NAG Library Routine Document

F08GGF (DOPMTR)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08GGF (DOPMTR) multiplies an arbitrary real matrix C by the real orthogonal matrix Q which was determined by F08GEF (DSPTRD) when reducing a real symmetric matrix to tridiagonal form.

2 Specification

```
SUBROUTINE F08GGF(SIDE, UPLO, TRANS, M, N, AP, TAU, C, LDC, WORK, INFO)
INTEGER            M, N, LDC, INFO
double precision   AP(*), TAU(*), C(LDC,*), WORK(*)
CHARACTER*1        SIDE, UPLO, TRANS
```

The routine may be called by its LAPACK name ***dopmtr***.

3 Description

F08GGF (DOPMTR) is intended to be used after a call to F08GEF (DSPTRD), which reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = QTQ^{\mathrm{T}}$. F08GEF (DSPTRD) represents the orthogonal matrix Q as a product of elementary reflectors.

This routine may be used to form one of the matrix products

$$QC, Q^{\mathrm{T}}C, CQ \text{ or } CQ^{\mathrm{T}},$$

overwriting the result on C (which may be any real rectangular matrix).

A common application of this routine is to transform a matrix Z of eigenvectors of T to the matrix QZ of eigenvectors of A.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: SIDE – CHARACTER*1 *Input*

On entry: indicates how Q or Q^{T} is to be applied to C.

SIDE = 'L'
: Q or Q^{T} is applied to C from the left.

SIDE = 'R'
: Q or Q^{T} is applied to C from the right.

Constraint: SIDE = 'L' or 'R'.

2: UPLO – CHARACTER*1 *Input*

On entry: this **must** be the same parameter UPLO as supplied to F08GEF (DSPTRD).

Constraint: UPLO = 'U' or 'L'.

3: TRANS – CHARACTER*1 *Input*

On entry: indicates whether Q or Q^{T} is to be applied to C.

TRANS = 'N'
Q is applied to C.

TRANS = 'T'
Q^{T} is applied to C.

Constraint: TRANS = 'N' or 'T'.

4: M – INTEGER *Input*

On entry: m, the number of rows of the matrix C; m is also the order of Q if SIDE = 'L'.

Constraint: $\mathrm{M} \geq 0$.

5: N – INTEGER *Input*

On entry: n, the number of columns of the matrix C; n is also the order of Q if SIDE = 'R'.

Constraint: $\mathrm{N} \geq 0$.

6: AP($*$) – ***double precision*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{M} \times (\mathrm{M}+1)/2)$ if SIDE = 'L' and at least $\max(1, \mathrm{N} \times (\mathrm{N}+1)/2)$ if SIDE = 'R'.

On entry: details of the vectors which define the elementary reflectors, as returned by F08GEF (DSPTRD).

On exit: is used as internal workspace prior to being restored and hence is unchanged.

7: TAU($*$) – ***double precision*** array *Input*

Note: the dimension of the array TAU must be at least $\max(1, \mathrm{M}-1)$ if SIDE = 'L' and at least $\max(1, \mathrm{N}-1)$ if SIDE = 'R'.

On entry: further details of the elementary reflectors, as returned by F08GEF (DSPTRD).

8: C(LDC,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1, \mathrm{N})$.

On entry: the m by n matrix C.

On exit: C is overwritten by QC or $Q^{\mathrm{T}}C$ or CQ or CQ^{T} as specified by SIDE and TRANS.

9: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08GGF (DOPMTR) is called.

Constraint: $\mathrm{LDC} \geq \max(1, \mathrm{M})$.

10: WORK($*$) – ***double precision*** array *Workspace*

Note: the dimension of the array WORK must be at least $\max(1, \mathrm{N})$ if SIDE = 'L' and at least $\max(1, \mathrm{M})$ if SIDE = 'R'.

11: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed result differs from the exact result by a matrix E such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $2m^2n$ if SIDE = 'L' and $2mn^2$ if SIDE = 'R'.

The complex analogue of this routine is F08GUF (ZUPMTR).

9 Example

This example computes the two smallest eigenvalues, and the associated eigenvectors, of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix},$$

using packed storage. Here A is symmetric and must first be reduced to tridiagonal form T by F08GEF (DSPTRD). The program then calls F08JJF (DSTEBZ) to compute the requested eigenvalues and F08JKF (DSTEIN) to compute the associated eigenvectors of T. Finally F08GGF (DOPMTR) is called to transform the eigenvectors to those of A.

9.1 Program Text

```
*     F08GGF Example Program Text
*     Mark 16 Release. NAG Copyright 1992.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDC
      PARAMETER        (NMAX=8,LDC=NMAX)
      DOUBLE PRECISION ZERO
      PARAMETER        (ZERO=0.0D0)
*     .. Local Scalars ..
      DOUBLE PRECISION VL, VU
      INTEGER          I, IFAIL, INFO, J, M, N, NSPLIT
      CHARACTER        UPLO
*     .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2), C(LDC,NMAX), D(NMAX),
     +                 E(NMAX), TAU(NMAX), W(NMAX), WORK(5*NMAX)
      INTEGER          IBLOCK(NMAX), IFAILV(NMAX), ISPLIT(NMAX),
     +                 IWORK(3*NMAX)
*     .. External Subroutines ..
      EXTERNAL         DOPMTR, DSPTRD, DSTEBZ, DSTEIN, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F08GGF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
```

```
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
         END IF
*
*        Reduce A to tridiagonal form T = (Q**T)*A*Q
*
         CALL DSPTRD(UPLO,N,AP,D,E,TAU,INFO)
*
*        Calculate the two smallest eigenvalues of T (same as A)
*
         CALL DSTEBZ('I','B',N,VL,VU,1,2,ZERO,D,E,M,NSPLIT,W,IBLOCK,
     +               ISPLIT,WORK,IWORK,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.GT.0) THEN
            WRITE (NOUT,*) 'Failure to converge.'
         ELSE
            WRITE (NOUT,*) 'Eigenvalues'
            WRITE (NOUT,99999) (W(I),I=1,M)
*
*           Calculate the eigenvectors of T, storing the result in C
*
            CALL DSTEIN(N,D,E,M,W,IBLOCK,ISPLIT,C,LDC,WORK,IWORK,IFAILV,
     +                  INFO)
*
            IF (INFO.GT.0) THEN
               WRITE (NOUT,*) 'Failure to converge.'
            ELSE
*
*              Calculate the eigenvectors of A = Q * (eigenvectors of T)
*
               CALL DOPMTR('Left',UPLO,'No transpose',N,M,AP,TAU,C,LDC,
     +                     WORK,INFO)
*
*              Print eigenvectors
*
               WRITE (NOUT,*)
               IFAIL = 0
*
               CALL X04CAF('General',' ',N,M,C,LDC,'Eigenvectors',IFAIL)
*
            END IF
         END IF
      END IF
*
99999 FORMAT (3X,(9F8.4))
      END
```

9.2 Program Data

```
F08GGF Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

9.3 Program Results

```
 F08GGF Example Program Results

 Eigenvalues
    -5.0034 -1.9987

 Eigenvectors
          1       2
 1   0.5658 -0.2328
 2  -0.3478  0.7994
 3  -0.4740 -0.4087
 4   0.5781  0.3737
```