NAG Library Routine Document

G05QAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G05QAF generates a random orthogonal matrix.

2 Specification

```
SUBROUTINE G05QAF(SIDE, INIT, M, N, A, LDA, IGEN, ISEED, WK, IFAIL)
INTEGER           M, N, LDA, IGEN, ISEED(4), IFAIL
double precision  A(LDA,N), WK(*)
CHARACTER*1       SIDE, INIT
```

3 Description

G05QAF pre- or post-multiplies an m by n matrix A by a random orthogonal matrix U, overwriting A. The matrix A may optionally be initialized to the identity matrix before multiplying by U, hence U is returned. U is generated using the method of Stewart (1980). The algorithm can be summarized as follows.

Let $x_1, x_2, \ldots, x_{n-1}$ follow independent multinormal distributions with zero mean and variance $I\sigma^2$ and dimensions $n, n-1, \ldots, 2$; let $H_j = \text{diag}\left(I_{j-1}, H_j^*\right)$, where I_{j-1} is the identity matrix and H_j^* is the Householder transformation that reduces x_j to $r_{jj}e_1$, e_1 being the vector with first element one and the remaining elements zero and r_{jj} being a scalar, and let $D = \text{diag}(\text{sign}(r_{11}), \text{sign}(r_{22}), \ldots, \text{sign}(r_{nn}))$. Then the product $U = DH_1H_2 \ldots H_{n-1}$ is a random orthogonal matrix distributed according to the Haar measure over the set of orthogonal matrices of n. See Theorem 3.3 in Stewart (1980).

One of the initialization routines G05KBF (for a repeatable sequence if computed sequentially) or G05KCF (for a non-repeatable sequence) must be called prior to the first call to G05QAF.

4 References

Stewart G W (1980) The efficient generation of random orthogonal matrices with an application to condition estimates *SIAM J. Numer. Anal.* **17** 403–409

5 Parameters

1: SIDE – CHARACTER*1 *Input*

On entry: indicates whether the matrix A is multiplied on the left or right by the random orthogonal matrix U.

SIDE = 'L'
The matrix A is multiplied on the left, i.e., pre-multiplied.

SIDE = 'R'
The matrix A is multiplied on the right, i.e., post-multiplied.

Constraint: SIDE = 'L' or 'R'.

2: INIT – CHARACTER*1 *Input*

On entry: indicates whether or not A should be initialized to the identity matrix.

INIT = 'I'
A is initialized to the identity matrix.

INIT = 'N'
A is not initialized and the matrix A must be supplied in A.

Constraint: INIT = 'I' or 'N'.

3: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraints:

if SIDE = 'L', M > 1;
otherwise M ≥ 1.

4: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraints:

if SIDE = 'R', N > 1;
otherwise N ≥ 1.

5: A(LDA,N) – ***double precision*** array *Input/Output*

On entry: if INIT = 'N', A must contain the matrix A.

On exit: the matrix UA when SIDE = 'L' or the matrix AU when SIDE = 'R'.

6: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which G05QAF is called.

Constraint: LDA $\geq$ M.

7: IGEN – INTEGER *Input*

On entry: must contain the identification number for the generator to be used to return a pseudorandom number and should remain unchanged following initialization by a prior call to G05KBF or G05KCF.

8: ISEED(4) – INTEGER array *Communication Array*

On entry: contains values which define the current state of the selected generator.

On exit: contains updated values defining the new state of the selected generator.

9: WK($*$) – ***double precision*** array *Workspace*

Note: the dimension of the array WK must be at least $2 \times$ M if SIDE = 'L' and at least $2 \times$ N if SIDE = 'R'.

10: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then

the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, M < 1,
or N < 1,
or LDA < M.

IFAIL = 2

On entry, SIDE ≠ 'L' or 'R',
or INIT ≠ 'I' or 'N'.

IFAIL = 3

On entry, an orthogonal matrix of dimension 1 has been requested.

7 Accuracy

The maximum error in $U^{\mathrm{T}}U$ should be a modest multiple of ***machine precision*** (see Chapter X02).

8 Further Comments

G05QBF computes a random correlation matrix from a random orthogonal matrix.

9 Example

Following initialization of the pseudorandom number generator by a call to G05KBF, a 4 by 4 orthogonal matrix is generated using the INIT = 'I' option and the result printed.

9.1 Program Text

```
*     G05QAF Example Program Text
*     Mark 20 Release. NAG Copyright 2001.
*     .. Parameters ..
      INTEGER          N, M, LDA
      PARAMETER        (N=4,M=4,LDA=10)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, IGEN, J
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,N), WK(2*N)
      INTEGER          ISEED(4)
*     .. External Subroutines ..
      EXTERNAL         G05KBF, G05QAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G05QAF Example Program Results'
      WRITE (NOUT,*)
*
*     IGEN identifies the stream.
      IGEN = 1
*     Initialize the seed to a repeatable sequence
      ISEED(1) = 1762543
```

```
      ISEED(2) = 9324783
      ISEED(3) = 423446
      ISEED(4) = 742355
      CALL G05KBF(IGEN,ISEED)
*
      IFAIL = 1
*
      CALL G05QAF('Right','Initialize',M,N,A,LDA,IGEN,ISEED,WK,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         DO 20 I = 1, M
            WRITE (NOUT,99999) (A(I,J),J=1,N)
   20    CONTINUE
      ELSE
         WRITE (NOUT,99998) IFAIL
      END IF
*
99999 FORMAT (1X,4F9.3)
99998 FORMAT (1X,' ** G05QAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 G05QAF Example Program Results

    -0.219   -0.197   -0.413   -0.862
     0.438    0.006    0.762   -0.478
     0.699   -0.627   -0.322    0.120
    -0.521   -0.754    0.381    0.122
```