

NAG Library Routine Document

S18AFF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S18AFF returns a value for the modified Bessel Function $I_1(x)$, via the routine name.

2 Specification

double precision FUNCTION S18AFF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S18AFF evaluates an approximation to the modified Bessel Function of the first kind $I_1(x)$.

Note: $I_1(-x) = -I_1(x)$, so the approximation need only consider $x \geq 0$.

The routine is based on three Chebyshev expansions:

For $0 < x \leq 4$,

$$I_1(x) = x \sum_{r=0}' a_r T_r(t), \quad \text{where } t = 2\left(\frac{x}{4}\right)^2 - 1;$$

For $4 < x \leq 12$,

$$I_1(x) = e^x \sum_{r=0}' b_r T_r(t), \quad \text{where } t = \frac{x-8}{4};$$

For $x > 12$,

$$I_1(x) = \frac{e^x}{\sqrt{x}} \sum_{r=0}' c_r T_r(t), \quad \text{where } t = 2\left(\frac{12}{x}\right) - 1.$$

For small x , $I_1(x) \simeq x$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For large x , the routine must fail because $I_1(x)$ cannot be represented without overflow.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

- 1: X – ***double precision*** *Input*
On entry: the argument x of the function.
- 2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

X is too large. On soft failure the routine returns the approximate value of $I_1(x)$ at the nearest valid argument. See also the Users' Note for your implementation.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{xI_0(x) - I_1(x)}{I_1(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{xI_0(x) - I_1(x)}{I_1(x)} \right|.$$

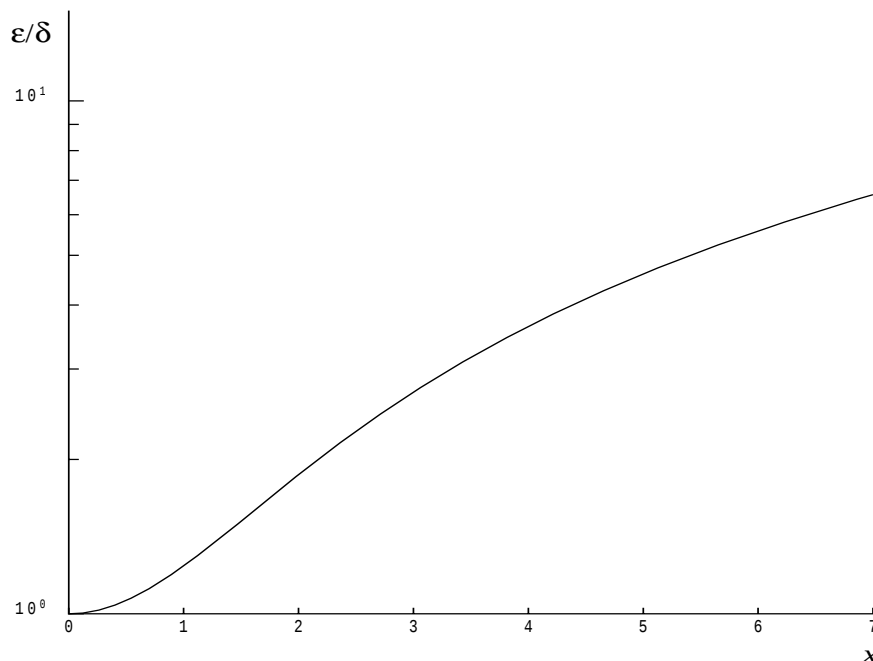


Figure 1

However, if δ is of the same order as *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , $\epsilon \simeq \delta$ and there is no amplification of errors.

For large x , $\epsilon \simeq x\delta$ and we have strong amplification of errors. However the routine must fail for quite moderate values of x because $I_1(x)$ would overflow; hence in practice the loss of accuracy for large x is not excessive. Note that for large x , the errors will be dominated by those of the standard function EXP.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S18AFF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S18AFF
      EXTERNAL         S18AFF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S18AFF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S18AFF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
40    CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S18AFF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S18AFF Example Program Data
      0.0
      0.5
      1.0
      3.0
      6.0
      8.0
     10.0
     15.0
     20.0
     -1.0
```

9.3 Program Results

S18AFF Example Program Results

X	Y	IFAIL
0.000E+00	0.000E+00	0
5.000E-01	2.579E-01	0
1.000E+00	5.652E-01	0
3.000E+00	3.953E+00	0
6.000E+00	6.134E+01	0
8.000E+00	3.999E+02	0
1.000E+01	2.671E+03	0
1.500E+01	3.281E+05	0
2.000E+01	4.245E+07	0
-1.000E+00	-5.652E-01	0
