

NAG Library Routine Document

F08KUF (ZUNMBR)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08KUF (ZUNMBR) multiplies an arbitrary complex m by n matrix C by one of the complex unitary matrices Q or P which were determined by F08KSF (ZGEBRD) when reducing a complex matrix to bidiagonal form.

2 Specification

```
SUBROUTINE F08KUF(VECT, SIDE, TRANS, M, N, K, A, LDA, TAU, C, LDC, WORK,
1                LWORK, INFO)
    INTEGER          M, N, K, LDA, LDC, LWORK, INFO
    complex*16      A(LDA,*), TAU(*), C(LDC,*), WORK(max(1,LWORK))
    CHARACTER*1      VECT, SIDE, TRANS
```

The routine may be called by its LAPACK name ***zunmbr***.

3 Description

F08KUF (ZUNMBR) is intended to be used after a call to F08KSF (ZGEBRD), which reduces a complex rectangular matrix A to real bidiagonal form B by a unitary transformation: $A = QB P^H$. F08KSF (ZGEBRD) represents the matrices Q and P^H as products of elementary reflectors.

This routine may be used to form one of the matrix products

$$QC, Q^H C, CQ, CQ^H, PC, P^H C, CP \text{ or } CP^H,$$

overwriting the result on C (which may be any complex rectangular matrix).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

Note: in the descriptions below, r denotes the order of Q or P^H : if SIDE = 'L', $r = M$ and if SIDE = 'R', $r = N$.

1: VECT – CHARACTER*1 *Input*

On entry: indicates whether Q or Q^H or P or P^H is to be applied to C .

VECT = 'Q'

Q or Q^H is applied to C .

VECT = 'P'

P or P^H is applied to C .

Constraint: VECT = 'Q' or 'P'.

- 2: SIDE – CHARACTER*1 *Input*
On entry: indicates how Q or Q^H or P or P^H is to be applied to C .
 SIDE = 'L'
 Q or Q^H or P or P^H is applied to C from the left.
 SIDE = 'R'
 Q or Q^H or P or P^H is applied to C from the right.
Constraint: SIDE = 'L' or 'R'.
- 3: TRANS – CHARACTER*1 *Input*
On entry: indicates whether Q or P or Q^H or P^H is to be applied to C .
 TRANS = 'N'
 Q or P is applied to C .
 TRANS = 'C'
 Q^H or P^H is applied to C .
Constraint: TRANS = 'N' or 'C'.
- 4: M – INTEGER *Input*
On entry: m , the number of rows of the matrix C .
Constraint: $M \geq 0$.
- 5: N – INTEGER *Input*
On entry: n , the number of columns of the matrix C .
Constraint: $N \geq 0$.
- 6: K – INTEGER *Input*
On entry: if VECT = 'Q', the number of columns in the original matrix A .
 If VECT = 'P', the number of rows in the original matrix A .
Constraint: $K \geq 0$.
- 7: A(LDA,*) – **complex*16** array *Input*
Note: the second dimension of the array A must be at least $\max(1, \min(r, K))$ if VECT = 'Q' and at least $\max(1, r)$ if VECT = 'P'.
On entry: details of the vectors which define the elementary reflectors, as returned by F08KSF (ZGEBRD).
- 8: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F08KUF (ZUNMBR) is called.
Constraints:
 if VECT = 'Q', $LDA \geq \max(1, r)$;
 if VECT = 'P', $LDA \geq \max(1, \min(r, K))$.
- 9: TAU(*) – **complex*16** array *Input*
Note: the dimension of the array TAU must be at least $\max(1, \min(r, K))$.
On entry: further details of the elementary reflectors, as returned by F08KSF (ZGEBRD) in its parameter TAUQ if VECT = 'Q', or in its parameter TAUP if VECT = 'P'.

- 10: C(LDC,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array C must be at least $\max(1, N)$.
On entry: the matrix C.
On exit: C is overwritten by QC or $Q^H C$ or CQ or $C^H Q$ or PC or $P^H C$ or CP or $C^H P$ as specified by VECT, SIDE and TRANS.
- 11: LDC – INTEGER *Input*
On entry: the first dimension of the array C as declared in the (sub)program from which F08KUF (ZUNMBR) is called.
Constraint: $LDC \geq \max(1, M)$.
- 12: WORK($\max(1, LWORK)$) – **complex*16** array *Workspace*
On exit: if INFO = 0, the real part of WORK(1) contains the minimum value of LWORK required for optimal performance.
- 13: LWORK – INTEGER *Input*
On entry: the dimension of the array WORK as declared in the (sub)program from which F08KUF (ZUNMBR) is called.
 If LWORK = -1, a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.
Suggested value: for optimal performance, $LWORK \geq N \times nb$ if SIDE = 'L' and at least $M \times nb$ if SIDE = 'R', where *nb* is the optimal **block size**.
Constraints:
 if SIDE = 'L', $LWORK \geq \max(1, N)$ or LWORK = -1;
 if SIDE = 'R', $LWORK \geq \max(1, M)$ or LWORK = -1.
- 14: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = -*i*, argument *i* had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed result differs from the exact result by a matrix *E* such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where ϵ is the **machine precision**.

8 Further Comments

The total number of real floating-point operations is approximately

if SIDE = 'L' and $m \geq k$, $8nk(2m - k)$;

if SIDE = 'R' and $n \geq k$, $8mk(2n - k)$;

if SIDE = 'L' and $m < k$, $8m^2n$;

if SIDE = 'R' and $n < k$, $8mn^2$,

where k is the value of the parameter K.

The real analogue of this routine is F08KGF (DORMBR).

9 Example

For this routine two examples are presented. Both illustrate how the reduction to bidiagonal form of a matrix A may be preceded by a QR or LQ factorization of A .

In the first example, $m > n$, and

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ -0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}.$$

The routine first performs a QR factorization of A as $A = Q_a R$ and then reduces the factor R to bidiagonal form B : $R = Q_b B P^H$. Finally it forms Q_a and calls F08KUF (ZUNMBR) to form $Q = Q_a Q_b$.

In the second example, $m < n$, and

$$A = \begin{pmatrix} 0.28 - 0.36i & 0.50 - 0.86i & -0.77 - 0.48i & 1.58 + 0.66i \\ -0.50 - 1.10i & -1.21 + 0.76i & -0.32 - 0.24i & -0.27 - 1.15i \\ 0.36 - 0.51i & -0.07 + 1.33i & -0.75 + 0.47i & -0.08 + 1.01i \end{pmatrix}.$$

The routine first performs an LQ factorization of A as $A = L P_a^H$ and then reduces the factor L to bidiagonal form B : $L = Q B P_b^H$. Finally it forms P_b^H and calls F08KUF (ZUNMBR) to form $P^H = P_b^H P_a^H$.

9.1 Program Text

```
*      F08KUF Example Program Text
*      Mark 16 Release. NAG Copyright 1992.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, LDA, LDPH, LDU, LWORK
      PARAMETER        (MMAX=8,NMAX=8,LDA=MMAX,LDPH=NMAX,LDU=MMAX,
+      LWORK=64*(MMAX+NMAX))
      COMPLEX *16      ZERO
      PARAMETER        (ZERO=(0.0D0,0.0D0))
*      .. Local Scalars ..
      INTEGER          I, IC, IFAIL, INFO, J, M, N
*      .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), PH(LDPH,NMAX), TAU(NMAX),
+      TAUP(NMAX), TAUQ(NMAX), U(LDU,NMAX), WORK(LWORK)
      DOUBLE PRECISION D(NMAX), E(NMAX-1)
      CHARACTER        CLABS(1), RLABS(1)
*      .. External Subroutines ..
      EXTERNAL         F06TFF, F06THF, X04DBF, ZGEBRD, ZGELQF, ZGEQRF,
+      ZUNGLQ, ZUNGQR, ZUNMBR
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F08KUF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      DO 20 IC = 1, 2
         READ (NIN,*) M, N
         IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
*
*           Read A from data file
*

```

```

      READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
*
      IF (M.GE.N) THEN
*
*         Compute the QR factorization of A
*
*         CALL ZGEQRF(M,N,A,LDA,TAU,WORK,LWORK,INFO)
*
*         Copy A to U
*
*         CALL F06TFF('Lower',M,N,A,LDA,U,LDU)
*
*         Form Q explicitly, storing the result in U
*
*         CALL ZUNGQR(M,M,N,U,LDU,TAU,WORK,LWORK,INFO)
*
*         Copy R to PH (used as workspace)
*
*         CALL F06TFF('Upper',N,N,A,LDA,PH,LDPH)
*
*         Set the strictly lower triangular part of R to zero
*
*         CALL F06THF('Lower',N-1,N-1,ZERO,ZERO,PH(2,1),LDPH)
*
*         Bidiagonalize R
*
*         CALL ZGEBRD(N,N,PH,LDPH,D,E,TAUQ,TAUP,WORK,LWORK,INFO)
*
*         Update Q, storing the result in U
*
*         CALL ZUNMBR('Q','Right','No transpose',M,N,N,PH,LDPH,
+           TAUQ,U,LDU,WORK,LWORK,INFO)
*
*         Print bidiagonal form and matrix Q
*
*         WRITE (NOUT,*)
*         WRITE (NOUT,*) 'Example 1: bidiagonal matrix B'
*         WRITE (NOUT,*) 'Diagonal'
*         WRITE (NOUT,99999) (D(I),I=1,N)
*         WRITE (NOUT,*) 'Super-diagonal'
*         WRITE (NOUT,99999) (E(I),I=1,N-1)
*         WRITE (NOUT,*)
*         IFAIL = 0
*
*         CALL X04DBF('General',' ',M,N,U,LDU,'Bracketed','F7.4',
+           'Example 1: matrix Q','Integer',RLABS,
+           'Integer',CLABS,80,0,IFAIL)
*
      ELSE
*
*         Compute the LQ factorization of A
*
*         CALL ZGELQF(M,N,A,LDA,TAU,WORK,LWORK,INFO)
*
*         Copy A to PH
*
*         CALL F06TFF('Upper',M,N,A,LDA,PH,LDPH)
*
*         Form Q explicitly, storing the result in PH
*
*         CALL ZUNGLQ(N,N,M,PH,LDPH,TAU,WORK,LWORK,INFO)
*
*         Copy L to U (used as workspace)
*
*         CALL F06TFF('Lower',M,M,A,LDA,U,LDU)
*
*         Set the strictly upper triangular part of L to zero
*
*         CALL F06THF('Upper',M-1,M-1,ZERO,ZERO,U(1,2),LDU)
*
*         Bidiagonalize L

```

```

*
*      CALL ZGEBRD(M,M,U,LDU,D,E,TAUQ,TAUP,WORK,LWORK,INFO)
*
*      Update P**H, storing the result in PH
*
*      CALL ZUNMBR('P','Left','Conjugate transpose',M,N,M,U,LDU,
+      TAUP,PH,LDPH,WORK,LWORK,INFO)
*
*      Print bidiagonal form and matrix P**H
*
*      WRITE (NOUT,*)
*      WRITE (NOUT,*) 'Example 2: bidiagonal matrix B'
*      WRITE (NOUT,*) 'Diagonal'
*      WRITE (NOUT,99999) (D(I),I=1,M)
*      WRITE (NOUT,*) 'Super-diagonal'
*      WRITE (NOUT,99999) (E(I),I=1,M-1)
*      WRITE (NOUT,*)
*      IFAIL = 0
*
*      CALL X04DBF('General',' ',M,N,PH,LDPH,'Bracketed','F7.4',
+      'Example 2: matrix P**H','Integer',RLABS,
+      'Integer',CLABS,80,0,IFAIL)
*
*      END IF
*      END IF
*      20 CONTINUE
*
*      99999 FORMAT (3X,(8F8.4))
*      END

```

9.2 Program Data

F08KUF Example Program Data

```

6 4                                     :Values of M and N, Example 1
( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
(-0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26) :End of matrix A
3 4                                     :Values of M and N, Example 2
( 0.28,-0.36) ( 0.50,-0.86) (-0.77,-0.48) ( 1.58, 0.66)
(-0.50,-1.10) (-1.21, 0.76) (-0.32,-0.24) (-0.27,-1.15)
( 0.36,-0.51) (-0.07, 1.33) (-0.75, 0.47) (-0.08, 1.01) :End of matrix A

```

9.3 Program Results

F08KUF Example Program Results

Example 1: bidiagonal matrix B

Diagonal

-3.0870 -2.0660 -1.8731 -2.0022

Super-diagonal

2.1126 -1.2628 1.6126

Example 1: matrix Q

```

1 2 3 4
1 (-0.3110, 0.2624) ( 0.6521, 0.5532) ( 0.0427, 0.0361) (-0.2634,-0.0741)
2 ( 0.3175,-0.6414) ( 0.3488, 0.0721) ( 0.2287, 0.0069) ( 0.1101,-0.0326)
3 (-0.2008, 0.1490) (-0.3103, 0.0230) ( 0.1855,-0.1817) (-0.2956, 0.5648)
4 ( 0.1199,-0.1231) (-0.0046,-0.0005) (-0.3305, 0.4821) (-0.0675, 0.3464)
5 (-0.2689,-0.1652) ( 0.1794,-0.0586) (-0.5235,-0.2580) ( 0.3927, 0.1450)
6 (-0.3499, 0.0907) ( 0.0829,-0.0506) ( 0.3202, 0.3038) ( 0.3174, 0.3241)

```

Example 2: bidiagonal matrix B

Diagonal

2.7615 1.6298 -1.3275

Super-diagonal

-0.9500 -1.0183

Example 2: matrix P**H

	1	2	3	4
1	(-0.1258, 0.1618)	(-0.2247, 0.3864)	(0.3460, 0.2157)	(-0.7099,-0.2966)
2	(0.4148, 0.1795)	(0.1368,-0.3976)	(0.6885, 0.3386)	(0.1667,-0.0494)
3	(0.4575,-0.4807)	(-0.2733, 0.4981)	(-0.0230, 0.3861)	(0.1730, 0.2395)
