

NAG Library Routine Document

S14BAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S14BAF computes values for the incomplete gamma functions $P(a, x)$ and $Q(a, x)$.

2 Specification

```
SUBROUTINE S14BAF(A, X, TOL, P, Q, IFAIL)
INTEGER          IFAIL
double precision A, X, TOL, P, Q
```

3 Description

S14BAF evaluates the incomplete gamma functions in the normalized form

$$P(a, x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt,$$

$$Q(a, x) = \frac{1}{\Gamma(a)} \int_x^\infty t^{a-1} e^{-t} dt,$$

with $x \geq 0$ and $a > 0$, to a user-specified accuracy. With this normalization, $P(a, x) + Q(a, x) = 1$.

Several methods are used to evaluate the functions depending on the arguments a and x , the methods including Taylor expansion for $P(a, x)$, Legendre's continued fraction for $Q(a, x)$, and power series for $Q(a, x)$. When both a and x are large, and $a \simeq x$, the uniform asymptotic expansion of Temme (1987) is employed for greater efficiency – specifically, this expansion is used when $a \geq 20$ and $0.7a \leq x \leq 1.4a$.

Once either P or Q is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of P and Q is computed first.

This routine is derived from the subroutine GAM in Gautschi (1979b).

4 References

Gautschi W (1979a) A computational procedure for incomplete gamma functions *ACM Trans. Math. Software* **5** 466–481

Gautschi W (1979b) Algorithm 542: Incomplete gamma functions *ACM Trans. Math. Software* **5** 482–489

Temme N M (1987) On the computation of the incomplete gamma functions for large values of the parameters *Algorithms for Approximation* (eds J C Mason and M G Cox) Oxford University Press

5 Parameters

1: **A – *double precision*** *Input*

On entry: the argument a of the functions.

Constraint: $A > 0.0$.

- 2: X – *double precision* *Input*
 On entry: the argument x of the functions.
 Constraint: $X \geq 0.0$.
- 3: TOL – *double precision* *Input*
 On entry: the relative accuracy required by you in the results. If S14BAF is entered with TOL greater than 1.0 or less than *machine precision*, then the value of *machine precision* is used instead.
- 4: P – *double precision* *Output*
 5: Q – *double precision* *Output*
 On exit: the values of the functions $P(a, x)$ and $Q(a, x)$ respectively.
- 6: $IFAIL$ – INTEGER *Input/Output*
 On entry: $IFAIL$ must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 On exit: $IFAIL = 0$ unless the routine detects an error (see Section 6).
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of $IFAIL$ on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $A \leq 0.0$.

$IFAIL = 2$

On entry, $X < 0.0$.

$IFAIL = 3$

Convergence of the Taylor series or Legendre continued fraction fails within 600 iterations. This error is extremely unlikely to occur; if it does, contact NAG.

7 Accuracy

There are rare occasions when the relative accuracy attained is somewhat less than that specified by parameter TOL . However, the error should never exceed more than one or two decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in the routine are given to this precision.

8 Further Comments

The time taken for a call of S14BAF depends on the precision requested through TOL , and also varies slightly with the input arguments a and x .

9 Example

This example reads values of the argument a and x from a file, evaluates the function and prints the results.

9.1 Program Text

```
*      S14BAF Example Program Text
*      Mark 14 Release. NAG Copyright 1989
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
DOUBLE PRECISION A, P, Q, TOL, X
INTEGER          IFAIL
*      .. External Functions ..
DOUBLE PRECISION X02AJF
EXTERNAL         X02AJF
*      .. External Subroutines ..
EXTERNAL         S14BAF
*      .. Executable Statements ..
WRITE (NOUT,*) 'S14BAF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
TOL = X02AJF()
WRITE (NOUT,*)
WRITE (NOUT,*) '          A          X          P          Q'
20 READ (NIN,*,END=40) A, X
   IFAIL = 1
*
   CALL S14BAF(A,X,TOL,P,Q,IFAIL)
*
   IF (IFAIL.GE.0) THEN
      WRITE (NOUT,99999) A, X, P, Q
      GO TO 20
   ELSE
      WRITE (NOUT,99998) IFAIL
   END IF
40 CONTINUE
*
99999 FORMAT (1X,4F12.4)
99998 FORMAT (1X,/1X,' ** S14BAF returned with IFAIL = ',I5)
END
```

9.2 Program Data

S14BAF Example Program Data

2.0	3.0
7.0	1.0
0.5	99.0
20.0	21.0
21.0	20.0

9.3 Program Results

S14BAF Example Program Results

A	X	P	Q
2.0000	3.0000	0.8009	0.1991
7.0000	1.0000	0.0001	0.9999
0.5000	99.0000	1.0000	0.0000
20.0000	21.0000	0.6157	0.3843
21.0000	20.0000	0.4409	0.5591
