

NAG Library Routine Document

G01FCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01FCF returns the deviate associated with the given lower tail probability of the χ^2 -distribution with real degrees of freedom, via the routine name.

2 Specification

```
double precision FUNCTION G01FCF(P, DF, IFAIL)
      INTEGER                                IFAIL
double precision                                P, DF
```

3 Description

The deviate, x_p , associated with the lower tail probability p of the χ^2 -distribution with ν degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{\nu/2-1} dX, \quad 0 \leq x_p < \infty; \nu > 0.$$

The required x_p is found by using the relationship between a χ^2 -distribution and a gamma distribution, i.e., a χ^2 -distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

For very large values of ν , greater than 10^5 , Wilson and Hilferty's normal approximation to the χ^2 is used; see Kendall and Stuart (1969).

4 References

Best D J and Roberts D E (1975) Algorithm AS 91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

5 Parameters

- | | | |
|----|--|--------------|
| 1: | P – <i>double precision</i> | <i>Input</i> |
| | <i>On entry:</i> p , the probability from the required χ^2 -distribution. | |
| | <i>Constraint:</i> $0.0 \leq P < 1.0$. | |
| 2: | DF – <i>double precision</i> | <i>Input</i> |
| | <i>On entry:</i> ν , the degrees of freedom of the χ^2 -distribution. | |
| | <i>Constraint:</i> $DF > 0.0$. | |

3: IFAIL – INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq$ 0 on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G01FCF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

If IFAIL = 1, 2, 3 or 5 on exit, then G01FCF returns 0.0.

IFAIL = 1

On entry, $P < 0.0$,
or $P \geq 1.0$.

IFAIL = 2

On entry, $DF \leq 0.0$.

IFAIL = 3

P is too close to 0 or 1 for the result to be calculated.

IFAIL = 4

The solution has failed to converge. The result should be a reasonable approximation.

IFAIL = 5

The series used to calculate the gamma probabilities has failed to converge. This is an unlikely error exit.

7 Accuracy

The results should be accurate to five significant digits for most parameter values. Some accuracy is lost for p close to 0.0.

8 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to G01FFF made.

9 Example

This example reads lower tail probabilities for several χ^2 -distributions, and calculates and prints the corresponding deviates until the end of data is reached.

9.1 Program Text

```

*      G01FCF Example Program Text
*      Mark 14 Release. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DF, P, X
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION G01FCF
      EXTERNAL          G01FCF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01FCF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          P          DF          X'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) P, DF
      IFAIL = 1
*
      X = G01FCF(P,DF,IFAIL)
*
      IF (IFAIL.GE.0) THEN
        IF (IFAIL.EQ.0) THEN
          WRITE (NOUT,99999) P, DF, X
        ELSE IF (IFAIL.GT.0) THEN
          WRITE (NOUT,99999) P, DF, X, ' NOTE: IFAIL = ', IFAIL
        END IF
        GO TO 20
      ELSE
        WRITE (NOUT,99998) ' ** G01FCF returned with IFAIL = ', IFAIL
      END IF
40    CONTINUE
*
99999 FORMAT (1X,3F8.3,A,I1)
99998 FORMAT (1X,A,I5)
      END

```

9.2 Program Data

```

G01FCF Example Program Data
0.0100 20.0          :P DF
0.4279 7.5           :P DF
0.8694 45.0          :P DF

```

9.3 Program Results

G01FCF Example Program Results

P	DF	X
0.010	20.000	8.260
0.428	7.500	6.200
0.869	45.000	55.759
