the arguments ϕ and m of the function.

Constraints:

$$0.0 \leq \text{PHI} \leq \frac{\pi}{2};$$

$$\text{DM} \times \sin^2(\text{PHI}) \leq 1.0.$$

3: IFAIL – INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

PHI lies outside the range $[0, \frac{\pi}{2}]$. On soft failure, the routine returns zero.

IFAIL = 2

On entry, $DM \times \sin^2(\text{PHI}) > 1.0$; the function is undefined. On soft failure, the routine returns zero.

7 Accuracy

In principle S21BFF is capable of producing full *machine precision*. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the *machine precision*.

8 Further Comments

You should consult the S Chapter Introduction, which shows the relationship between this routine and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute R_F and R_D , see the routine documents for S21BBF and S21BCF, respectively.

If you wish to input a value of PHI outside the range allowed by this routine you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities. For example, $E(-\phi|m) = -E(\phi|m)$. A parameter $m > 1$ can be replaced by one less than unity using $E(\phi|m) = \sqrt{m}E(\phi\sqrt{m}|\frac{1}{m}) - (m-1)\phi$.

9 Example

This example simply generates a small set of non-extreme arguments that are used with the routine to produce the table of results.

9.1 Program Text

```
*      S21BFF Example Program Text
*      Mark 22 Release. NAG Copyright 2006.
*      .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DM, F, PHI, PI
```

```

      INTEGER          IFAIL, IX
*    .. External Functions ..
      DOUBLE PRECISION S21BFF, X01AAF
      EXTERNAL          S21BFF, X01AAF
*    .. Executable Statements ..
      WRITE (NOUT,*) 'S21BFF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      PHI      DM      S21BFF      IFAIL'
      WRITE (NOUT,*)

*
      PI = X01AAF(1.0D0)
*
      DO 20 IX = 1, 3
        PHI = IX*PI/6.0D0
        DM = IX*0.25D0
        IFAIL = 1

*
        F = S21BFF(PHI,DM,IFAIL)
*
        IF (IFAIL.GE.0) THEN
          WRITE (NOUT,99999) PHI, DM, F, IFAIL
        ELSE
          WRITE (NOUT,99998) IFAIL
          GO TO 40
        END IF
      20 CONTINUE
*
      40 CONTINUE
*
      99999 FORMAT (1X,2F7.2,F12.4,I5)
      99998 FORMAT (1X,' ** S21BFF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

None.

9.3 Program Results

S21BFF Example Program Results

PHI	DM	S21BFF	IFAIL
0.52	0.25	0.5179	0
1.05	0.50	0.9650	0
1.57	0.75	1.2111	0
