

NAG Library Routine Document

F07VGF (DTBCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07VGF (DTBCON) estimates the condition number of a real triangular band matrix.

2 Specification

```

SUBROUTINE F07VGF(NORM, UPLO, DIAG, N, KD, AB, LDAB, RCOND, WORK, IWORK,
1                      INFO)
    INTEGER          N, KD, LDAB, IWORK(N), INFO
    double precision AB(LDAB,*), RCOND, WORK(3*N)
    CHARACTER*1      NORM, UPLO, DIAG

```

The routine may be called by its LAPACK name ***dtbcon***.

3 Description

F07VGF (DTBCON) estimates the condition number of a real triangular band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

- 1: NORM – CHARACTER*1 *Input*
On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.
 NORM = '1' or 'O'
 $\kappa_1(A)$ is estimated.
 NORM = 'I'
 $\kappa_\infty(A)$ is estimated.
Constraint: NORM = '1', 'O' or 'I'.
- 2: UPLO – CHARACTER*1 *Input*
On entry: indicates whether A is upper or lower triangular.
 UPLO = 'U'
 A is upper triangular.

- UPLO = 'L'
 A is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 3: DIAG – CHARACTER*1 *Input*
On entry: indicates whether A is a nonunit or unit triangular matrix.
 DIAG = 'N'
 A is a nonunit triangular matrix.
 DIAG = 'U'
 A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: DIAG = 'N' or 'U'.
- 4: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 5: KD – INTEGER *Input*
On entry: k_d , the number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.
Constraint: $KD \geq 0$.
- 6: AB(LDAB,*) – **double precision** array *Input*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the n by n triangular band matrix A .
 The matrix is stored in rows 1 to $k_d + 1$, more precisely,
 if UPLO = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $AB(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;
 if UPLO = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $AB(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.
 If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.
- 7: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07VGF (DTBCON) is called.
Constraint: $LDAB \geq KD + 1$.
- 8: RCOND – **double precision** *Output*
On exit: an estimate of the reciprocal of the condition number of A . RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than **machine precision**, A is singular to working precision.
- 9: WORK($3 \times N$) – **double precision** array *Workspace*
- 10: IWORK(N) – INTEGER array *Workspace*
- 11: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07VGF (DTBCON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to F07VEF (DTBTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07VUF (ZTBCON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal. The true condition number in the 1-norm is 69.62.

9.1 Program Text

```
*      F07VGF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KDMAX, LDAB
      PARAMETER        (NMAX=8,KDMAX=NMAX,LDAB=KDMAX+1)
      CHARACTER        NORM, DIAG
      PARAMETER        (NORM='1',DIAG='N')
*      .. Local Scalars ..
      DOUBLE PRECISION RCOND
      INTEGER          I, INFO, J, KD, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION AB(LDAB,NMAX), WORK(3*NMAX)
      INTEGER          IWORK(NMAX)
*      .. External Functions ..
      DOUBLE PRECISION X02AJF
      EXTERNAL         X02AJF
*      .. External Subroutines ..
      EXTERNAL         DTBCON
*      .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07VGF Example Program Results'
*      Skip heading in data file
```

```

      READ (NIN,*)
      READ (NIN,*) N, KD
      IF (N.LE.NMAX .AND. KD.LE.KDMAX) THEN
*
*       Read A from data file
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
        DO 20 I = 1, N
          READ (NIN,*) (AB(KD+1+I-J,J),J=I,MIN(N,I+KD))
20      CONTINUE
      ELSE IF (UPLO.EQ.'L') THEN
        DO 40 I = 1, N
          READ (NIN,*) (AB(1+I-J,J),J=MAX(1,I-KD),I)
40      CONTINUE
      END IF
*
*       Estimate condition number
*
      CALL DTBCON(NORM,UPLO,DIAG,N,KD,AB,LDAB,RCOND,WORK,IWORK,INFO)
*
      WRITE (NOUT,*)
      IF (RCOND.GE.X02AJF()) THEN
        WRITE (NOUT,99999) 'Estimate of condition number =',
+       1.0D0/RCOND
      ELSE
        WRITE (NOUT,*) 'A is singular to working precision'
      END IF
      END IF
*
99999 FORMAT (1X,A,1P,E10.2)
      END

```

9.2 Program Data

F07VGF Example Program Data

4	1				:Values of N and KD
'L'					:Value of UPLO
-4.16					
-2.25	4.78				
	5.86	6.32			
	-4.82	0.16			:End of matrix A

9.3 Program Results

F07VGF Example Program Results

Estimate of condition number = 6.96E+01
