NAG Library Routine Document

F11JBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F11JBF solves a system of linear equations involving the incomplete Cholesky preconditioning matrix generated by F11JAF.

2 Specification

```
SUBROUTINE F11JBF(N, A, LA, IROW, ICOL, IPIV, ISTR, CHECK, Y, X, IFAIL)
INTEGER           N, LA, IROW(LA), ICOL(LA), IPIV(N), ISTR(N+1), IFAIL
double precision  A(LA), Y(N), X(N)
CHARACTER*1       CHECK
```

3 Description

F11JBF solves a system of linear equations

$$Mx = y$$

involving the preconditioning matrix $M = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, corresponding to an incomplete Cholesky decomposition of a sparse symmetric matrix stored in symmetric co-ordinate storage (SCS) format (see Section 2.1.2 in the F11 Chapter Introduction), as generated by F11JAF.

In the above decomposition L is a lower triangular sparse matrix with unit diagonal, D is a diagonal matrix and P is a permutation matrix. L and D are supplied to F11JBF through the matrix

$$C = L + D^{-1} - I$$

which is a lower triangular N by N sparse matrix, stored in SCS format, as returned by F11JAF. The permutation matrix P is returned from F11JAF via the array IPIV.

It is envisaged that a common use of F11JBF will be to carry out the preconditioning step required in the application of F11GEF to sparse symmetric linear systems. F11JBF is used for this purpose by the Black Box routine F11JCF.

F11JBF may also be used in combination with F11JAF to solve a sparse symmetric positive-definite system of linear equations directly (see Section 8.4 in F11JAF). This use of F11JBF is demonstrated in Section 9.

4 References

None.

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix M. This **must** be the same value as was supplied in the preceding call to F11JAF.

Constraint: N ≥ 1.

2: A(LA) – ***double precision*** array *Input*

On entry: the values returned in the array A by a previous call to F11DAF.

3: LA – INTEGER *Input*

On entry: the dimension of the arrays A, IROW and ICOL as declared in the (sub)program from which F11JBF is called. This **must** be the same value as was supplied in the preceding call to F11JAF.

4: IROW(LA) – INTEGER array *Input*
5: ICOL(LA) – INTEGER array *Input*
6: IPIV(N) – INTEGER array *Input*
7: ISTR(N + 1) – INTEGER array *Input*

On entry: the values returned in arrays IROW, ICOL, IPIV and ISTR by a previous call to F11JAF.

8: CHECK – CHARACTER*1 *Input*

On entry: specifies whether or not the input data should be checked.

CHECK = 'C'
Checks are carried out on the values of N, IROW, ICOL, IPIV and ISTR.

CHECK = 'N'
None of these checks are carried out.

See also Section 8.2.

Constraint: CHECK = 'C' or 'N'.

9: Y(N) – ***double precision*** array *Input*

On entry: the right-hand side vector y.

10: X(N) – ***double precision*** array *Output*

On exit: the solution vector x.

11: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CHECK ≠ 'C' or 'N'.

IFAIL = 2

On entry, N < 1.

IFAIL = 3

On entry, the SCS representation of the preconditioning matrix M is invalid. Further details are given in the error message. Check that the call to F11JBF has been preceded by a valid call to F11JAF and that the arrays A, IROW, ICOL, IPIV and ISTR have not been corrupted between the two calls.

7 Accuracy

The computed solution x is the exact solution of a perturbed system of equations $(M + \delta M)x = y$, where

$$|\delta M| \leq c(n)\epsilon P|L||D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}},$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

8.1 Timing

The time taken for a call to F11JBF is proportional to the value of NNZC returned from F11JAF.

8.2 Use of CHECK

It is expected that a common use of F11JBF will be to carry out the preconditioning step required in the application of F11GEF to sparse symmetric linear systems. In this situation F11JBF is likely to be called many times with the same matrix M. In the interests of both reliability and efficiency, you are recommended to set CHECK to 'C' for the first of such calls, and to 'N' for all subsequent calls.

9 Example

This example reads in a symmetric positive-definite sparse matrix A and a vector y. It then calls F11JAF, with LFILL $= -1$ and DTOL $= 0.0$, to compute the **complete** Cholesky decomposition of A:

$$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}.$$

Finally it calls F11JBF to solve the system

$$PLDL^{\mathrm{T}}P^{\mathrm{T}}x = y.$$

9.1 Program Text

```
*     F11JBF Example Program Text
*     Mark 17 Release. NAG Copyright 1995.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LA, LIWORK
      PARAMETER        (NMAX=1000,LA=10000,LIWORK=2*LA+7*NMAX+1)
*     .. Local Scalars ..
      DOUBLE PRECISION DSCALE, DTOL
      INTEGER          I, IFAIL, LFILL, N, NNZ, NNZC, NPIVM
      CHARACTER        CHECK, MIC, PSTRAT
*     .. Local Arrays ..
      DOUBLE PRECISION A(LA), X(NMAX), Y(NMAX)
      INTEGER          ICOL(LA), IPIV(NMAX), IROW(LA), ISTR(NMAX+1),
     +                 IWORK(LIWORK)
*     .. External Subroutines ..
      EXTERNAL         F11JAF, F11JBF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F11JBF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
*
*     Read order of matrix and number of non-zero entries
*
      READ (NIN,*) N
```

```
      IF (N.LE.NMAX) THEN
         READ (NIN,*) NNZ
*
*        Read the matrix A
*
         DO 20 I = 1, NNZ
            READ (NIN,*) A(I), IROW(I), ICOL(I)
   20    CONTINUE
*
*        Read the vector y
*
         READ (NIN,*) (Y(I),I=1,N)
*
*        Calculate Cholesky factorization
*
         LFILL = -1
         DTOL = 0.0D0
         MIC = 'N'
         DSCALE = 0.0D0
         PSTRAT = 'M'
         IFAIL = 1
*
         CALL F11JAF(N,NNZ,A,LA,IROW,ICOL,LFILL,DTOL,MIC,DSCALE,PSTRAT,
     +               IPIV,ISTR,NNZC,NPIVM,IWORK,LIWORK,IFAIL)
         IF (IFAIL.EQ.0) THEN
*
*           Check the output value of NPIVM
*
            IF (NPIVM.NE.0) THEN
*
               WRITE (NOUT,*) 'Factorization is not complete'
*
            ELSE
*                           T T
*              Solve P L D L P x = y
*
               CHECK = 'C'
*
               IFAIL = 1
               CALL F11JBF(N,A,LA,IROW,ICOL,IPIV,ISTR,CHECK,Y,X,IFAIL)
               IF (IFAIL.EQ.0) THEN
*
*                 Output results
*
                  WRITE (NOUT,*) ' Solution of linear system'
                  DO 40 I = 1, N
                     WRITE (NOUT,99999) X(I)
   40             CONTINUE
*
               ELSE
                  WRITE (NOUT,99998) IFAIL
               END IF
            END IF
         ELSE
            WRITE (NOUT,99997) IFAIL
         END IF
      END IF
*
99999 FORMAT (1X,E16.4)
99998 FORMAT (1X,/1X,' ** F11JBF returned with IFAIL = ',I5)
99997 FORMAT (1X,/1X,' ** F11JAF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

```
F11JBF Example Program Data
  9                  N
 23                  NNZ
  4.   1    1
 -1.   2    1
```

```
 6.   2    2
 1.   3    2
 2.   3    3
 3.   4    4
 2.   5    1
 4.   5    5
 1.   6    3
 2.   6    4
 6.   6    6
-4.   7    2
 1.   7    5
-1.   7    6
 6.   7    7
-1.   8    4
-1.   8    6
 3.   8    8
 1.   9    1
 1.   9    5
-1.   9    6
 1.   9    8
 4.   9    9          A(I), IROW(I), ICOL(I), I=1,...,NNZ
 4.10 -2.94  1.41
 2.53  4.35  1.29
 5.01  0.52  4.57     Y(I), I=1,...,N
```

9.3 Program Results

```
F11JBF Example Program Results
 Solution of linear system
      0.7000E+00
      0.1600E+00
      0.5200E+00
      0.7700E+00
      0.2800E+00
      0.2100E+00
      0.9300E+00
      0.2000E+00
      0.9000E+00
```