

NAG Library Routine Document

S15ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S15ABF returns the value of the cumulative Normal distribution function, $P(x)$, via the routine name.

2 Specification

double precision FUNCTION S15ABF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S15ABF evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du.$$

The routine is based on the fact that

$$P(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{-x}{\sqrt{2}}\right)$$

and it calls S15ADF to obtain a value of *erfc* for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***double precision*** *Input*
On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

There are no failure exits from this routine. The parameter IFAIL is included for consistency with other routines in this chapter.

7 Accuracy

Because of its close relationship with *erfc*, the accuracy of this routine is very similar to that in S15ADF. If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)} \delta \right|$$

so that the relative error in the argument, x , is amplified by a factor, $\frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by **machine precision**.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E , is given by

$$|E| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \delta \right|$$

so absolute accuracy can be guaranteed for all x .

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S15ABF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S15ABF
      EXTERNAL          S15ABF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S15ABF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X      Y'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S15ABF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y
        GO TO 20
      ELSE
        WRITE (NOUT,99998) IFAIL
```

```
      END IF
40    CONTINUE
*
99999  FORMAT (1X,1P,2E12.3)
99998  FORMAT (1X,' ** S15ABF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
S15ABF Example Program Data
      -20.0
      -1.0
      0.0
      1.0
      2.0
      20.0
```

9.3 Program Results

```
S15ABF Example Program Results
```

X	Y
-2.000E+01	2.754E-89
-1.000E+00	1.587E-01
0.000E+00	5.000E-01
1.000E+00	8.413E-01
2.000E+00	9.772E-01
2.000E+01	1.000E+00
