

NAG Library Routine Document

F07THF (DTRRFS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07THF (DTRRFS) returns error bounds for the solution of a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^T X = B$.

2 Specification

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SUBROUTINE F07THF(UPLO, TRANS, DIAG, N, NRHS, A, LDA, B, LDB, X, LDX,
1              FERR, BERR, WORK, IWORK, INFO)
    INTEGER      N, NRHS, LDA, LDB, LDX, IWORK(N), INFO
    double precision A(LDA,*), B(LDB,*), X(LDX,*), FERR(NRHS), BERR(NRHS),
1              WORK(3*N)
    CHARACTER*1   UPLO, TRANS, DIAG

```

The routine may be called by its LAPACK name ***dtrrfs***.

3 Description

F07THF (DTRRFS) returns the backward errors and estimated bounds on the forward errors for the solution of a real triangular system of linear equations with multiple right-hand sides $AX = B$ or $A^T X = B$. The routine handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of F07THF (DTRRFS) in terms of a single right-hand side b and solution x .

Given a computed solution x , the routine computes the *component-wise backward error* β . This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$| \delta a_{ij} | \leq \beta | a_{ij} | \quad \text{and} \quad | \delta b_i | \leq \beta | b_i |.$$

Then the routine estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the F07 Chapter Introduction.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*
On entry: indicates whether A is upper or lower triangular.
UPLO = 'U'
 A is upper triangular.

- UPLO = 'L'
 A is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: TRANS – CHARACTER*1 *Input*
On entry: indicates the form of the equations.
 TRANS = 'N'
 The equations are of the form $AX = B$.
 TRANS = 'T' or 'C'
 The equations are of the form $A^T X = B$.
Constraint: TRANS = 'N', 'T' or 'C'.
- 3: DIAG – CHARACTER*1 *Input*
On entry: indicates whether A is a nonunit or unit triangular matrix.
 DIAG = 'N'
 A is a nonunit triangular matrix.
 DIAG = 'U'
 A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: DIAG = 'N' or 'U'.
- 4: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 5: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: NRHS ≥ 0 .
- 6: A(LDA,*) – **double precision** array *Input*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the n by n triangular matrix A .
 If UPLO = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.
 If UPLO = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.
 If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.
- 7: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F07THF (DTRRFS) is called.
Constraint: LDA $\geq \max(1, N)$.
- 8: B(LDB,*) – **double precision** array *Input*
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
On entry: the n by r right-hand side matrix B .

- 9: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07THF (DTRRFS) is called.
Constraint: $LDB \geq \max(1, N)$.
- 10: X(LDX,*) – **double precision** array *Input*
Note: the second dimension of the array X must be at least $\max(1, NRHS)$.
On entry: the n by r solution matrix X , as returned by F07TEF (DTRTRS).
- 11: LDX – INTEGER *Input*
On entry: the first dimension of the array X as declared in the (sub)program from which F07THF (DTRRFS) is called.
Constraint: $LDX \geq \max(1, N)$.
- 12: FERR(NRHS) – **double precision** array *Output*
On exit: FERR(j) contains an estimated error bound for the j th solution vector, that is, the j th column of X , for $j = 1, 2, \dots, r$.
- 13: BERR(NRHS) – **double precision** array *Output*
On exit: BERR(j) contains the component-wise backward error bound β for the j th solution vector, that is, the j th column of X , for $j = 1, 2, \dots, r$.
- 14: WORK($3 \times N$) – **double precision** array *Workspace*
- 15: IWORK(N) – INTEGER array *Workspace*
- 16: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The bounds returned in FERR are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

A call to F07THF (DTRRFS), for each right-hand side, involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating-point operations.

The complex analogue of this routine is F07TVF (ZTRRFS).

9 Example

This example solves the system of equations $AX = B$ and to compute forward and backward error bounds, where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -12.90 & -21.50 \\ 16.75 & 14.93 \\ -17.55 & 6.33 \\ -11.04 & 8.09 \end{pmatrix}.$$

9.1 Program Text

```
*      F07THF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NRHMAX, LDA, LDB, LDX
      PARAMETER        (NMAX=8,NRHMAX=NMAX,LDA=NMAX,LDB=NMAX,LDX=NMAX)
      CHARACTER        TRANS, DIAG
      PARAMETER        (TRANS='N',DIAG='N')
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), B(LDB,NRHMAX), BERR(NRHMAX),
+      FERR(NRHMAX), WORK(3*NMAX), X(LDX,NMAX)
      INTEGER          IWORK(NMAX)
*      .. External Subroutines ..
      EXTERNAL         DTRRFS, DTRTRS, F06QFF, X04CAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07THF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*         Read A and B from data file, and copy B to X
*
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
         READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
      ELSE IF (UPLO.EQ.'L') THEN
         READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
      END IF
      READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
      CALL F06QFF('General',N,NRHS,B,LDB,X,LDX)
*
*      Compute solution in the array X
*
      CALL DTRTRS(UPLO,TRANS,DIAG,N,NRHS,A,LDA,X,LDX,INFO)
*
*      Compute backward errors and estimated bounds on the
*      forward errors
*
      CALL DTRRFS(UPLO,TRANS,DIAG,N,NRHS,A,LDA,B,LDB,X,LDX,FERR,BERR,
+      WORK,IWORK,INFO)
*
*      Print solution
*
      WRITE (NOUT,*)
      IFAIL = 0
      CALL X04CAF('General',' ',N,NRHS,X,LDX,'Solution(s)',IFAIL)
      WRITE (NOUT,*)
      WRITE (NOUT,*) 'Backward errors (machine-dependent)'
      WRITE (NOUT,99999) (BERR(J),J=1,NRHS)
      WRITE (NOUT,*)
+      'Estimated forward error bounds (machine-dependent)'
```

```

      WRITE (NOUT,99999) (FERR(J),J=1,NRHS)
    END IF
*
99999 FORMAT ((3X,1P,7E11.1))
END

```

9.2 Program Data

F07THF Example Program Data

```

  4  2      :Values of N and NRHS
  'L'      :Value of UPLO
  4.30
 -3.96 -4.87
  0.40  0.31 -8.02
 -0.27  0.07 -5.95  0.12 :End of matrix A
-12.90 -21.50
 16.75 14.93
-17.55  6.33
-11.04  8.09      :End of matrix B

```

9.3 Program Results

F07THF Example Program Results

Solution(s)

	1	2
1	-3.0000	-5.0000
2	-1.0000	1.0000
3	2.0000	-1.0000
4	1.0000	6.0000

Backward errors (machine-dependent)

6.9E-17	0.0E+00
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Estimated forward error bounds (machine-dependent)

8.3E-14	2.6E-14
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