NAG Library Routine Document

G13AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13AAF carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

2 Specification

```
SUBROUTINE G13AAF(X, NX, ND, NDS, NS, XD, NXD, IFAIL)
INTEGER           NX, ND, NDS, NS, NXD, IFAIL
double precision  X(NX), XD(NX)
```

3 Description

Let $\nabla^d \nabla_s^D x_i$ be the ith value of a time series x_i, for $i = 1, 2, \ldots, n$ after non-seasonal differencing of order d and seasonal differencing of order D (with period or seasonality s). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i && d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i && D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order d is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i && \text{for } i = 1, 2, \ldots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i && \text{for } i = 1, 2, \ldots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i && \text{for } i = 1, 2, \ldots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order D is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i && \text{for } i = 1, 2, \ldots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i && \text{for } i = 1, 2, \ldots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i && \text{for } i = 1, 2, \ldots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of $m = n - d - D \times s$ values.

4 References

None.

5 Parameters

1: X(NX) – ***double precision*** array *Input*

On entry: the undifferenced time series, x_i, for $i = 1, 2, \ldots, n$.

2: NX – INTEGER *Input*

On entry: n, the number of values in the undifferenced time series.

Constraint: NX $>$ ND $+$ (NDS $\times$ NS).

3: ND – INTEGER *Input*

On entry: d, the order of non-seasonal differencing.

Constraint: ND ≥ 0.

4: NDS – INTEGER *Input*

On entry: D, the order of seasonal differencing.

Constraint: NDS ≥ 0.

5: NS – INTEGER *Input*

On entry: s, the seasonality.

Constraints:

if NDS > 0, NS > 0;
if NDS $= 0$, NS ≥ 0.

6: XD(NX) – ***double precision*** array *Output*

On exit: the differenced values in elements 1 to NXD, and reconstitution data in the remainder of the array.

7: NXD – INTEGER *Output*

On exit: the number of differenced values in the array XD.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, ND < 0,
or NDS < 0,
or NS < 0,
or NS $= 0$ when NDS > 0.

IFAIL $= 2$

On entry, NX $\leq$ ND $+$ (NDS $\times$ NS).

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by G13AAF is approximately proportional to $(\mathrm{ND} + \mathrm{NDS}) \times \mathrm{NX}$.

9 Example

This example reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

9.1 Program Text

```
*     G13AAF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXMAX
      PARAMETER        (NXMAX=20)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, ND, NDS, NS, NX, NXD
*     .. Local Arrays ..
      DOUBLE PRECISION X(NXMAX), XD(NXMAX)
*     .. External Subroutines ..
      EXTERNAL         G13AAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13AAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NX, ND, NDS, NS
      IF (NX.GT.0 .AND. NX.LE.NXMAX) THEN
         READ (NIN,*) (X(I),I=1,NX)
         WRITE (NOUT,*)
         WRITE (NOUT,99999) 'Non-seasonal differencing of order ', ND,
     +     ' and seasonal differencing'
         WRITE (NOUT,99999) 'of order ', NDS, ' with seasonality ', NS,
     +     ' are applied'
         IFAIL = 0
*
         CALL G13AAF(X,NX,ND,NDS,NS,XD,NXD,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,99998) 'The output array holds ', NX,
     +     ' values, of which the first ', NXD,
     +     ' are differenced values'
         WRITE (NOUT,*)
         WRITE (NOUT,99997) (XD(I),I=1,NX)
      END IF
*
99999 FORMAT (1X,A,I1,A,I1,A)
99998 FORMAT (1X,A,I2,A,I2,A)
99997 FORMAT (1X,5F9.1)
      END
```

9.2 Program Data

```
G13AAF Example Program Data
 20  2  1  4
 120.0 108.0  98.0 118.0 135.0
 131.0 118.0 125.0 121.0 100.0
  82.0  82.0  89.0  88.0  86.0
  96.0 108.0 110.0  99.0 105.0
```

9.3 Program Results

```
 G13AAF Example Program Results

 Non-seasonal differencing of order 2 and seasonal differencing
 of order 1 with seasonality 4 are applied

 The output array holds 20 values, of which the first 14 are differenced values

     -11.0    -10.0     -8.0      4.0     12.0
      -2.0     18.0      9.0     -4.0     -6.0
      -5.0     -2.0    -12.0      5.0      2.0
     -10.0    -13.0     17.0      6.0    105.0
```