

NAG Library Routine Document

S17AEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S17AEF returns the value of the Bessel Function $J_0(x)$, via the routine name.

2 Specification

double precision FUNCTION S17AEF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S17AEF evaluates an approximation to the Bessel Function of the first kind $J_0(x)$.

Note: $J_0(-x) = J_0(x)$, so the approximation need only consider $x \geq 0$.

The routine is based on three Chebyshev expansions:

For $0 < x \leq 8$,

$$J_0(x) = \sum_{r=0}' a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$J_0(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_0(x) \cos\left(x - \frac{\pi}{4}\right) - Q_0(x) \sin\left(x - \frac{\pi}{4}\right) \right\},$$

where $P_0(x) = \sum_{r=0}' b_r T_r(t)$,

and $Q_0(x) = \frac{8}{x} \sum_{r=0}' c_r T_r(t)$,

with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $J_0(x) \simeq 1$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For very large x , it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the routine fails. Such arguments contain insufficient information to determine the phase of oscillation of $J_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi|x|}}$, can be determined and this is returned on soft failure.

The range for which this occurs is roughly related to ***machine precision***; the routine will fail if $|x| \gtrsim 1/\text{machine precision}$ (see the Users' Note for your implementation for details).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

X is too large. On soft failure the routine returns the amplitude of the J_0 oscillation, $\sqrt{\frac{2}{\pi|x|}}$.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $J_0(x)$ oscillates about zero, absolute error and not relative error is significant.)

If δ is somewhat larger than the *machine precision* (e.g., if δ is due to data errors etc.), then E and δ are approximately related by:

$$E \simeq |xJ_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xJ_1(x)|$.

However, if δ is of the same order as *machine precision*, then rounding errors could make E slightly larger than the above relation predicts.

For very large x , the above relation ceases to apply. In this region, $J_0(x) \simeq \sqrt{\frac{2}{\pi|x|}} \cos\left(x - \frac{\pi}{4}\right)$. The

amplitude $\sqrt{\frac{2}{\pi|x|}}$ can be calculated with reasonable accuracy for all x , but $\cos\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is

written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\cos\left(x - \frac{\pi}{4}\right)$ is determined by θ only. If $x \gtrsim \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of, the inverse of the *machine precision*, it is impossible to calculate the phase of $J_0(x)$ and the routine must fail.

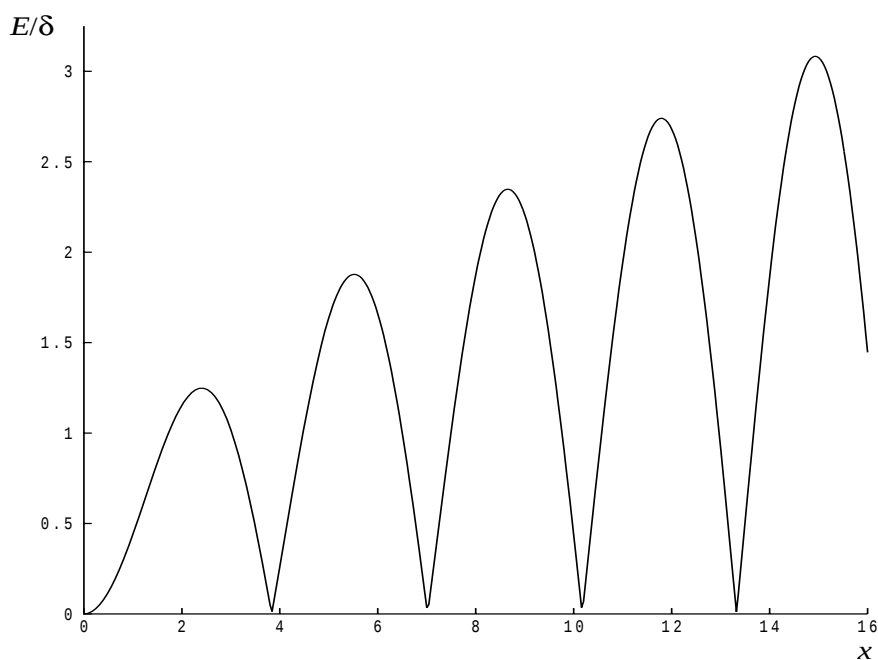


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S17AEF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S17AEF
      EXTERNAL          S17AEF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S17AEF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S17AEF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
```

```

        WRITE (NOUT,99998) IFAIL
      END IF
40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S17AEF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

S17AEF Example Program Data

```

0.0
0.5
1.0
3.0
6.0
8.0
10.0
-1.0
1000.0

```

9.3 Program Results

S17AEF Example Program Results

X	Y	IFAIL
0.000E+00	1.000E+00	0
5.000E-01	9.385E-01	0
1.000E+00	7.652E-01	0
3.000E+00	-2.601E-01	0
6.000E+00	1.506E-01	0
8.000E+00	1.717E-01	0
1.000E+01	-2.459E-01	0
-1.000E+00	7.652E-01	0
1.000E+03	2.479E-02	0
