

NAG Library Routine Document

S17ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S17ADF returns the value of the Bessel Function $Y_1(x)$, via the routine name.

2 Specification

double precision FUNCTION S17ADF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S17ADF evaluates an approximation to the Bessel Function of the second kind $Y_1(x)$.

Note: $Y_1(x)$ is undefined for $x \leq 0$ and the routine will fail for such arguments.

The routine is based on four Chebyshev expansions:

For $0 < x \leq 8$,

$$Y_1(x) = \frac{2}{\pi} \ln x \sum_{r=0}' a_r T_r(t) - \frac{2}{\pi x} + \frac{x}{8} \sum_{r=0}' b_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$Y_1(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_1(x) \sin\left(x - 3\frac{\pi}{4}\right) + Q_1(x) \cos\left(x - 3\frac{\pi}{4}\right) \right\}$$

where $P_1(x) = \sum_{r=0}' c_r T_r(t)$,

and $Q_1(x) = \frac{8}{x} \sum_{r=0}' d_r T_r(t)$, with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $Y_1(x) \simeq -\frac{2}{\pi x}$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***. For extremely small x , there is a danger of overflow in calculating $-\frac{2}{\pi x}$ and for such arguments the routine will fail.

For very large x , it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the routine fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_1(x)$; only the amplitude, $\sqrt{\frac{2}{\pi x}}$, can be determined and this is returned on soft failure. The range for which this occurs is roughly related to ***machine precision***; the routine will fail if $x \gtrsim 1/\text{machine precision}$ (see the Users' Note for your implementation for details).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO

5 Parameters

1: X – *double precision* *Input*

On entry: the argument x of the function.

Constraint: $X > 0.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

X is too large. On soft failure the routine returns the amplitude of the Y_1 oscillation, $\sqrt{\frac{2}{\pi x}}$.

IFAIL = 2

$X \leq 0.0$, Y_1 is undefined. On soft failure the routine returns zero.

IFAIL = 3

X is too close to zero, there is a danger of overflow. On soft failure, the routine returns the value of $Y_1(x)$ at the smallest valid argument.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_1(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x .)

If δ is somewhat larger than the *machine precision* (e.g., if δ is due to data errors etc.), then E and δ are approximately related by:

$$E \simeq |xY_0(x) - Y_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xY_0(x) - Y_1(x)|$.

However, if δ is of the same order as *machine precision*, then rounding errors could make E slightly larger than the above relation predicts.

For very small x , absolute error becomes large, but the relative error in the result is of the same order as δ .

For very large x , the above relation ceases to apply. In this region, $Y_1(x) \simeq \frac{2}{\pi x} \sin\left(x - \frac{3\pi}{4}\right)$. The amplitude $\frac{2}{\pi x}$ can be calculated with reasonable accuracy for all x , but $\sin\left(x - \frac{3\pi}{4}\right)$ cannot. If $x - \frac{3\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin\left(x - \frac{3\pi}{4}\right)$ is determined by θ only. If

$x > \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of, the inverse of the **machine precision**, it is impossible to calculate the phase of $Y_1(x)$ and the routine must fail.

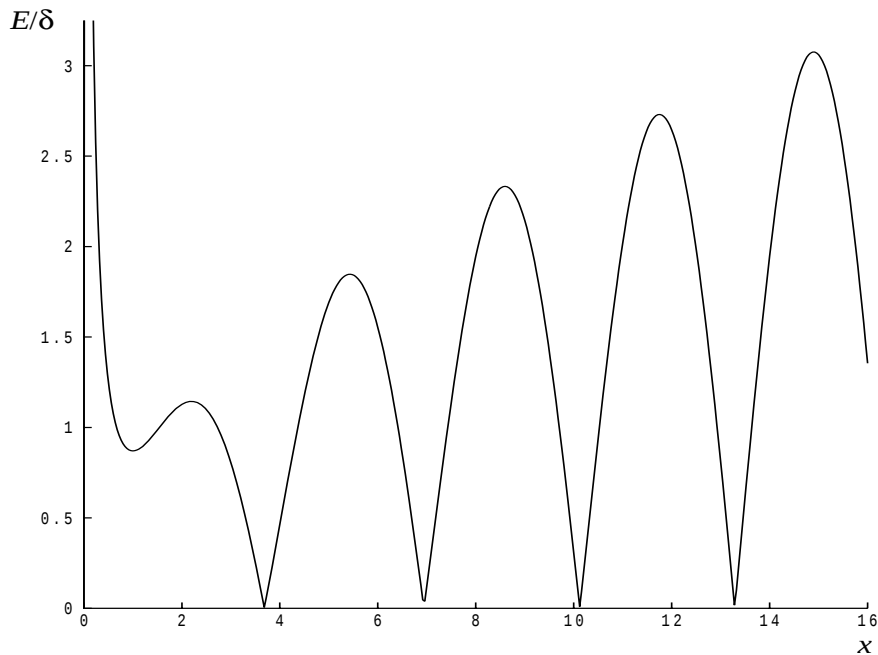


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S17ADF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
INTEGER                NIN, NOUT
PARAMETER              (NIN=5,NOUT=6)
*      .. Local Scalars ..
DOUBLE PRECISION X, Y
INTEGER                IFAIL
*      .. External Functions ..
DOUBLE PRECISION S17ADF
EXTERNAL               S17ADF
*      .. Executable Statements ..
WRITE (NOUT,*) 'S17ADF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
WRITE (NOUT,*)
WRITE (NOUT,*) '      X              Y              IFAIL'
WRITE (NOUT,*)
20 READ (NIN,*,END=40) X
   IFAIL = 1
*
   Y = S17ADF(X,IFAIL)
*
   IF (IFAIL.GE.0) THEN
```

```

        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
    ELSE
        WRITE (NOUT,99998) IFAIL
    END IF
40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S17ADF returned with IFAIL = ',I5)
END

```

9.2 Program Data

S17ADF Example Program Data

```

0.0
0.5
1.0
3.0
6.0
8.0
10.0
-1.0
1000.0

```

9.3 Program Results

S17ADF Example Program Results

X	Y	IFAIL
0.000E+00	0.000E+00	2
5.000E-01	-1.471E+00	0
1.000E+00	-7.812E-01	0
3.000E+00	3.247E-01	0
6.000E+00	-1.750E-01	0
8.000E+00	-1.581E-01	0
1.000E+01	2.490E-01	0
-1.000E+00	0.000E+00	2
1.000E+03	-2.478E-02	0
