NAG Library Routine Document

G01EBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EBF returns the lower tail, upper tail or two tail probability for the Student's t-distribution with real degrees of freedom, via the routine name.

2 Specification

double precision `FUNCTION G01EBF(TAIL, T, DF, IFAIL)`

```
INTEGER                  IFAIL
double precision         T, DF
CHARACTER*1              TAIL
```

3 Description

The lower tail probability for the Student's t-distribution with ν degrees of freedom, $P(T \leq t : \nu)$ is defined by:

$$P(T \leq t : \nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)} \int_{-\infty}^{t} \left[1 + \frac{T^2}{\nu}\right]^{-(\nu+1)/2} dT, \qquad \nu \geq 1.$$

Computationally, there are two situations:

(i) when $\nu < 20$, a transformation of the beta distribution, $P_\beta(B \leq \beta : a, b)$ is used

$$P(T \leq t : \nu) = \tfrac{1}{2} P_\beta\left(B \leq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \qquad \text{when } t < 0.0$$

or

$$P(T \leq t : \nu) = \tfrac{1}{2} + \tfrac{1}{2} P_\beta\left(B \geq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \qquad \text{when } t > 0.0;$$

(ii) when $\nu \geq 20$, an asymptotic normalizing expansion of the Cornish–Fisher type is used to evaluate the probability, see Hill (1970).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's t-distribution *Comm. ACM* **13 (10)** 617–619

5 Parameters

1: TAIL – CHARACTER*1 *Input*

On entry: indicates which tail the returned probability should represent.

TAIL = 'U'
The upper tail probability is returned, i.e., $P(T \geq t : \nu)$.

TAIL = 'S'
The two tail (significance level) probability is returned, i.e., $P(T \geq |t| : \nu) + P(T \leq -|t| : \nu)$.

TAIL = 'C'
The two tail (confidence interval) probability is returned, i.e., $P(T \leq |t| : \nu) - P(T \leq -|t| : \nu)$.

TAIL = 'L'
The lower tail probability is returned, i.e., $P(T \leq t : \nu)$.

Constraint: TAIL = 'U', 'S', 'C' or 'L'.

2: T – ***double precision*** *Input*

On entry: t, the value of the Student's t variate.

3: DF – ***double precision*** *Input*

On entry: ν, the degrees of freedom of the Student's t-distribution.

Constraint: DF $\geq$ 1.0.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL $\neq$ 0, then G01EBF returns 0.0.

IFAIL = 1

On entry, TAIL $\neq$ 'U', 'S', 'C' or 'L'.

IFAIL = 2

On entry, DF $<$ 1.0.

7 Accuracy

The computed probability should be accurate to five significant places for reasonable probabilities but there will be some loss of accuracy for very low probabilities (less than 10^{-10}), see Hastings and Peacock (1975).

8 Further Comments

The probabilities could also be obtained by using the appropriate transformation to a beta distribution (see Abramowitz and Stegun (1972)) and using G01EEF. G01EBF allows you to set the required accuracy.

9 Example

This example reads values from, and degrees of freedom for Student's t-distributions along with the required tail. The probabilities are calculated and printed until the end of data is reached.

9.1 Program Text

```
*     G01EBF Example Program Text
*     Mark 14 Release. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      DOUBLE PRECISION DF, PROB, T
      INTEGER          IFAIL
      CHARACTER        TAIL
*     .. External Functions ..
      DOUBLE PRECISION G01EBF
      EXTERNAL         G01EBF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01EBF Example Program Results'
*     Skip heading in data file
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   T      DF     PROB  TAIL'
      WRITE (NOUT,*)
      READ (NIN,*)
   20 READ (NIN,*,END=40) T, DF, TAIL
      IFAIL = 1
*
      PROB = G01EBF(TAIL,T,DF,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) T, DF, PROB, TAIL
         GO TO 20
      ELSE
         WRITE (NOUT,99998) ' ** G01EBF returned with IFAIL = ', IFAIL
      END IF
*
   40 CONTINUE
*
99999 FORMAT (1X,F6.3,F8.3,F8.4,2X,A1)
99998 FORMAT (1X,A,I5)
      END
```

9.2 Program Data

```
G01EBF Example Program Data
0.85  20.0  'L'    :T DF TAIL
0.85  20.0  'S'    :T DF TAIL
0.85  20.0  'C'    :T DF TAIL
0.85  20.0  'U'    :T DF TAIL
```

9.3 Program Results

```
 G01EBF Example Program Results

    T      DF     PROB  TAIL

  0.850  20.000  0.7973  L
  0.850  20.000  0.4054  S
  0.850  20.000  0.5946  C
  0.850  20.000  0.2027  U
```