

NAG Library Routine Document

S07AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S07AAF returns the value of the circular tangent, $\tan x$, via the routine name.

2 Specification

double precision FUNCTION S07AAF(X, IFAIL)
 INTEGER IFAIL
double precision X

3 Description

S07AAF calculates an approximate value for the circular tangent of its argument, $\tan x$. It is based on the Chebyshev expansion

$$\tan \theta = \theta y(t) = \theta \sum_{r=0}' c_r T_r(t)$$

where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ and $-1 < t < +1$, $t = 2\left(\frac{4\theta}{\pi}\right)^2 - 1$.

The reduction to the standard range is accomplished by taking

$$x = N\pi/2 + \theta$$

where N is an integer and $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$,

i.e., $\theta = x - \left(\frac{2x}{\pi}\right)\frac{\pi}{2}$ where $N = \left[\frac{2x}{\pi}\right] =$ the nearest integer to $\frac{2x}{\pi}$.

From the properties of $\tan x$ it follows that

$$\tan x = \begin{cases} \tan \theta, & N \text{ even} \\ -1/\tan \theta, & N \text{ odd} \end{cases}$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

- 1: X – ***double precision*** *Input*
On entry: the argument x of the function.
- 2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

The routine has been called with an argument that is larger in magnitude than F ; the default result returned is zero. The value of F is given in the Users' Note for your implementation.

$IFAIL = 2$

The routine has been called with an argument that is too close (as determined using the relative tolerance F) to an odd multiple of $\pi/2$, at which the function is infinite; the routine returns a value with the correct sign but a more or less arbitrary but large magnitude (see Section 7). The value of F is given in the Users' Note for your implementation.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$\epsilon \geq \frac{2x}{\sin 2x} \delta.$$

That is a relative error in the argument, x , is amplified by at least a factor $2x/\sin 2x$ in the result.

Similarly if E is the absolute error in the result this is given by

$$E \geq \frac{x}{\cos^2 x} \delta.$$

The equalities should hold if δ is greater than the *machine precision* (δ is a result of data errors etc.) but if δ is simply the round-off error in the machine it is possible that internal calculation rounding will lose an extra figure.

The graphs below show the behaviour of these amplification factors.

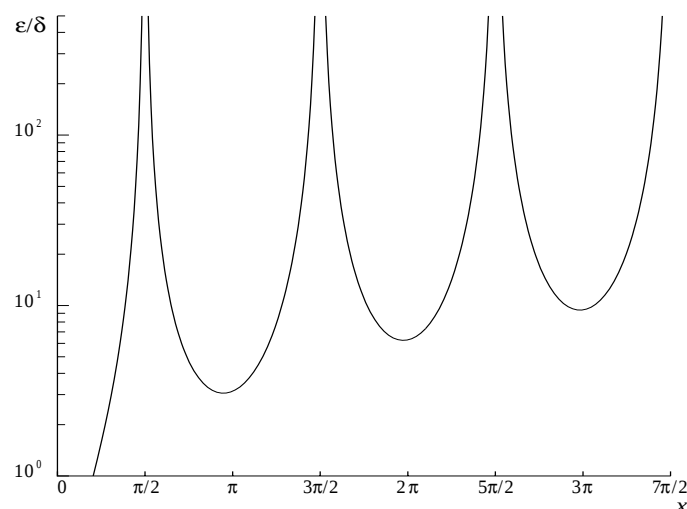


Figure 1

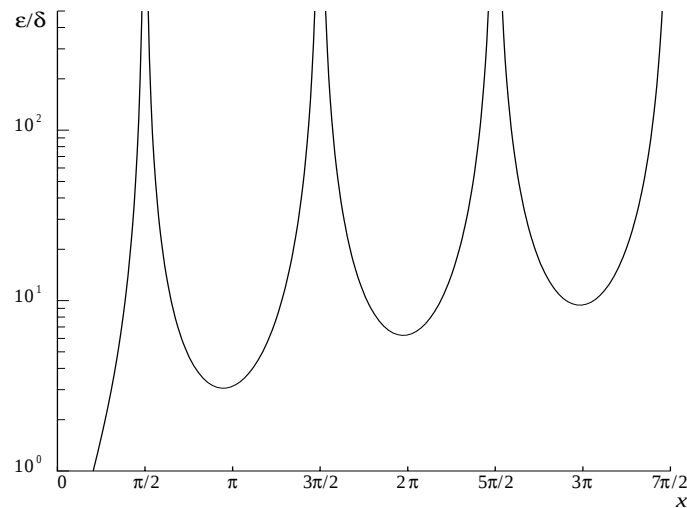


Figure 2

In the principal range it is possible to preserve relative accuracy even near the zero of $\tan x$ at $x = 0$ but at the other zeros only absolute accuracy is possible. Near the infinities of $\tan x$ both the relative and absolute errors become infinite and the routine must fail (error 2).

If N is odd and $|\theta| \leq xF_2$ the routine could not return better than two figures and in all probability would produce a result that was in error in its most significant figure. Therefore the routine fails and it returns the value

$$-\operatorname{sign} \theta \left(\frac{1}{|xF_2|} \right) \simeq -\operatorname{sign} \theta \tan \left(\frac{\pi}{2} - |xF_2| \right)$$

which is the value of the tangent at the nearest argument for which a valid call could be made.

Accuracy is also unavoidably lost if the routine is called with a large argument. If $|x| > F_1$ the routine fails (error 1) and returns zero. (See the Users' Note for your implementation for specific values of F_1 and F_2 .)

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
*      S07AAF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION X, Y
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION S07AAF
      EXTERNAL         S07AAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S07AAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
```

```

      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X          Y          IFAIL'
      WRITE (NOUT,*)
20  READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S07AAF(X,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        WRITE (NOUT,99999) X, Y, IFAIL
        GO TO 20
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
40  CONTINUE
*
99999 FORMAT (1X,1P,2E12.3,I7)
99998 FORMAT (1X,' ** S07AAF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

S07AAF Example Program Data

```

      -2.0
      -0.5
      1.0
      3.0
      1.5708

```

9.3 Program Results

S07AAF Example Program Results

X	Y	IFAIL
-2.000E+00	2.185E+00	0
-5.000E-01	-5.463E-01	0
1.000E+00	1.557E+00	0
3.000E+00	-1.425E-01	0
1.571E+00	-2.722E+05	0
