NAG Library Routine Document

S17AKF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S17AKF returns a value for the derivative of the Airy function $\mathrm{Bi}(x)$, via the routine name.

2 Specification

```
double precision FUNCTION S17AKF(X, IFAIL)
INTEGER                         IFAIL
double precision                X
```

3 Description

S17AKF calculates an approximate value for the derivative of the Airy function $\mathrm{Bi}(x)$. It is based on a number of Chebyshev expansions.

For $x < -5$,

$$\mathrm{Bi}'(x) = \sqrt[4]{-x}\left[-a(t)\sin z + \frac{b(t)}{\zeta}\cos z\right],$$

where $z = \frac{\pi}{4} + \zeta$, $\zeta = \frac{2}{3}\sqrt{-x^3}$ and $a(t)$ and $b(t)$ are expansions in the variable $t = -2\left(\frac{5}{x}\right)^3 - 1$.

For $-5 \leq x \leq 0$,

$$\mathrm{Bi}'(x) = \sqrt{3}\big(x^2 f(t) + g(t)\big),$$

where f and g are expansions in $t = -2\left(\frac{x}{5}\right)^3 - 1$.

For $0 < x < 4.5$,

$$\mathrm{Bi}'(x) = e^{3x/2} y(t),$$

where $y(t)$ is an expansion in $t = 4x/9 - 1$.

For $4.5 \leq x < 9$,

$$\mathrm{Bi}'(x) = e^{21x/8} u(t),$$

where $u(t)$ is an expansion in $t = 4x/9 - 3$.

For $x \geq 9$,

$$\mathrm{Bi}'(x) = \sqrt[4]{x} e^{z} v(t),$$

where $z = \frac{2}{3}\sqrt{x^3}$ and $v(t)$ is an expansion in $t = 2\left(\frac{18}{z}\right) - 1$.

For $|x| <$ the square of the ***machine precision***, the result is set directly to $\mathrm{Bi}'(0)$. This saves time and avoids possible underflows in calculation.

For large negative arguments, it becomes impossible to calculate a result for the oscillating function with any accuracy so the routine must fail. This occurs for $x < -\left(\frac{\sqrt{\pi}}{\epsilon}\right)^{4/7}$, where ϵ is the ***machine precision***.

For large positive arguments, where Bi' grows in an essentially exponential manner, there is a danger of overflow so the routine must fail.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: $\mathrm{IFAIL} = 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $\mathrm{IFAIL} = 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$\mathrm{IFAIL} = 1$

X is too large and positive. On soft failure, the routine returns zero. (see the Users' Note for your implementation for details)

$\mathrm{IFAIL} = 2$

X is too large and negative. On soft failure the routine returns zero. (see the Users' Note for your implementation for details)

7 Accuracy

For negative arguments the function is oscillatory and hence absolute error is appropriate. In the positive region the function has essentially exponential behaviour and hence relative error is needed. The absolute error, E, and the relative error ϵ, are related in principle to the relative error in the argument δ, by

$$E \simeq \left|x^2\,\mathrm{Bi}(x)\right|\delta \qquad \epsilon \simeq \left|\frac{x^2\,\mathrm{Bi}(x)}{\mathrm{Bi}'(x)}\right|\delta.$$

In practice, approximate equality is the best that can be expected. When δ, ϵ or E is of the order of the ***machine precision***, the errors in the result will be somewhat larger.

For small x, positive or negative, errors are strongly attenuated by the function and hence will effectively be bounded by the ***machine precision***.

For moderate to large negative x, the error is, like the function, oscillatory. However, the amplitude of the absolute error grows like $\frac{|x|^{7/4}}{\sqrt{\pi}}$. Therefore it becomes impossible to calculate the function with any accuracy if $|x|^{7/4} > \frac{\sqrt{\pi}}{\delta}$.

For large positive x, the relative error amplification is considerable: $\frac{\epsilon}{\delta} \sim \sqrt{x^3}$. However, very large arguments are not possible due to the danger of overflow. Thus in practice the actual amplification that occurs is limited.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*     S17AKF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      DOUBLE PRECISION S17AKF
      EXTERNAL         S17AKF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S17AKF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S17AKF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
         WRITE (NOUT,99999) X, Y, IFAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
   40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S17AKF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S17AKF Example Program Data
             -10.0
             -1.0
             0.0
             1.0
             5.0
             10.0
             20.0
```

9.3 Program Results

```
 S17AKF Example Program Results

      X           Y        IFAIL

   -1.000E+01   1.194E-01      0
   -1.000E+00   5.924E-01      0
    0.000E+00   4.483E-01      0
    1.000E+00   9.324E-01      0
    5.000E+00   1.436E+03      0
    1.000E+01   1.429E+09      0
    2.000E+01   9.382E+25      0
```