NAG Library Routine Document

F08LEF (DGBBRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08LEF (DGBBRD) reduces a real m by n band matrix to upper bidiagonal form.

2 Specification

```
      SUBROUTINE F08LEF(VECT, M, N, NCC, KL, KU, AB, LDAB, D, E, Q, LDQ, PT,
     1                  LDPT, C, LDC, WORK, INFO)
      INTEGER           M, N, NCC, KL, KU, LDAB, LDQ, LDPT, LDC, INFO
      double precision  AB(LDAB,*), D(min(M,N)), E(min(M,N)-1), Q(LDQ,*),
     1                  PT(LDPT,*), C(LDC,*), WORK(2*max(M,N))
      CHARACTER*1       VECT
```

The routine may be called by its LAPACK name ***dgbbrd***.

3 Description

F08LEF (DGBBRD) reduces a real m by n band matrix to upper bidiagonal form B by an orthogonal transformation: $A = QBP^{\mathrm{T}}$. The orthogonal matrices Q and P^{T}, of order m and n respectively, are determined as a product of Givens rotation matrices, and may be formed explicitly by the routine if required. A matrix C may also be updated to give $\tilde{C} = Q^{\mathrm{T}}C$.

The routine uses a vectorizable form of the reduction.

4 References

None.

5 Parameters

1: VECT – CHARACTER*1 *Input*

On entry: indicates whether the matrices Q and/or P^{T} are generated.

VECT = 'N'
: Neither Q nor P^{T} is generated.

VECT = 'Q'
: Q is generated.

VECT = 'P'
: P^{T} is generated.

VECT = 'B'
: Both Q and P^{T} are generated.

Constraint: VECT = 'N', 'Q', 'P' or 'B'.

2: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: M ≥ 0.

3: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

4: NCC – INTEGER *Input*

On entry: n_C, the number of columns of the matrix C.

Constraint: $\mathrm{NCC} \geq 0$.

5: KL – INTEGER *Input*

On entry: the number of subdiagonals, k_l, within the band of A.

Constraint: $\mathrm{KL} \geq 0$.

6: KU – INTEGER *Input*

On entry: the number of superdiagonals, k_u, within the band of A.

Constraint: $\mathrm{KU} \geq 0$.

7: AB(LDAB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array AB must be at least $\max(1,\mathrm{N})$.

On entry: the original m by n band matrix A.

The matrix is stored in rows 1 to $k_l + k_u + 1$, more precisely, the element A_{ij} must be stored in

$$\mathrm{AB}(k_u + 1 + i - j, j) \qquad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$

On exit: AB is overwritten by values generated during the reduction.

8: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraint: $\mathrm{LDAB} \geq \mathrm{KL} + \mathrm{KU} + 1$.

9: D($\min(\mathrm{M},\mathrm{N})$) – ***double precision*** array *Output*

On exit: the diagonal elements of the bidiagonal matrix B.

10: E($\min(\mathrm{M},\mathrm{N}) - 1$) – ***double precision*** array *Output*

On exit: the superdiagonal elements of the bidiagonal matrix B.

11: Q(LDQ,$*$) – ***double precision*** array *Output*

Note: the second dimension of the array Q must be at least $\max(1,\mathrm{M})$ if VECT = 'Q' or 'B', and at least 1 otherwise.

On exit: if VECT = 'Q' or 'B', contains the m by m orthogonal matrix Q.

If VECT = 'N' or 'P', Q is not referenced.

12: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if VECT = 'Q' or 'B', $\mathrm{LDQ} \geq \max(1,\mathrm{M})$;
otherwise $\mathrm{LDQ} \geq 1$.

13: PT(LDPT,$*$) – ***double precision*** array *Output*

Note: the second dimension of the array PT must be at least $\max(1, \mathrm{N})$ if VECT = 'P' or 'B', and at least 1 otherwise.

On exit: the n by n orthogonal matrix P^{T}, if VECT = 'P' or 'B'. If VECT = 'N' or 'Q', PT is not referenced.

14: LDPT – INTEGER *Input*

On entry: the first dimension of the array PT as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if VECT = 'P' or 'B', LDPT $\geq \max(1, \mathrm{N})$;
otherwise LDPT ≥ 1.

15: C(LDC,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1, \mathrm{NCC})$.

On entry: an m by n_C matrix C.

On exit: C is overwritten by $Q^{\mathrm{T}}C$. If NCC $= 0$, C is not referenced.

16: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08LEF (DGBBRD) is called.

Constraints:

if NCC > 0, LDC $\geq \max(1, \mathrm{M})$;
if NCC $= 0$, LDC ≥ 1.

17: WORK($2 \times \max(\mathrm{M}, \mathrm{N})$) – ***double precision*** array *Workspace*

18: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed bidiagonal form B satisfies $QBP^{\mathrm{T}} = A + E$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of B themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the singular values and vectors.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix F such that

$$\|F\|_2 = O(\epsilon).$$

A similar statement holds for the computed matrix P^{T}.

8 Further Comments

The total number of real floating-point operations is approximately the sum of:

$6n^2k$, if VECT = 'N' and NCC = 0, and

$3n^2n_C(k-1)/k$, if C is updated, and

$3n^3(k-1)/k$, if either Q or P^T is generated (double this if both),

where $k = k_l + k_u$, assuming $n \gg k$. For this section we assumed that $m = n$.

The complex analogue of this routine is F08LSF (ZGBBRD).

9 Example

This example reduces the matrix A to upper bidiagonal form, where

$$A = \begin{pmatrix} -0.57 & -1.28 & 0.00 & 0.00 \\ -1.93 & 1.08 & -0.31 & 0.00 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ 0.00 & 0.64 & -0.66 & 0.08 \\ 0.00 & 0.00 & 0.15 & -2.13 \\ -0.00 & 0.00 & 0.00 & 0.50 \end{pmatrix}.$$

9.1 Program Text

```
*     F08LEF Example Program Text
*     Mark 19 Release. NAG Copyright 1999.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, NCCMAX, KLMAX, KUMAX, LDAB, LDQ,
     +                 LDPT, LDC
      PARAMETER        (MMAX=8,NMAX=8,NCCMAX=8,KLMAX=8,KUMAX=8,
     +                 LDAB=KLMAX+KUMAX+1,LDQ=MMAX,LDPT=NMAX,LDC=MMAX)
      CHARACTER        VECT
      PARAMETER        (VECT='N')
*     .. Local Scalars ..
      INTEGER          I, INFO, J, KL, KU, M, N, NCC
*     .. Local Arrays ..
      DOUBLE PRECISION AB(LDAB,NMAX), C(MMAX,NCCMAX), D(NMAX),
     +                 E(NMAX-1), PT(LDPT,NMAX), Q(LDQ,MMAX),
     +                 WORK(2*MMAX+2*NMAX)
*     .. External Subroutines ..
      EXTERNAL         DGBBRD
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F08LEF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N, KL, KU, NCC
      IF (M.LE.MMAX .AND. N.LE.NMAX .AND. KL.LE.KLMAX .AND. KU.LE.
     +    KUMAX .AND. NCC.LE.NCCMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) ((AB(KU+1+I-J,J),J=MAX(I-KL,1),MIN(I+KU,N)),I=1,M)
*
*        Reduce A to upper bidiagonal form
*
         CALL DGBBRD(VECT,M,N,NCC,KL,KU,AB,LDAB,D,E,Q,LDQ,PT,LDPT,C,LDC,
     +               WORK,INFO)
*
*        Print bidiagonal form
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'Diagonal'
```

```
         WRITE (NOUT,99999) (D(I),I=1,MIN(M,N))
         WRITE (NOUT,*) 'Super-diagonal'
         WRITE (NOUT,99999) (E(I),I=1,MIN(M,N)-1)
      END IF
*
99999 FORMAT (1X,8F9.4)
      END
```

9.2 Program Data

```
F08LEF Example Program Data
  6  4  2  1  0               :Values of M, N, KL, KU and NCC
 -0.57  -1.28
 -1.93   1.08  -0.31
  2.30   0.24   0.40  -0.35
         0.64  -0.66   0.08
                0.15  -2.13
                       0.50   :End of matrix A
```

9.3 Program Results

```
 F08LEF Example Program Results

 Diagonal
    3.0561   1.5259   0.9690   1.5685
 Super-diagonal
    0.6206  -1.2353  -1.1240
```