

NAG Library Routine Document

F08UTF (ZPBSTF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08UTF (ZPBSTF) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix.

2 Specification

```
SUBROUTINE F08UTF(UPLO, N, KB, BB, LDBB, INFO)
INTEGER          N, KB, LDBB, INFO
complex*16      BB(LDBB,*)
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name *zpbstf*.

3 Description

F08UTF (ZPBSTF) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix B . It is designed to be used in conjunction with F08USF (ZHBGST).

The factorization has the form $B = S^H S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n + k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & & & s_{66} & & & \\ & & & & s_{64} & s_{65} & s_{66} & & \\ & & & & & s_{75} & s_{76} & s_{77} & \\ & & & & & & s_{86} & s_{87} & s_{88} \\ & & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

1: UPLO – CHARACTER*1

Input

On entry: indicates whether the upper or lower triangular part of B is stored.

UPLO = 'U'

The upper triangular part of B is stored.

UPLO = 'L'

The lower triangular part of B is stored.

Constraint: UPLO = 'U' or 'L'.

- 2: N – INTEGER Input
On entry: n , the order of the matrix B .
Constraint: $N \geq 0$.
- 3: KB – INTEGER Input
On entry: if UPLO = 'U', the number of superdiagonals, k_b , of the matrix B .
 If UPLO = 'L', the number of subdiagonals, k_b , of the matrix B .
Constraint: $KB \geq 0$.
- 4: BB(LDBB,*) – **complex*16** array Input/Output
Note: the second dimension of the array BB must be at least $\max(1, N)$.
On entry: the n by n Hermitian positive-definite band matrix B .
 The matrix is stored in rows 1 to $k_b + 1$, more precisely,
 if UPLO = 'U', the elements of the upper triangle of B within the band must be stored with
 element B_{ij} in $BB(k_b + 1 + i - j, j)$ for $\max(1, j - k_b) \leq i \leq j$;
 if UPLO = 'L', the elements of the lower triangle of B within the band must be stored with
 element B_{ij} in $BB(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_b)$.
On exit: B is overwritten by the elements of its split Cholesky factor S .
- 5: LDBB – INTEGER Input
On entry: the first dimension of the array BB as declared in the (sub)program from which F08UTF (ZPBSTF) is called.
Constraint: $LDBB \geq KB + 1$.
- 6: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the factorization could not be completed, because the updated element $b(i, i)$ would be the square root of a negative number. Hence B is not positive-definite. This may indicate an error in forming the matrix B .

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $(B + E)$, where

$$|E| \leq c(k+1)\epsilon |S^H| |S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the **machine precision**. It follows that $|e_{ij}| \leq c(k+1)\epsilon \sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $4n(k+1)^2$, assuming $n \gg k$.

A call to F08UTF (ZPBSTF) may be followed by a call to F08USF (ZHBGST) to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The real analogue of this routine is F08UFF (DPBSTF).

9 Example

See Section 9 in F08USF (ZHBGST).
