

NAG Library Routine Document

F08QHF (DTRSYL)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08QHF (DTRSYL) solves the real quasi-triangular Sylvester matrix equation.

2 Specification

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SUBROUTINE F08QHF(TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC,
1          SCALE, INFO)
    INTEGER          ISGN, M, N, LDA, LDB, LDC, INFO
    double precision A(LDA,*), B(LDB,*), C(LDC,*), SCALE
    CHARACTER*1      TRANA, TRANB

```

The routine may be called by its LAPACK name ***dtrsyl***.

3 Description

F08QHF (DTRSYL) solves the real Sylvester matrix equation

$$\text{op}(A)X \pm X\text{op}(B) = \alpha C,$$

where $\text{op}(A) = A$ or A^T , and the matrices A and B are upper quasi-triangular matrices in canonical Schur form (as returned by F08PEF (DHSEQR)); α is a scale factor (≤ 1) determined by the routine to avoid overflow in X ; A is m by m and B is n by n while the right-hand side matrix C and the solution matrix X are both m by n . The matrix X is obtained by a straightforward process of back-substitution (see Golub and Van Loan (1996)).

Note that the equation has a unique solution if and only if $\alpha_i \pm \beta_j \neq 0$, where $\{\alpha_i\}$ and $\{\beta_j\}$ are the eigenvalues of A and B respectively and the sign (+ or $-$) is the same as that used in the equation to be solved.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1992) Perturbation theory and backward error for $AX - XB = C$ *Numerical Analysis Report* University of Manchester

5 Parameters

1: TRANA – CHARACTER*1 *Input*

On entry: specifies the option $\text{op}(A)$.

TRANA = 'N'

$\text{op}(A) = A$.

TRANA = 'T' or 'C'

$\text{op}(A) = A^T$.

Constraint: TRANA = 'N', 'T' or 'C'.

- 2: TRANB – CHARACTER*1 *Input*
On entry: specifies the option $\text{op}(B)$.
TRANB = 'N'
 $\text{op}(B) = B$.
TRANB = 'T' or 'C'
 $\text{op}(B) = B^T$.
Constraint: TRANB = 'N', 'T' or 'C'.
- 3: ISGN – INTEGER *Input*
On entry: indicates the form of the Sylvester equation.
ISGN = +1
The equation is of the form $\text{op}(A)X + X \text{op}(B) = \alpha C$.
ISGN = -1
The equation is of the form $\text{op}(A)X - X \text{op}(B) = \alpha C$.
Constraint: ISGN = +1 or -1.
- 4: M – INTEGER *Input*
On entry: m , the order of the matrix A , and the number of rows in the matrices X and C .
Constraint: $M \geq 0$.
- 5: N – INTEGER *Input*
On entry: n , the order of the matrix B , and the number of columns in the matrices X and C .
Constraint: $N \geq 0$.
- 6: A(LDA,*) – **double precision** array *Input*
Note: the second dimension of the array A must be at least $\max(1, M)$.
On entry: the m by m upper quasi-triangular matrix A in canonical Schur form, as returned by F08PEF (DHSEQR).
- 7: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F08QHF (DTRSYL) is called.
Constraint: $LDA \geq \max(1, M)$.
- 8: B(LDB,*) – **double precision** array *Input*
Note: the second dimension of the array B must be at least $\max(1, N)$.
On entry: the n by n upper quasi-triangular matrix B in canonical Schur form, as returned by F08PEF (DHSEQR).
- 9: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F08QHF (DTRSYL) is called.
Constraint: $LDB \geq \max(1, N)$.
- 10: C(LDC,*) – **double precision** array *Input/Output*
Note: the second dimension of the array C must be at least $\max(1, N)$.
On entry: the m by n right-hand side matrix C .

On exit: C is overwritten by the solution matrix X .

11: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08QHF (DTRSYL) is called.

Constraint: $LDC \geq \max(1, M)$.

12: SCALE – **double precision** *Output*

On exit: the value of the scale factor α .

13: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO = 1

A and B have common or close eigenvalues, perturbed values of which were used to solve the equation.

7 Accuracy

Consider the equation $AX - XB = C$. (To apply the remarks to the equation $AX + XB = C$, simply replace B by $-B$.)

Let $\tilde{X}$ be the computed solution and R the residual matrix:

$$R = C - (A\tilde{X} - \tilde{X}B).$$

Then the residual is always small:

$$\|R\|_F = O(\epsilon)(\|A\|_F + \|B\|_F)\|\tilde{X}\|_F.$$

However, $\tilde{X}$ is **not** necessarily the exact solution of a slightly perturbed equation; in other words, the solution is not backwards stable.

For the forward error, the following bound holds:

$$\|\tilde{X} - X\|_F \leq \frac{\|R\|_F}{\text{sep}(A, B)}$$

but this may be a considerable overestimate. See Golub and Van Loan (1996) for a definition of $\text{sep}(A, B)$, and Higham (1992) for further details.

These remarks also apply to the solution of a general Sylvester equation, as described in Section 8.

8 Further Comments

The total number of floating-point operations is approximately $mn(m + n)$.

To solve the **general** real Sylvester equation

$$AX \pm XB = C$$

where A and B are general nonsymmetric matrices, A and B must first be reduced to Schur form (by calling F08PAF (DGEES), for example):

$$A = Q_1 \tilde{A} Q_1^T$$

and

$$B = Q_2 \tilde{B} Q_2^T$$

where $\tilde{A}$ and $\tilde{B}$ are upper quasi-triangular and Q_1 and Q_2 are orthogonal. The original equation may then be transformed to:

$$\tilde{A} \tilde{X} \pm \tilde{X} \tilde{B} = \tilde{C}$$

where $\tilde{X} = Q_1^T X Q_2$ and $\tilde{C} = Q_1^T C Q_2$. $\tilde{C}$ may be computed by matrix multiplication; F08QHF (DTRSYL) may be used to solve the transformed equation; and the solution to the original equation can be obtained as $X = Q_1 \tilde{X} Q_2^T$.

The complex analogue of this routine is F08QVF (ZTRSYL).

9 Example

This example solves the Sylvester equation $AX + XB = C$, where

$$A = \begin{pmatrix} 0.10 & 0.50 & 0.68 & -0.21 \\ -0.50 & 0.10 & -0.24 & 0.67 \\ 0.00 & 0.00 & 0.19 & -0.35 \\ 0.00 & 0.00 & 0.00 & -0.72 \end{pmatrix}, \quad B = \begin{pmatrix} -0.99 & -0.17 & 0.39 & 0.58 \\ 0.00 & 0.48 & -0.84 & -0.15 \\ 0.00 & 0.00 & 0.75 & 0.25 \\ 0.00 & 0.00 & -0.25 & 0.75 \end{pmatrix}$$

and

$$C = \begin{pmatrix} 0.63 & -0.56 & 0.08 & -0.23 \\ -0.45 & -0.31 & 0.27 & 1.21 \\ 0.20 & -0.35 & 0.41 & 0.84 \\ 0.49 & -0.05 & -0.52 & -0.08 \end{pmatrix}.$$

9.1 Program Text

```
*      F08QHF Example Program Text
*      Mark 16 Release. NAG Copyright 1992.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, LDA, LDB, LDC
      PARAMETER        (MMAX=8,NMAX=8,LDA=MMAX,LDB=NMAX,LDC=MMAX)
*      .. Local Scalars ..
      DOUBLE PRECISION SCALE
      INTEGER          I, IFAIL, INFO, J, M, N
*      .. Local Arrays ..
      DOUBLE PRECISION A(LDA,MMAX), B(LDB,NMAX), C(LDC,NMAX)
*      .. External Subroutines ..
      EXTERNAL         DTRSYL, X04CAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F08QHF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N
      IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
*
*         Read A, B and C from data file
*
*         READ (NIN,*) ((A(I,J),J=1,M),I=1,M)
*         READ (NIN,*) ((B(I,J),J=1,N),I=1,N)
*         READ (NIN,*) ((C(I,J),J=1,N),I=1,M)
*
*         Solve the Sylvester equation A*X + X*B = C for X
*
*         CALL DTRSYL('No transpose','No transpose',1,M,N,A,LDA,B,LDB,C,
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      +          LDC,SCALE,INFO)
      IF (INFO.EQ.1) THEN
        WRITE (NOUT,99999)
        WRITE (NOUT,*)
      END IF
*
*      Print the solution matrix X
*
      IFAIL = 0
      CALL X04CAF('General',' ',M,N,C,LDC,'Solution matrix X',IFAIL)
*
      WRITE (NOUT,*)
      WRITE (NOUT,99998) 'SCALE = ', SCALE
    ELSE
      WRITE (NOUT,*) 'MMAX and/or NMAX too small'
    END IF
*
99999 FORMAT (' A and B have common or very close eigenvalues.',/' Pe',
+          'rturbed values were used to solve the equations')
99998 FORMAT (1X,A,1P,E10.2)
      END

```

9.2 Program Data

F08QHF Example Program Data

4	4				:Values of M and N
0.10	0.50	0.68	-0.21		
-0.50	0.10	-0.24	0.67		
0.00	0.00	0.19	-0.35		
0.00	0.00	0.00	-0.72		:End of matrix A
-0.99	-0.17	0.39	0.58		
0.00	0.48	-0.84	-0.15		
0.00	0.00	0.75	0.25		
0.00	0.00	-0.25	0.75		:End of matrix B
0.63	-0.56	0.08	-0.23		
-0.45	-0.31	0.27	1.21		
0.20	-0.35	0.41	0.84		
0.49	-0.05	-0.52	-0.08		:End of matrix C

9.3 Program Results

F08QHF Example Program Results

Solution matrix X

	1	2	3	4
1	-0.4209	0.1764	0.2438	-0.9577
2	0.5600	-0.8337	-0.7221	0.5386
3	-0.1246	-0.3392	0.6221	0.8691
4	-0.2865	0.4113	0.5535	0.3174

SCALE = 1.00E+00
