

NAG Library Routine Document

G01FAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01FAF returns the deviate associated with the given probability of the standard Normal distribution, via the routine name.

2 Specification

double precision FUNCTION G01FAF(TAIL, P, IFAIL)
 INTEGER IFAIL
double precision P
 CHARACTER*1 TAIL

3 Description

The deviate, x_p associated with the lower tail probability, p , for the standard Normal distribution is defined as the solution to

$$P(X \leq x_p) = p = \int_{-\infty}^{x_p} Z(X) dX,$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \quad -\infty < X < \infty.$$

The method used is an extension of that of Wichura (1988). p is first replaced by $q = p - 0.5$.

(a) If $|q| \leq 0.3$, x_p is computed by a rational Chebyshev approximation

$$x_p = s \frac{A(s^2)}{B(s^2)},$$

where $s = \sqrt{2\pi}q$ and A, B are polynomials of degree 7.

(b) If $0.3 < |q| \leq 0.42$, x_p is computed by a rational Chebyshev approximation

$$x_p = \text{sign } q \left(\frac{C(t)}{D(t)} \right),$$

where $t = |q| - 0.3$ and C, D are polynomials of degree 5.

(c) If $|q| > 0.42$, x_p is computed as

$$x_p = \text{sign } q \left[\left(\frac{E(u)}{F(u)} \right) + u \right],$$

where $u = \sqrt{-2 \times \log(\min(p, 1-p))}$ and E, F are polynomials of degree 6.

For the upper tail probability $-x_p$ is returned, while for the two tail probabilities the value x_{p^*} is returned, where p^* is the required tail probability computed from the input value of p .

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Wichura (1988) Algorithm AS 241: The percentage points of the Normal distribution *Appl. Statist.* **37** 477–484

5 Parameters

1: TAIL – CHARACTER*1 *Input*

On entry: indicates which tail the supplied probability represents.

TAIL = 'L'

The lower probability, i.e., $P(X \leq x_p)$.

TAIL = 'U'

The upper probability, i.e., $P(X \geq x_p)$.

TAIL = 'S'

The two tail (significance level) probability, i.e., $P(X \geq |x_p|) + P(X \leq -|x_p|)$.

TAIL = 'C'

The two tail (confidence interval) probability, i.e., $P(X \leq |x_p|) - P(X \leq -|x_p|)$.

Constraint: TAIL = 'L', 'U', 'S' or 'C'.

2: P – *double precision* *Input*

On entry: p , the probability from the standard Normal distribution as defined by TAIL.

Constraint: $0.0 < P < 1.0$.

3: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If on exit IFAIL $\neq$ 0, then G01FAF returns 0.0.

IFAIL = 1

On entry, TAIL $\neq$ 'L', 'U', 'S' or 'C'.

IFAIL = 2

On entry, $P \leq 0.0$,
or $P \geq 1.0$.

7 Accuracy

The accuracy is mainly limited by the *machine precision*.

8 Further Comments

None.

9 Example

Four values of TAIL and P are input and the deviates calculated and printed.

9.1 Program Text

```
*      G01FAF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DEV, P
      INTEGER          I, IFAIL
      CHARACTER        TAIL
*      .. External Functions ..
      DOUBLE PRECISION G01FAF
      EXTERNAL          G01FAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01FAF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) ' Tail      Probability      Deviate '
      WRITE (NOUT,*)
      DO 20 I = 1, 4
         READ (NIN,*) TAIL, P
         IFAIL = 1
*
         DEV = G01FAF(TAIL,P,IFAIL)
*
         IF (IFAIL.EQ.0) THEN
*
            WRITE (NOUT,99999) TAIL, P, DEV
*
         ELSE
            WRITE (NOUT,99998) IFAIL
            GO TO 40
         END IF
      20 CONTINUE
      40 CONTINUE
*
99999 FORMAT (3X,A1,9X,F5.3,9X,F6.4)
99998 FORMAT (1X,/1X,' ** G01FAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
G01FAF Example Program Data
'L' 0.975
'U' 0.025
'C' 0.95
'S' 0.05
```

9.3 Program Results

G01FAF Example Program Results

Tail	Probability	Deviate
L	0.975	1.9600
U	0.025	1.9600
C	0.950	1.9600
S	0.050	1.9600
