NAG Library Routine Document

S21BEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21BEF returns a value of the classical (Legendre) form of the incomplete elliptic integral of the first kind, via the routine name.

2 Specification

double precision `FUNCTION S21BEF(PHI, DM, IFAIL)`

```
INTEGER                  IFAIL
double precision         PHI, DM
```

3 Description

S21BEF calculates an approximation to the integral

$$F(\phi \mid m) = \int_0^{\phi} \left(1 - m \sin^2 \theta\right)^{-\frac{1}{2}} d\theta,$$

where $0 \leq \phi \leq \frac{\pi}{2}$, $m \sin^2 \phi \leq 1$ and m and $\sin \phi$ may not both equal one.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$F(\phi \mid m) = R_F(q, r, 1) \sin \phi,$$

where $q = \cos^2 \phi$, $r = 1 - m \sin^2 \phi$ and R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see S21BBF).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

1: PHI – ***double precision*** *Input*

2: DM – ***double precision*** *Input*

On entry: the arguments ϕ and m of the function.

Constraints:

$0.0 \leq \text{PHI} \leq \frac{\pi}{2}$;
$\text{DM} \times \sin^2(\text{PHI}) \leq 1.0$;
Only one of sin(PHI) and DM may be 1.0.

Note that $\text{DM} \times \sin^2(\text{PHI}) = 1.0$ is allowable, as long as $\text{DM} \neq 1.0$.

3: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

PHI lies outside the range $\left[0, \frac{\pi}{2}\right]$. On soft failure, the routine returns zero.

IFAIL $= 2$

On entry, DM $\times \sin^2$(PHI) > 1.0; the function is undefined. On soft failure, the routine returns zero.

IFAIL $= 3$

On entry, sin(PHI) $= 1.0$ and DM $= 1.0$; the function is infinite. On soft failure, the routine returns the largest machine number (see X02ALF).

7 Accuracy

In principle S21BEF is capable of producing full ***machine precision***. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

8 Further Comments

You should consult the S Chapter Introduction, which shows the relationship between this routine and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithm used to compute R_F, see the routine document for S21BBF.

If you wish to input a value of PHI outside the range allowed by this routine you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities. For example, $F(-\phi|m) = -F(\phi|m)$ and $F(s\pi \pm \phi|m) = 2sK(m) \pm F(\phi|m)$ where s is an integer and $K(m)$ is the complete elliptic integral given by S21BHF.

A parameter $m > 1$ can be replaced by one less than unity using $F(\phi|m) = \frac{1}{\sqrt{m}} F\left(\theta|\frac{1}{m}\right)$, $\sin\theta = \sqrt{m}\sin\phi$.

9 Example

This example simply generates a small set of non-extreme arguments that are used with the routine to produce the table of results.

9.1 Program Text

```
*     S21BEF Example Program Text
*     Mark 22 Release. NAG Copyright 2006.
*     .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION DM, F, PHI, PI
      INTEGER          IFAIL, IX
*     .. External Functions ..
      DOUBLE PRECISION S21BEF, X01AAF
      EXTERNAL         S21BEF, X01AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S21BEF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '    PHI    DM      S21BEF   IFAIL'
      WRITE (NOUT,*)
*
      PI = X01AAF(1.0D0)
*
      DO 20 IX = 1, 3
         PHI = IX*PI/6.0D0
         DM = IX*0.25D0
         IFAIL = 1
*
         F = S21BEF(PHI,DM,IFAIL)
*
         IF (IFAIL.GE.0) THEN
            WRITE (NOUT,99999) PHI, DM, F, IFAIL
         ELSE
            WRITE (NOUT,99998) IFAIL
            GO TO 40
         END IF
   20 CONTINUE
*
   40 CONTINUE
*
99999 FORMAT (1X,2F7.2,F12.4,I5)
99998 FORMAT (1X,' ** S21BEF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 S21BEF Example Program Results

     PHI    DM      S21BEF   IFAIL

    0.52   0.25      0.5294    0
    1.05   0.50      1.1424    0
    1.57   0.75      2.1565    0
```