

NAG Library Routine Document

G01DBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01DBF calculates an approximation to the set of Normal Scores, i.e., the expected values of an ordered set of independent observations from a Normal distribution with mean 0.0 and standard deviation 1.0.

2 Specification

SUBROUTINE G01DBF(N, PP, IFAIL)

INTEGER N, IFAIL

double precision PP(N)

3 Description

G01DBF is an adaptation of the Applied Statistics Algorithm AS 177.3, see Royston (1982). If you are particularly concerned with the accuracy with which G01DBF computes the expected values of the order statistics (see Section 7), then G01DAF which is more accurate should be used instead at a cost of increased storage and computing time.

Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be the order statistics from a random sample of size n from the standard Normal distribution. Defining

$$P_{r,n} = \Phi(-E(x_{(r)}))$$

and

$$Q_{r,n} = \frac{r - \epsilon}{n + \gamma}, \quad r = 1, 2, \dots, n,$$

where $E(x_{(r)})$ is the expected value of $x_{(r)}$, the current routine approximates the Normal upper tail area corresponding to $E(x_{(r)})$ as,

$$\tilde{P}_{r,n} = Q_{r,n} + \frac{\delta_1}{n} Q_{r,n}^\lambda + \frac{\delta_2}{n} Q_{r,n}^{2\lambda} - C_{r,n}.$$

Estimates of ϵ , γ , δ_1 , δ_2 and λ are obtained for $r = 1, 2, 3$, and $r \geq 4$. A small correction $C_{r,n}$ to $\tilde{P}_{r,n}$ is necessary when $r \leq 7$ and $n \leq 20$.

The approximation to $E(X_{(r)})$ is thus given by

$$E(x_{(r)}) = -\Phi^{-1}(\tilde{P}_{r,n}), \quad r = 1, 2, \dots, n.$$

Values of the inverse Normal probability integral Φ^{-1} are obtained from G01FAF.

4 References

Royston J P (1982) Algorithm AS 177: Expected normal order statistics (exact and approximate) *Appl. Statist.* **31** 161–165

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the size of the sample.
Constraint: $N \geq 1$.
- 2: PP(N) – **double precision** array *Output*
On exit: the Normal scores. PP(i) contains the value $E(x_{(i)})$, for $i = 1, 2, \dots, n$.
- 3: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, –1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
For environments where it might be inappropriate to halt program execution when an error is detected, the value –1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value –1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or –1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $N < 1$.

7 Accuracy

For $n \leq 2000$, the maximum error is 0.0001, but G01DBF is usually accurate to 5 or 6 decimal places. For n up to 5000, comparison with the exact scores calculated by G01DAF shows that the maximum error is 0.001.

8 Further Comments

The time taken by G01DBF is proportional to n .

9 Example

A program to calculate the expected values of the order statistics for a sample of size 10.

9.1 Program Text

```

*      G01DBF Example Program Text
*      Mark 14 Revised. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          N
      PARAMETER        (N=10)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*      .. Local Scalars ..
      INTEGER          I, IFAIL
*      .. Local Arrays ..
      DOUBLE PRECISION PP(N)
*      .. External Subroutines ..
      EXTERNAL         G01DBF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01DBF Example Program Results'
      IFAIL = 1

*
      CALL G01DBF(N,PP,IFAIL)
*
      IF (IFAIL.NE.0) THEN
        WRITE (NOUT,*)
        WRITE (NOUT,99999) ' ** G01DBF returned with IFAIL = ', IFAIL
      ELSE
        WRITE (NOUT,*)
        WRITE (NOUT,99998) 'Sample size = ', N
        WRITE (NOUT,*) 'Normal scores'
        WRITE (NOUT,99997) (PP(I),I=1,N)
      END IF
*
99999  FORMAT (1X,A,I5)
99998  FORMAT (1X,A,I2)
99997  FORMAT (10X,5F12.4)
      END

```

9.2 Program Data

None.

9.3 Program Results

G01DBF Example Program Results

Sample size = 10

Normal scores

-1.5388	-1.0014	-0.6561	-0.3757	-0.1227
0.1227	0.3757	0.6561	1.0014	1.5388
