

NAG Library Routine Document

S13ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S13ADF returns the value of the sine integral

$$\text{Si}(x) = \int_0^x \frac{\sin u}{u} du,$$

via the routine name.

2 Specification

```
double precision FUNCTION S13ADF(X, IFAIL)
      INTEGER                                IFAIL
double precision                                X
```

3 Description

S13ADF calculates an approximate value for $\text{Si}(x)$.

For $|x| \leq 16.0$ it is based on the Chebyshev expansion

$$\text{Si}(x) = x \sum_{r=0}^l a_r T_r(t), \quad t = 2 \left(\frac{x}{16} \right)^2 - 1.$$

For $16 < |x| < x_{\text{hi}}$, where x_{hi} is an implementation-dependent number,

$$\text{Si}(x) = \text{sign}(x) \left\{ \frac{\pi}{2} - \frac{f(x) \cos x}{x} - \frac{g(x) \sin x}{x^2} \right\}$$

where $f(x) = \sum_{r=0}' f_r T_r(t)$ and $g(x) = \sum_{r=0}' g_r T_r(t)$, $t = 2 \left(\frac{16}{x} \right)^2 - 1$.

For $|x| \geq x_{\text{hi}}$, $\text{Si}(x) = \frac{1}{2}\pi \text{sign } x$ to within ***machine precision***.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

- 1: X – ***double precision*** *Input*
On entry: the argument x of the function.
- 2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

There are no failure exits from S13ADF. The parameter IFAIL has been included for consistency with other routines in this chapter.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{\delta \sin x}{\text{Si}(x)} \right|.$$

The equality may hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation, then since the factor relating δ to ϵ is always less than one, the accuracy will be limited by *machine precision*.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S13ADF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S13ADF
      EXTERNAL          S13ADF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S13ADF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S13ADF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
```

```

        ELSE
            WRITE (NOUT,99998) IFAIL
        END IF
    40 CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S13ADF returned with IFAIL = ',I5)
END

```

9.2 Program Data

S13ADF Example Program Data

```

0.0
0.2
0.4
0.6
0.8
1.0

```

9.3 Program Results

S13ADF Example Program Results

X	Y	IFAIL
0.000E+00	0.000E+00	0
2.000E-01	1.996E-01	0
4.000E-01	3.965E-01	0
6.000E-01	5.881E-01	0
8.000E-01	7.721E-01	0
1.000E+00	9.461E-01	0
