

# NAG Library Routine Document

## S10AAF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

S10AAF returns a value for the hyperbolic tangent,  $\tanh x$ , via the routine name.

### 2 Specification

***double precision*** FUNCTION S10AAF(X, IFAIL)  
 INTEGER IFAIL  
***double precision*** X

### 3 Description

S10AAF calculates an approximate value for the hyperbolic tangent of its argument,  $\tanh x$ .

For  $|x| \leq 1$  it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x \sum_{r=0}' a_r T_r(t)$$

where  $-1 \leq x \leq 1$ ,  $-1 \leq t \leq 1$ , and  $t = 2x^2 - 1$ .

For  $1 < |x| < E_1$  (see the Users' Note for your implementation for value of  $E_1$ )

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For  $|x| \geq E_1$ ,  $\tanh x = \text{sign } x$  to within the representation accuracy of the machine and so this approximation is used.

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

### 5 Parameters

1: X – ***double precision*** *Input*  
*On entry:* the argument  $x$  of the function.

2: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0,  $-1$  or  $1$ . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.  
*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value  $-1$  or  $1$  is recommended. If the output of error messages is undesirable, then the value  $1$  is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is  $0$ . **When the value  $-1$  or  $1$  is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

None.

## 7 Accuracy

If  $\delta$  and  $\epsilon$  are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{2x}{\sinh 2x} \delta \right|.$$

That is, a relative error in the argument,  $x$ , is amplified by a factor approximately  $\frac{2x}{\sinh 2x}$ , in the result.

The equality should hold if  $\delta$  is greater than the *machine precision* ( $\delta$  due to data errors etc.) but if  $\delta$  is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

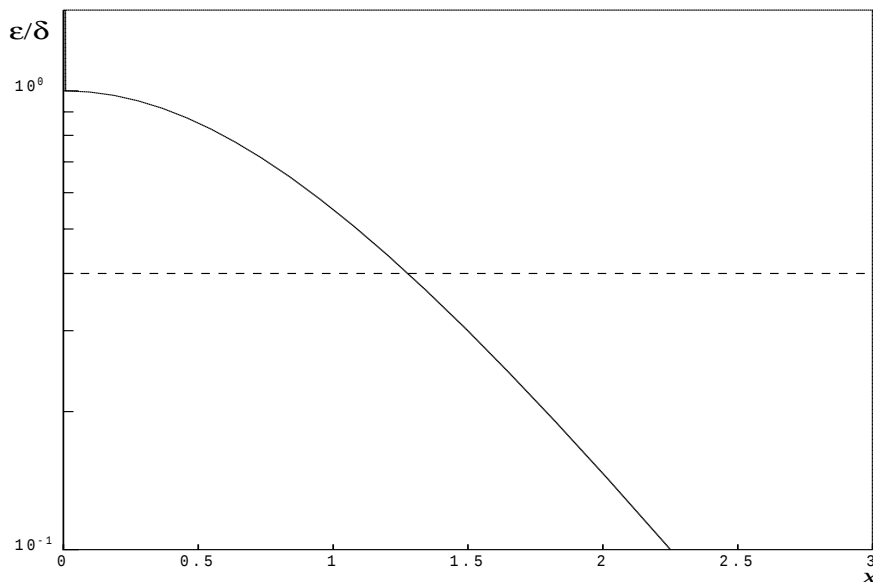


Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from  $x = 0$  the accuracy will eventually be limited entirely by the precision of machine representation.

## 8 Further Comments

None.

## 9 Example

This example reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

## 9.1 Program Text

```

*      S10AAF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S10AAF
      EXTERNAL         S10AAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S10AAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S10AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
40    CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S10AAF returned with IFAIL = ',I5)
      END

```

## 9.2 Program Data

```

S10AAF Example Program Data
      -20.0
      -5.0
      0.5
      5.0

```

## 9.3 Program Results

S10AAF Example Program Results

| X          | Y          | IFAIL |
|------------|------------|-------|
| -2.000E+01 | -1.000E+00 | 0     |
| -5.000E+00 | -9.999E-01 | 0     |
| 5.000E-01  | 4.621E-01  | 0     |
| 5.000E+00  | 9.999E-01  | 0     |

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