

NAG Library Routine Document

G05RAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G05RAF sets up a reference vector and generates an array of pseudorandom numbers from a Normal (Gaussian) copula with covariance matrix C .

2 Specification

```
SUBROUTINE G05RAF(MODE, M, C, LDC, N, X, LDX, IGEN, ISEED, R, LR, IFAIL)
INTEGER          MODE, M, LDC, N, LDX, IGEN, ISEED(4), LR, IFAIL
double precision C(LDC,M), X(LDX,M), R(LR)
```

3 Description

The Gaussian copula, c , is defined by

$$c(u_1, u_2, \dots, u_m; C) = \Phi_C(\phi_{C_{11}}^{-1}(u_1), \phi_{C_{22}}^{-1}(u_2), \dots, \phi_{C_{mm}}^{-1}(u_m))$$

where m is the number of dimensions, Φ_C is the multivariate Normal density function with mean zero and covariance matrix C and $\phi_{C_{ii}}^{-1}$ is the inverse of the univariate Normal density function with mean zero and variance C_{ii} .

Routine G05LYF is used to generate a vector from a multivariate Normal distribution and routine G01EAF is used to convert each element of that vector into a uniformly distributed value between zero and one.

One of the initialization routines G05KBF (for a repeatable sequence if computed sequentially) or G05KCF (for a non-repeatable sequence) must be called prior to the first call to G05RAF.

4 References

Nelsen R B (1998) *An Introduction to Copulas. Lecture Notes in Statistics 139* Springer

Sklar A (1973) Random Variables: Joint Distribution Functions and Copulas *Kybernetika* **9** 499–460

5 Parameters

- 1: MODE – INTEGER *Input*
- On entry:* selects the operation to be performed.
- MODE = 0
 Initialize and generate random numbers.
- MODE = 1
 Initialize only (i.e., set up reference vector).
- MODE = 2
 Generate random numbers using previously set up reference vector.
- Constraint:* MODE = 0, 1 or 2.

- 2: M – INTEGER *Input*
On entry: m , the number of dimensions of the distribution.
Constraint: $M > 0$.
- 3: C(LDC,M) – **double precision** array *Input*
On entry: the covariance matrix of the distribution. Only the upper triangle need be set.
Constraint: C must be positive semi-definite to **machine precision**
- 4: LDC – INTEGER *Input*
On entry: the first dimension of the array C as declared in the (sub)program from which G05RAF is called.
Constraint: $LDC \geq M$.
- 5: N – INTEGER *Input*
On entry: n , the number of random variates required.
Constraint: $N \geq 1$.
- 6: X(LDX,M) – **double precision** array *Output*
On exit: the array of pseudorandom multivariate Normal vectors.
- 7: LDX – INTEGER *Input*
On entry: the first dimension of the array X as declared in the (sub)program from which G05RAF is called.
Constraint: $LDX \geq N$.
- 8: IGEN – INTEGER *Input*
On entry: must contain the identification number for the generator to be used to return a pseudorandom number and should remain unchanged following initialization by a prior call to G05KBF or G05KCF.
- 9: ISEED(4) – INTEGER array *Communication Array*
On entry: contains values which define the current state of the selected generator.
On exit: contains updated values defining the new state of the selected generator.
- 10: R(LR) – **double precision** array *Communication Array*
On entry: if $MODE = 2$, the reference vector as set up by G05RAF in a previous call with $MODE = 0$ or 1.
On exit: if $MODE = 0$ or 1, the reference vector that can be used in subsequent calls to G05RAF with $MODE = 2$.
- 11: LR – INTEGER *Input*
On entry: the dimension of the array R as declared in the (sub)program from which G05RAF is called. If $MODE = 2$, it must be the same as the value of LR specified in the prior call to G05RAF with $MODE = 0$ or 1.
Constraint: $LR > M \times (M + 1)$.
- 12: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $\text{MODE} \neq 0, 1 \text{ or } 2$.

IFAIL = 2

On entry, $M < 1$.

IFAIL = 3

The covariance matrix C is not positive semi-definite to *machine precision*.

IFAIL = 4

On entry, $\text{LDC} < M$.

IFAIL = 5

On entry, $N < 1$.

IFAIL = 7

On entry, $\text{LDX} < N$.

IFAIL = 8

On entry, invalid value for IGEN. IGEN must be the same as the value as specified in the prior call to G05RAF with $\text{MODE} = 0$ or 1 .

IFAIL = 10

The reference vector R has been corrupted or M has changed since R was set up in a previous call with $\text{MODE} = 0$ or 1 .

IFAIL = 11

On entry, $\text{LR} < M \times (M + 1)$.

7 Accuracy

See Section 7 in G05LYF for an indication of the accuracy of the underlying multivariate Normal distribution.

8 Further Comments

None.

9 Example

This example prints ten pseudorandom observations from a Normal copula with covariance matrix

$$\begin{bmatrix} 1.69 & 0.39 & -1.86 & 0.07 \\ 0.39 & 98.01 & -7.07 & -0.71 \\ -1.86 & -7.07 & 11.56 & 0.03 \\ 0.07 & -0.71 & 0.03 & 0.01 \end{bmatrix},$$

generated by G05RAF. All ten observations are generated by a single call to G05RAF with MODE = 0. The random number generator is initialized by G05KBF.

9.1 Program Text

```
*      G05RAF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NOUT, LDC, LDX, LR
      PARAMETER        (NOUT=6,LDC=5,LDX=100,LR=LDC*LDC+LDC+1)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, IGEN, J, M, MODE, N
*      .. Local Arrays ..
      DOUBLE PRECISION C(LDC,LDC), R(LR), X(LDX,LDC)
      INTEGER          ISEED(4)
*      .. External Subroutines ..
      EXTERNAL         G05KBF, G05RAF
*      .. Executable Statements ..
      CONTINUE

*
      WRITE (NOUT,*) 'G05RAF Example Program Results'
      WRITE (NOUT,*)

*
*      Initialize the seed to a repeatable sequence
      ISEED(1) = 1762543
      ISEED(2) = 9324783
      ISEED(3) = 42344
      ISEED(4) = 742355
*
*      Choose the random generator to use
      IGEN = 1
*
*      Initialize the random generator
      CALL G05KBF(IGEN,ISEED)
*
*      Set the number of variables and variates
      M = 4
      N = 10
*
*      Input the upper triangle portion of the covariance matrix
      C(1,1) = 1.69D0
      C(1,2) = 0.39D0
      C(1,3) = -1.86D0
      C(1,4) = 0.07D0
      C(2,2) = 98.01D0
      C(2,3) = -7.07D0
      C(2,4) = -0.71D0
      C(3,3) = 11.56D0
      C(3,4) = 0.03D0
      C(4,4) = 0.01D0
*
*      MODE = 0
      IFAIL = 1
*
*      Set up reference vector and generate N numbers
      CALL G05RAF(MODE,M,C,LDC,N,X,LDX,IGEN,ISEED,R,LR,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
*          Display the results
          DO 20 I = 1, N
```

```
        WRITE (NOUT,99999) (X(I,J),J=1,M)
20      CONTINUE
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
*
99999 FORMAT (1X,10F10.4)
99998 FORMAT (1X,' ** G05RAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

None.

9.3 Program Results

G05RAF Example Program Results

0.9819	0.1689	0.0712	0.9428
0.2525	0.7025	0.5261	0.1134
0.4771	0.8504	0.8684	0.4064
0.0900	0.4690	0.7162	0.2592
0.3835	0.0400	0.8975	0.7915
0.5492	0.9685	0.4275	0.1749
0.2064	0.3430	0.9595	0.5140
0.7369	0.6728	0.0191	0.3639
0.8970	0.0732	0.5617	0.9151
0.2198	0.0157	0.8901	0.8911
