NAG Library Routine Document

G01EMF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EMF returns the probability associated with the lower tail of the distribution of the Studentized range statistic, via the routine name.

2 Specification

double precision `FUNCTION G01EMF(Q, V, IR, IFAIL)`
`INTEGER` `IR, IFAIL`
double precision `Q, V`

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as:

$$q = \frac{\max x_i - \min x_i}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - qx)]^{r-1}\, dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \qquad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \qquad \text{and} \qquad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

The above two-dimensional integral is evaluated using D01DAF with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - q)]^{r-1}\, dy.$$

This integral is evaluated using D01AMF.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32 (2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

1: Q – ***double precision*** *Input*

On entry: q, the Studentized range statistic.

Constraint: Q > 0.0.

2: V – ***double precision*** *Input*

On entry: v, the number of degrees of freedom for the experimental error.

Constraint: V ≥ 1.0.

3: IR – INTEGER *Input*

On entry: r, the number of groups.

Constraint: IR ≥ 2.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

If on exit IFAIL $= 1$, then G01EMF returns to 0.0.

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G01EMF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, Q ≤ 0.0,
or V < 1.0,
or IR < 2.

IFAIL $= 2$

There is some doubt as to whether full accuracy has been achieved.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless IFAIL = 2. When IFAIL = 2 it is usual that the returned value will be a good estimate of the true value.

8 Further Comments

None.

9 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q, ν and r.

9.1 Program Text

```
*     G01EMF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION Q, V, VALP
      INTEGER          I, IFAIL, IR
*     .. External Functions ..
      DOUBLE PRECISION G01EMF
      EXTERNAL         G01EMF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01EMF Example Program Results '
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      DO 20 I = 1, 3
         READ (NIN,*) Q, V, IR
         IFAIL = 1
*
         VALP = G01EMF(Q,V,IR,IFAIL)
*
         IF (IFAIL.EQ.0) THEN
            IF (I.EQ.1) THEN
               WRITE (NOUT,*) '  Q       V    IR    Quantile '
               WRITE (NOUT,*)
            END IF
            WRITE (NOUT,99999) Q, V, IR, VALP
         ELSE
            WRITE (NOUT,99998) IFAIL
            GO TO 40
         END IF
   20 CONTINUE
   40 CONTINUE
*
99999 FORMAT (1X,F7.4,2X,F4.1,1X,I3,1X,F10.4)
99998 FORMAT (1X,' ** G01EMF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
G01EMF Example Program Data
4.6543  10.0   5
2.8099  60.0  12
4.2636   5.0   4
```

9.3 Program Results

```
 G01EMF Example Program Results

   Q       V    IR    Quantile

 4.6543  10.0   5      0.9500
 2.8099  60.0  12      0.3000
 4.2636   5.0   4      0.9000
```