NAG Library Routine Document

G13DXF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13DXF calculates the zeros of a vector autoregressive (or moving average) operator. This routine is likely to be used in conjunction with G05PJF, G13ASF, G13DDF or G13DSF.

2 Specification

```
SUBROUTINE G13DXF(K, IP, PAR, RR, RI, RMOD, WORK, IWORK, IFAIL)
INTEGER            K, IP, IWORK(K*IP), IFAIL
double precision   PAR(IP*K*K), RR(K*IP), RI(K*IP), RMOD(K*IP),
1                  WORK(K*K*IP*IP)
```

3 Description

Consider the vector autoregressive moving average (VARMA) model

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p\big(W_{t-p} - \mu\big) + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}, \tag{1}$$

where W_t denotes a vector of k time series and ϵ_t is a vector of k residual series having zero mean and a constant variance-covariance matrix. The components of ϵ_t are also assumed to be uncorrelated at non-simultaneous lags. $\phi_1, \phi_2, \ldots, \phi_p$ denotes a sequence of k by k matrices of autoregressive (AR) parameters and $\theta_1, \theta_2, \ldots, \theta_q$ denotes a sequence of k by k matrices of moving average (MA) parameters. μ is a vector of length k containing the series means. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk}$$

where I denotes the k by k identity matrix.

The model (1) is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle. Similarly let

$$B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}.$$

Then the model is said to be invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle.

G13DXF returns the pk eigenvalues of $A(\phi)$ (or the qk eigenvalues of $B(\theta)$) along with their moduli, in descending order of magnitude. Thus to check for stationarity or invertibility you should check whether the modulus of the largest eigenvalue is less than one.

4 References

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Parameters

1: K – INTEGER *Input*

On entry: k, the dimension of the multivariate time series.

Constraint: K ≥ 1.

2: IP – INTEGER *Input*

On entry: the number of AR (or MA) parameter matrices, p (or q).

Constraint: IP ≥ 1.

3: PAR(IP $\times$ K $\times$ K) – ***double precision*** array *Input*

On entry: the AR (or MA) parameter matrices read in row by row in the order $\phi_1, \phi_2, \ldots, \phi_p$ (or $\theta_1, \theta_2, \ldots, \theta_q$). That is, PAR$((l-1) \times k \times k + (i-1) \times k + j - 1)$ must be set equal to the (i,j)th element of ϕ_l, for $l = 1, 2, \ldots, p$ (or the (i,j)th element of θ_l, for $l = 1, 2, \ldots, q$).

4: RR(K $\times$ IP) – ***double precision*** array *Output*

On exit: the real parts of the eigenvalues.

5: RI(K $\times$ IP) – ***double precision*** array *Output*

On exit: the imaginary parts of the eigenvalues.

6: RMOD(K $\times$ IP) – ***double precision*** array *Output*

On exit: the moduli of the eigenvalues.

7: WORK(K $\times$ K $\times$ IP $\times$ IP) – ***double precision*** array *Workspace*

8: IWORK(K $\times$ IP) – INTEGER array *Workspace*

9: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, K < 1,
or IP < 1.

IFAIL = 2

An excessive number of iterations are needed to evaluate the eigenvalues of $A(\phi)$ (or $B(\theta)$). This is an unlikely exit. All output parameters are undefined.

7 Accuracy

The accuracy of the results depends on the original matrix and the multiplicity of the roots.

8 Further Comments

The time taken is approximately proportional to kp^3 (or kq^3).

9 Example

This example finds the eigenvalues of $A(\phi)$ where $k = 2$ and $p = 1$ and $\phi_1 = \begin{bmatrix} 0.802 & 0.065 \\ 0.000 & 0.575 \end{bmatrix}$.

9.1 Program Text

```
*     G13DXF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          KMAX, IPMAX
      PARAMETER        (KMAX=6,IPMAX=3)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, IP, K, NPAR
*     .. Local Arrays ..
      DOUBLE PRECISION PAR(KMAX*KMAX*IPMAX), RI(KMAX*IPMAX),
     +                 RMOD(KMAX*IPMAX), RR(KMAX*IPMAX),
     +                 WORK(KMAX*KMAX*IPMAX*IPMAX)
      INTEGER          IWORK(KMAX*IPMAX)
*     .. External Subroutines ..
      EXTERNAL         G13DXF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13DXF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) K, IP
      IF (K.GT.0 .AND. K.LE.KMAX .AND. IP.GT.0 .AND. IP.LE.IPMAX) THEN
*        Read the AR (or MA) parameters
         NPAR = IP*K*K
         READ (NIN,*) (PAR(I),I=1,NPAR)
         IFAIL = 0
*
         CALL G13DXF(K,IP,PAR,RR,RI,RMOD,WORK,IWORK,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) '        Eigenvalues       Moduli'
         WRITE (NOUT,*) '        -----------       ------'
*
         DO 20 I = 1, K*IP
            IF (RI(I).GE.0.0D0) THEN
               WRITE (NOUT,99999) RR(I), RI(I), RMOD(I)
            ELSE
               WRITE (NOUT,99998) RR(I), -RI(I), RMOD(I)
            END IF
   20    CONTINUE
      ELSE
         WRITE (NOUT,*) ' Either K or IP is out of range'
```

```
      END IF
*
99999 FORMAT (' ',F10.3,'  + ',F6.3,' i  ',F8.3)
99998 FORMAT (' ',F10.3,'  - ',F6.3,' i  ',F8.3)
      END
```

9.2 Program Data

```
G13DXF Example Program Data
 2 1
 0.802 0.065
 0.000 0.575
```

9.3 Program Results

```
 G13DXF Example Program Results

         Eigenvalues       Moduli
         -----------       ------
      0.802  +  0.000 i     0.802
      0.575  +  0.000 i     0.575
```