

NAG Library Routine Document

S18AEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S18AEF returns the value of the modified Bessel Function $I_0(x)$, via the routine name.

2 Specification

double precision FUNCTION S18AEF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S18AEF evaluates an approximation to the modified Bessel Function of the first kind $I_0(x)$.

Note: $I_0(-x) = I_0(x)$, so the approximation need only consider $x \geq 0$.

The routine is based on three Chebyshev expansions:

For $0 < x \leq 4$,

$$I_0(x) = e^x \sum_{r=0}' a_r T_r(t), \quad \text{where } t = 2\left(\frac{x}{4}\right) - 1.$$

For $4 < x \leq 12$,

$$I_0(x) = e^x \sum_{r=0}' b_r T_r(t), \quad \text{where } t = \frac{x-8}{4}.$$

For $x > 12$,

$$I_0(x) = \frac{e^x}{\sqrt{x}} \sum_{r=0}' c_r T_r(t), \quad \text{where } t = 2\left(\frac{12}{x}\right) - 1.$$

For small x , $I_0(x) \simeq 1$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For large x , the routine must fail because of the danger of overflow in calculating e^x .

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

- 1: X – ***double precision*** *Input*
On entry: the argument x of the function.
- 2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

X is too large. On soft failure the routine returns the approximate value of $I_0(x)$ at the nearest valid argument. (see the Users' Note for your implementation for details)

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x I_1(x)}{I_0(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{x I_1(x)}{I_0(x)} \right|.$$

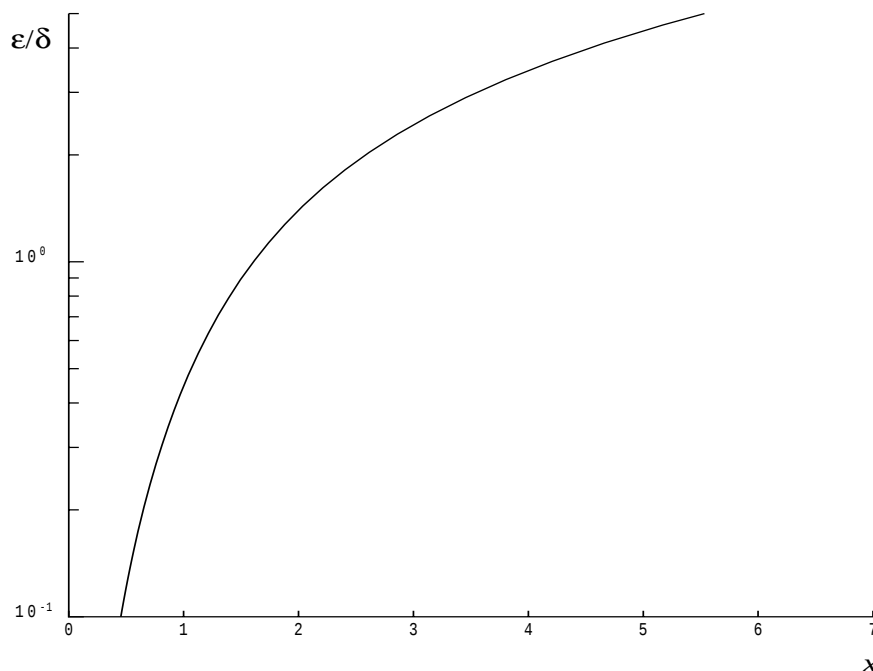


Figure 1

However if δ is of the same order as *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x the amplification factor is approximately $\frac{x^2}{2}$, which implies strong attenuation of the error, but in general ϵ can never be less than the *machine precision*.

For large x , $\epsilon \simeq x\delta$ and we have strong amplification of errors. However the routine must fail for quite moderate values of x , because $I_0(x)$ would overflow; hence in practice the loss of accuracy for large x is not excessive. Note that for large x the errors will be dominated by those of the standard function EXP.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S18AEF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S18AEF
      EXTERNAL          S18AEF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S18AEF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S18AEF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
      40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S18AEF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S18AEF Example Program Data
      0.0
      0.5
      1.0
      3.0
      6.0
```

8.0
10.0
15.0
20.0
-1.0

9.3 Program Results

S18AEF Example Program Results

X	Y	IFAIL
0.000E+00	1.000E+00	0
5.000E-01	1.063E+00	0
1.000E+00	1.266E+00	0
3.000E+00	4.881E+00	0
6.000E+00	6.723E+01	0
8.000E+00	4.276E+02	0
1.000E+01	2.816E+03	0
1.500E+01	3.396E+05	0
2.000E+01	4.356E+07	0
-1.000E+00	1.266E+00	0
