

NAG Library Routine Document

G01EFF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EFF returns the lower or upper tail probability of the gamma distribution, with parameters α and β , via the routine name.

2 Specification

double precision FUNCTION G01EFF(TAIL, G, A, B, IFAIL)

INTEGER

IFAIL

double precision

G, A, B

CHARACTER*1

TAIL

3 Description

The lower tail probability for the gamma distribution with parameters α and β , $P(G \leq g)$, is defined by:

$$P(G \leq g; \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^g G^{\alpha-1} e^{-G/\beta} dG, \quad \alpha > 0.0, \beta > 0.0.$$

The mean of the distribution is $\alpha\beta$ and its variance is $\alpha\beta^2$. The transformation $Z = \frac{G}{\beta}$ is applied to yield the following incomplete gamma function in normalized form,

$$P(G \leq g; \alpha, \beta) = P(Z \leq g/\beta : \alpha, 1.0) = \frac{1}{\Gamma(\alpha)} \int_0^{g/\beta} Z^{\alpha-1} e^{-Z} dZ.$$

This is then evaluated using S14BAF.

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Parameters

1: TAIL – CHARACTER*1

Input

On entry: indicates whether an upper or lower tail probability is required.

TAIL = 'L'

The lower tail probability is returned, that is $P(G \leq g : \alpha, \beta)$.

TAIL = 'U'

The upper tail probability is returned, that is $P(G \geq g : \alpha, \beta)$.

Constraint: TAIL = 'L' or 'U'.

2: G – ***double precision***

Input

On entry: g, the value of the gamma variate.

Constraint: G $\geq$ 0.0.

- 3: A – *double precision* *Input*
 On entry: the parameter α of the gamma distribution.
 Constraint: $A > 0.0$.
- 4: B – *double precision* *Input*
 On entry: the parameter β of the gamma distribution.
 Constraint: $B > 0.0$.
- 5: IFAIL – INTEGER *Input/Output*
 On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL = 1, 2, 3 or 4 on exit, then G01EFF returns 0.0.

IFAIL = 1

On entry, TAIL $\neq$ 'L' or 'U'.

IFAIL = 2

On entry, $G < 0.0$.

IFAIL = 3

On entry, $A \leq 0.0$,
 or $B \leq 0.0$.

IFAIL = 4

The solution did not converge in 600 iterations. See S14BAF. The probability returned should be a reasonable approximation to the solution.

7 Accuracy

The result should have a relative accuracy of *machine precision*. There are rare occasions when the relative accuracy attained is somewhat less than *machine precision* but the error should not exceed more than 1 or 2 decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in S14BAF are given to this precision.

8 Further Comments

The time taken by G01EFF varies slightly with the input parameters G, A and B.

9 Example

This example reads in values for several gamma distributions, computes and prints the lower probabilities for each case, until the end of data is reached.

9.1 Program Text

```
*      G01EFF Example Program Text
*      Mark 14 Release. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION A, B, G, P
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION G01EFF
      EXTERNAL          G01EFF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01EFF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*)
+ 'Gamma deviate      Alpha      Beta      Lower tail prob.'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
20 READ (NIN,*,END=40) G, A, B
      IFAIL = 1
*
      P = G01EFF('Lower',G,A,B,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
        WRITE (NOUT,99998) G, A, B, P
        GO TO 20
      ELSE
        WRITE (NOUT,99999) ' ** G01EFF returned with IFAIL = ', IFAIL
      END IF
40 CONTINUE
*
99999 FORMAT (1X,A,I5)
99998 FORMAT (1X,F9.2,F13.2,F9.2,F14.4)
      END
```

9.2 Program Data

```
G01EFF Example Program Data
15.5   4.0   2.0           :G A B
 0.5   4.0   1.0           :G A B
10.0   1.0   2.0           :G A B
 5.0   2.0   2.0           :G A B
```

9.3 Program Results

G01EFF Example Program Results

Gamma deviate	Alpha	Beta	Lower tail prob.
15.50	4.00	2.00	0.9499
0.50	4.00	1.00	0.0018
10.00	1.00	2.00	0.9933
5.00	2.00	2.00	0.7127
