NAG Library Routine Document

F08GSF (ZHPTRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08GSF (ZHPTRD) reduces a complex Hermitian matrix to tridiagonal form, using packed storage.

2 Specification

```
SUBROUTINE F08GSF(UPLO, N, AP, D, E, TAU, INFO)
INTEGER                  N, INFO
double precision         D(N), E(N-1)
complex*16               AP(*), TAU(N-1)
CHARACTER*1              UPLO
```

The routine may be called by its LAPACK name ***zhptrd***.

3 Description

F08GSF (ZHPTRD) reduces a complex Hermitian matrix A, held in packed storage, to real symmetric tridiagonal form T by a unitary similarity transformation: $A = QTQ^{\mathrm{H}}$.

The matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors (see the F08 Chapter Introduction for details). Routines are provided to work with Q in this representation (see Section 8).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored.

UPLO = 'U'
The upper triangular part of A is stored.

UPLO = 'L'
The lower triangular part of A is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: AP$(*)$ – ***complex*16*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{N} \times (\mathrm{N}+1)/2)$.

On entry: the n by n Hermitian matrix A, packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in AP$(i+j(j-1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in AP$(i+(2n-j)(j-1)/2)$ for $i \geq j$.

On exit: AP is overwritten by the tridiagonal matrix T and details of the unitary matrix Q.

4: D(N) – ***double precision*** array *Output*

On exit: the diagonal elements of the tridiagonal matrix T.

5: E(N − 1) – ***double precision*** array *Output*

On exit: the off-diagonal elements of the tridiagonal matrix T.

6: TAU(N − 1) – ***complex*16*** array *Output*

On exit: further details of the unitary matrix Q.

7: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A+E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{16}{3}n^3$.

To form the unitary matrix Q F08GSF (ZHPTRD) may be followed by a call to F08GTF (ZUPGTR):

```
CALL ZUPGTR(UPLO,N,AP,TAU,Q,LDQ,WORK,INFO)
```

To apply Q to an n by p complex matrix C F08GSF (ZHPTRD) may be followed by a call to F08GUF (ZUPMTR). For example,

```
CALL ZUPMTR('Left',UPLO,'No Transpose',N,P,AP,TAU,C,LDC,WORK,
+            INFO)
```

forms the matrix product QC.

The real analogue of this routine is F08GEF (DSPTRD).

9 Example

This example reduces the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} -2.28+0.00i & 1.78-2.03i & 2.26+0.10i & -0.12+2.53i \\ 1.78+2.03i & -1.12+0.00i & 0.01+0.43i & -1.07+0.86i \\ 2.26-0.10i & 0.01-0.43i & -0.37+0.00i & 2.31-0.92i \\ -0.12-2.53i & -1.07-0.86i & 2.31+0.92i & -0.73+0.00i \end{pmatrix},$$

using packed storage.

9.1 Program Text

```
*     F08GSF Example Program Text
*     Mark 16 Release. NAG Copyright 1992.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*     .. Local Scalars ..
      INTEGER          I, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      COMPLEX *16      AP(NMAX*(NMAX+1)/2), TAU(NMAX-1)
      DOUBLE PRECISION D(NMAX), E(NMAX-1)
*     .. External Subroutines ..
      EXTERNAL         ZHPTRD
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F08GSF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
         END IF
*
*        Reduce A to tridiagonal form
*
         CALL ZHPTRD(UPLO,N,AP,D,E,TAU,INFO)
*
*        Print tridiagonal form
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'Diagonal'
         WRITE (NOUT,99999) (D(I),I=1,N)
         WRITE (NOUT,*) 'Off-diagonal'
         WRITE (NOUT,99999) (E(I),I=1,N-1)
      END IF
*
99999 FORMAT (1X,8F9.4)
      END
```

9.2 Program Data

```
F08GSF Example Program Data
  4                                                          :Value of N
  'L'                                                        :Value of UPLO
 (-2.28, 0.00)
 ( 1.78, 2.03) (-1.12, 0.00)
 ( 2.26,-0.10) ( 0.01,-0.43) (-0.37, 0.00)
 (-0.12,-2.53) (-1.07,-0.86) ( 2.31, 0.92) (-0.73, 0.00)   :End of matrix A
```

9.3 Program Results

```
 F08GSF Example Program Results

 Diagonal
   -2.2800  -0.1285  -0.1666  -1.9249
 Off-diagonal
   -4.3385  -2.0226  -1.8023
```