

NAG Library Routine Document

S18ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S18ACF returns the value of the modified Bessel Function $K_0(x)$, via the routine name.

2 Specification

double precision FUNCTION S18ACF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S18ACF evaluates an approximation to the modified Bessel Function of the second kind $K_0(x)$.

Note: $K_0(x)$ is undefined for $x \leq 0$ and the routine will fail for such arguments.

The routine is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_0(x) = -\ln x \sum_{r=0}' a_r T_r(t) + \sum_{r=0}' b_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$

For $1 < x \leq 2$,

$$K_0(x) = e^{-x} \sum_{r=0}' c_r T_r(t), \quad \text{where } t = 2x - 3.$$

For $2 < x \leq 4$,

$$K_0(x) = e^{-x} \sum_{r=0}' d_r T_r(t), \quad \text{where } t = x - 3.$$

For $x > 4$,

$$K_0(x) = \frac{e^{-x}}{\sqrt{x}} \sum_{r=0}' e_r T_r(t), \quad \text{where } t = \frac{9-x}{1+x}.$$

For x near zero, $K_0(x) \simeq -\gamma - \ln\left(\frac{x}{2}\right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For large x , where there is a danger of underflow due to the smallness of K_0 , the result is set exactly to zero.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

Constraint: $X > 0.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

$X \leq 0.0$, K_0 is undefined. On soft failure the routine returns zero.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x K_1(x)}{K_0(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{x K_1(x)}{K_0(x)} \right|.$$

However, if δ is of the same order as *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , the amplification factor is approximately $\left| \frac{1}{\ln x} \right|$, which implies strong attenuation of the error, but in general ϵ can never be less than the *machine precision*.

For large x , $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually K_0 , which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the routine will return zero. Note that for large x the errors will be dominated by those of the standard function EXP.

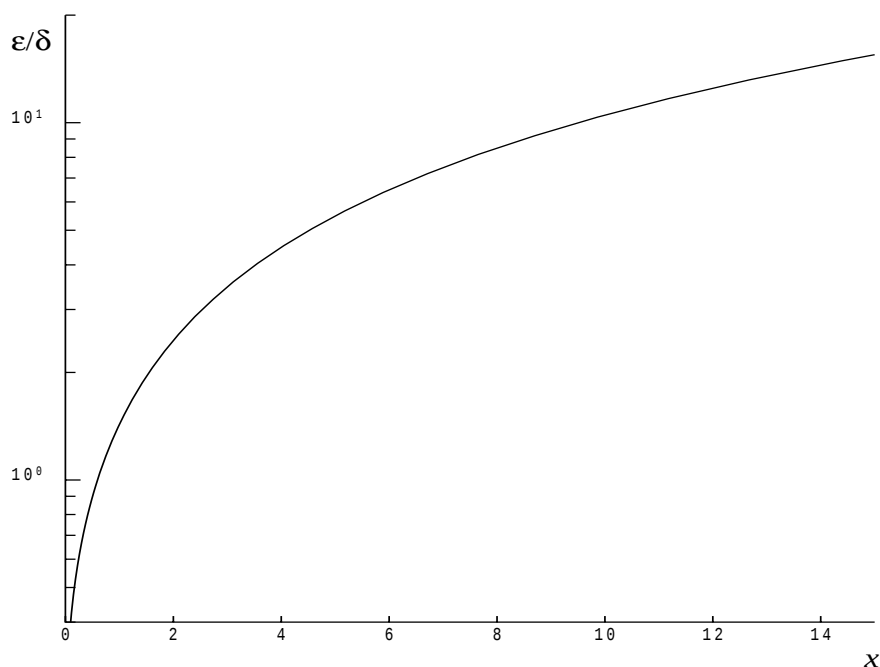


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S18ACF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S18ACF
      EXTERNAL          S18ACF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S18ACF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S18ACF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
```

```

        WRITE (NOUT,99998) IFAIL
      END IF
40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S18ACF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

S18ACF Example Program Data

```

0.0
0.4
0.6
1.4
1.6
2.5
3.5
6.0
8.0
10.0
-1.0
1000.0

```

9.3 Program Results

S18ACF Example Program Results

X	Y	IFAIL
0.000E+00	0.000E+00	1
4.000E-01	1.115E+00	0
6.000E-01	7.775E-01	0
1.400E+00	2.437E-01	0
1.600E+00	1.880E-01	0
2.500E+00	6.235E-02	0
3.500E+00	1.960E-02	0
6.000E+00	1.244E-03	0
8.000E+00	1.465E-04	0
1.000E+01	1.778E-05	0
-1.000E+00	0.000E+00	1
1.000E+03	0.000E+00	0
