NAG Library Routine Document

F07UEF (DTPTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07UEF (DTPTRS) solves a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^{\mathrm{T}}X = B$, using packed storage.

2 Specification

```
SUBROUTINE F07UEF(UPLO, TRANS, DIAG, N, NRHS, AP, B, LDB, INFO)
INTEGER           N, NRHS, LDB, INFO
double precision  AP(*), B(LDB,*)
CHARACTER*1       UPLO, TRANS, DIAG
```

The routine may be called by its LAPACK name ***dtptrs***.

3 Description

F07UEF (DTPTRS) solves a real triangular system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, using packed storage.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether A is upper or lower triangular.

UPLO = 'U'
A is upper triangular.

UPLO = 'L'
A is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: TRANS – CHARACTER*1 *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
The equations are of the form $AX = B$.

TRANS = 'T' or 'C'
The equations are of the form $A^{\mathrm{T}}X = B$.

Constraint: TRANS = 'N', 'T' or 'C'.

3: DIAG – CHARACTER*1 *Input*

On entry: indicates whether A is a nonunit or unit triangular matrix.

DIAG = 'N'
A is a nonunit triangular matrix.

DIAG = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: DIAG = 'N' or 'U'.

4: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

5: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathrm{NRHS} \geq 0$.

6: AP$(*)$ – ***double precision*** array *Input*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{N} \times (\mathrm{N}+1)/2)$.

On entry: the n by n triangular matrix A, packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathrm{AP}(i+j(j-1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathrm{AP}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether DIAG = 'N' or 'U'.

7: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07UEF (DTPTRS) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

9: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $a(i,i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon, \qquad \text{provided} \qquad c(n)\,\mathrm{cond}(A,x)\epsilon < 1,$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A,x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07UHF (DTPRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07UGF (DTPCON) with NORM = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $n^2 r$.

The complex analogue of this routine is F07USF (ZTPTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -12.90 & -21.50 \\ 16.75 & 14.93 \\ -17.55 & 6.33 \\ -11.04 & 8.09 \end{pmatrix},$$

using packed storage for A.

9.1 Program Text

```
*     F07UEF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NRHMAX, LDB
      PARAMETER        (NMAX=8,NRHMAX=NMAX,LDB=NMAX)
      CHARACTER        TRANS, DIAG
      PARAMETER        (TRANS='N',DIAG='N')
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
      CHARACTER        UPLO
*     .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2), B(LDB,NRHMAX)
*     .. External Subroutines ..
      EXTERNAL         DTPTRS, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07UEF Example Program Results'
```

```
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*        Read A and B from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
         END IF
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Compute solution
*
         CALL DTPTRS(UPLO,TRANS,DIAG,N,NRHS,AP,B,LDB,INFO)
*
*        Print solution
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
            IFAIL = 0
            CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
         ELSE
            WRITE (NOUT,*) 'A is singular'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07UEF Example Program Data
  4  2                        :Values of N and NRHS
  'L'                         :Value of UPLO
  4.30
 -3.96  -4.87
  0.40   0.31  -8.02
 -0.27   0.07  -5.95   0.12   :End of matrix A
-12.90 -21.50
 16.75  14.93
-17.55   6.33
-11.04   8.09                 :End of matrix B
```

9.3 Program Results

```
 F07UEF Example Program Results

 Solution(s)
                1          2
 1     -3.0000    -5.0000
 2     -1.0000     1.0000
 3      2.0000    -1.0000
 4      1.0000     6.0000
```