

NAG Library Routine Document

F07TGF (DTRCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07TGF (DTRCON) estimates the condition number of a real triangular matrix.

2 Specification

```
SUBROUTINE F07TGF(NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, IWORK, INFO)
INTEGER          N, LDA, IWORK(N), INFO
double precision A(LDA,*), RCOND, WORK(3*N)
CHARACTER*1      NORM, UPLO, DIAG
```

The routine may be called by its LAPACK name ***dtrcon***.

3 Description

F07TGF (DTRCON) estimates the condition number of a real triangular matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: NORM – CHARACTER*1 *Input*

On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

NORM = '1' or 'O'

$\kappa_1(A)$ is estimated.

NORM = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: NORM = '1', 'O' or 'I'.

2: UPLO – CHARACTER*1 *Input*

On entry: indicates whether A is upper or lower triangular.

UPLO = 'U'

A is upper triangular.

- UPLO = 'L'
A is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 3: DIAG – CHARACTER*1 *Input*
On entry: indicates whether *A* is a nonunit or unit triangular matrix.
 DIAG = 'N'
A is a nonunit triangular matrix.
 DIAG = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: DIAG = 'N' or 'U'.
- 4: N – INTEGER *Input*
On entry: *n*, the order of the matrix *A*.
Constraint: $N \geq 0$.
- 5: A(LDA,*) – **double precision** array *Input*
Note: the second dimension of the array *A* must be at least $\max(1, N)$.
On entry: the *n* by *n* triangular matrix *A*.
 If UPLO = 'U', *A* is upper triangular and the elements of the array below the diagonal are not referenced.
 If UPLO = 'L', *A* is lower triangular and the elements of the array above the diagonal are not referenced.
 If DIAG = 'U', the diagonal elements of *A* are assumed to be 1, and are not referenced.
- 6: LDA – INTEGER *Input*
On entry: the first dimension of the array *A* as declared in the (sub)program from which F07TGF (DTRCON) is called.
Constraint: $LDA \geq \max(1, N)$.
- 7: RCOND – **double precision** *Output*
On exit: an estimate of the reciprocal of the condition number of *A*. RCOND is set to zero if exact singularity is detected or if the estimate underflows. If RCOND is less than **machine precision**, then *A* is singular to working precision.
- 8: WORK($3 \times N$) – **double precision** array *Workspace*
- 9: IWORK(N) – INTEGER array *Workspace*
- 10: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the *i*th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07TGF (DTRCON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating-point operations but takes considerably longer than a call to F07TEF (DTRTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07TUF (ZTRCON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix}.$$

The true condition number in the 1-norm is 116.41.

9.1 Program Text

```
*      F07TGF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA
      PARAMETER        (NMAX=8,LDA=NMAX)
      CHARACTER        NORM, DIAG
      PARAMETER        (NORM='1',DIAG='N')
*      .. Local Scalars ..
      DOUBLE PRECISION RCOND
      INTEGER          I, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), WORK(3*NMAX)
      INTEGER          IWORK(NMAX)
*      .. External Functions ..
      DOUBLE PRECISION X02AJF
      EXTERNAL         X02AJF
*      .. External Subroutines ..
      EXTERNAL         DTRCON
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07TGF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A from data file
*
*          READ (NIN,*) UPLO
*          IF (UPLO.EQ.'U') THEN
*              READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
*          ELSE IF (UPLO.EQ.'L') THEN
*              READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
*          END IF
*
*          Estimate condition number
*
```

```

      CALL DTRCON(NORM,UPLO,DIAG,N,A,LDA,RCOND,WORK,IWORK,INFO)
*
      WRITE (NOUT,*)
      IF (RCOND.GE.X02AJF()) THEN
        WRITE (NOUT,99999) 'Estimate of condition number =',
+       1.0D0/RCOND
      ELSE
        WRITE (NOUT,*) 'A is singular to working precision'
      END IF
    END IF
*
99999 FORMAT (1X,A,1P,E10.2)
      END

```

9.2 Program Data

F07TGF Example Program Data

```

4                               :Value of N
'L'                             :Value of UPLO
4.30
-3.96  -4.87
0.40   0.31  -8.02
-0.27   0.07  -5.95   0.12   :End of matrix A

```

9.3 Program Results

F07TGF Example Program Results

Estimate of condition number = 1.16E+02
