

NAG Library Routine Document

F07QUF (ZSPCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07QUF (ZSPCON) estimates the condition number of a complex symmetric matrix A , where A has been factorized by F07QRF (ZSPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07QUF(UPLO, N, AP, IPIV, ANORM, RCOND, WORK, INFO)
INTEGER          N, IPIV(*), INFO
double precision ANORM, RCOND
complex*16      AP(*), WORK(2*N)
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***zspcon***.

3 Description

F07QUF (ZSPCON) estimates the condition number (in the 1-norm) of a complex symmetric matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The routine should be preceded by a call to F06UGF to compute $\|A\|_1$ and a call to F07QRF (ZSPTRF) to compute the Bunch–Kaufman factorization of A . The routine then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

- 1: UPLO – CHARACTER*1 *Input*
On entry: indicates how A has been factorized.
 UPLO = 'U'
 $A = PUDU^T P^T$, where U is upper triangular.
 UPLO = 'L'
 $A = PLDL^T P^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.

- 2: N – INTEGER Input
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: AP(*) – **complex*16** array Input
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the factorization of A stored in packed form, as returned by F07QRF (ZSPTRF).
- 4: IPIV(*) – INTEGER array Input
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: details of the interchanges and the block structure of D , as returned by F07QRF (ZSPTRF).
- 5: ANORM – **double precision** Input
On entry: the 1-norm of the **original** matrix A , which may be computed by calling F06UGF with its parameter NORM = '1'. ANORM must be computed either **before** calling F07QRF (ZSPTRF) or else from a **copy** of the original matrix A .
Constraint: ANORM ≥ 0.0 .
- 6: RCOND – **double precision** Output
On exit: an estimate of the reciprocal of the condition number of A . RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than **machine precision**, A is singular to working precision.
- 7: WORK($2 \times N$) – **complex*16** array Workspace
- 8: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07QUF (ZSPCON) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to F07QSF (ZSPTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this routine is F07PGF (DSPCON).

9 Example

This example estimates the condition number in the 1-norm (or ∞ -norm) of the matrix A , where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}.$$

Here A is symmetric, stored in packed form, and must first be factorized by F07QRF (ZSPTRF). The true condition number in the 1-norm is 32.92.

9.1 Program Text

```
*      F07QUF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      DOUBLE PRECISION ANORM, RCOND
      INTEGER          I, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      COMPLEX *16      AP(NMAX*(NMAX+1)/2), WORK(2*NMAX)
      DOUBLE PRECISION RWORK(NMAX)
      INTEGER          IPIV(NMAX)
*      .. External Functions ..
      DOUBLE PRECISION F06UGF, X02AJF
      EXTERNAL          F06UGF, X02AJF
*      .. External Subroutines ..
      EXTERNAL          ZSPCON, ZSPTRF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07QUF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A from data file
*
*          READ (NIN,*) UPLO
*          IF (UPLO.EQ.'U') THEN
*              READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
*          ELSE IF (UPLO.EQ.'L') THEN
*              READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
*          END IF
*
*          Compute norm of A
*
*          ANORM = F06UGF('1-norm',UPLO,N,AP,RWORK)
*
*          Factorize A
*
*          CALL ZSPTRF(UPLO,N,AP,IPIV,INFO)
*
*          WRITE (NOUT,*)
*          IF (INFO.EQ.0) THEN
*
*              Estimate condition number
*
*              CALL ZSPCON(UPLO,N,AP,IPIV,ANORM,RCOND,WORK,INFO)
*
*              IF (RCOND.GE.X02AJF()) THEN
*                  WRITE (NOUT,99999) 'Estimate of condition number =',
+                  1.0D0/RCOND
*              ELSE
```

```

        WRITE (NOUT,*) 'A is singular to working precision'
      END IF
    ELSE
      WRITE (NOUT,*) 'The factor D is singular'
    END IF
  END IF
*
99999 FORMAT (1X,A,1P,E10.2)
END

```

9.2 Program Data

F07QUF Example Program Data

4	:Value of N
'L'	:Value of UPLO
(-0.39,-0.71)	
(5.14,-0.64) (8.86, 1.81)	
(-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)	
(3.80, 0.92) (5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12)	:End of matrix A

9.3 Program Results

F07QUF Example Program Results

Estimate of condition number = 2.06E+01
