

# NAG Library Routine Document

## G01ECF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

G01ECF returns the lower or upper tail probability for the  $\chi^2$ -distribution with real degrees of freedom, via the routine name.

### 2 Specification

***double precision*** FUNCTION G01ECF(TAIL, X, DF, IFAIL)

INTEGER

IFAIL

***double precision***

X, DF

CHARACTER\*1

TAIL

### 3 Description

The lower tail probability for the  $\chi^2$ -distribution with  $\nu$  degrees of freedom,  $P(X \leq x : \nu)$  is defined by:

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_{0.0}^x X^{\nu/2-1} e^{-X/2} dX, \quad x \geq 0, \nu > 0.$$

To calculate  $P(X \leq x : \nu)$  a transformation of a gamma distribution is employed, i.e., a  $\chi^2$ -distribution with  $\nu$  degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter  $\nu/2$ .

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

### 5 Parameters

1: TAIL – CHARACTER\*1 *Input*

*On entry:* indicates whether the upper or lower tail probability is required.

TAIL = 'L'

The lower tail probability is returned, i.e.,  $P(X \leq x : \nu)$ .

TAIL = 'U'

The upper tail probability is returned, i.e.,  $P(X \geq x : \nu)$ .

*Constraint:* TAIL = 'L' or 'U'.

2: X – ***double precision*** *Input*

*On entry:*  $x$ , the value of the  $\chi^2$  variate with  $\nu$  degrees of freedom.

*Constraint:*  $X \geq 0.0$ .

3: DF – *double precision**Input*

*On entry:*  $\nu$ , the degrees of freedom of the  $\chi^2$ -distribution.

*Constraint:*  $DF > 0.0$ .

## 4: IFAIL – INTEGER

*Input/Output*

*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL  $\neq$  0 on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

**Note:** G01ECF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

If IFAIL = 1, 2 or 3 on exit, then G01ECF returns 0.0.

IFAIL = 1

On entry, TAIL  $\neq$  'L' or 'U'.

IFAIL = 2

On entry,  $X < 0.0$ .

IFAIL = 3

On entry,  $DF \leq 0.0$ .

IFAIL = 4

The solution has failed to converge while calculating the Gamma variate. The result returned should represent an approximation to the solution.

## 7 Accuracy

A relative accuracy of five significant figures is obtained in most cases.

## 8 Further Comments

For higher accuracy the transformation described in Section 3 may be used with a direct call to S14BAF.

## 9 Example

Values from various  $\chi^2$ -distributions are read, the lower-tail probabilities calculated, and all these values printed out, until the end of data is reached.

## 9.1 Program Text

```

*      G01ECF Example Program Text
*      Mark 14 Release. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      DOUBLE PRECISION DF, PROB, X
      INTEGER          IFAIL
*      .. External Functions ..
      DOUBLE PRECISION G01ECF
      EXTERNAL          G01ECF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01ECF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      X          DF      PROB'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
20    READ (NIN,*,END=40) X, DF
      IFAIL = 1

*
      PROB = G01ECF('Lower',X,DF,IFAIL)
*
      IF (IFAIL.GE.0) THEN
*
          IF (IFAIL.EQ.0) THEN
              WRITE (NOUT,99999) X, DF, PROB
          ELSE
              WRITE (NOUT,99999) X, DF, PROB, ' NOTE: IFAIL = ', IFAIL
          END IF
          GO TO 20
*
      ELSE
          WRITE (NOUT,99998) ' ** G01ECF returned with IFAIL = ', IFAIL
      END IF
      40 CONTINUE
*
99999 FORMAT (1X,F6.3,F8.3,F8.4,A,I1)
99998 FORMAT (1X,A,I5)
      END

```

## 9.2 Program Data

```

G01ECF Example Program Data
  8.26   20.0
  6.2    7.5
 55.76  45.0

```

## 9.3 Program Results

G01ECF Example Program Results

| X      | DF     | PROB   |
|--------|--------|--------|
| 8.260  | 20.000 | 0.0100 |
| 6.200  | 7.500  | 0.4279 |
| 55.760 | 45.000 | 0.8694 |

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